# Theory of Machine

- V- thread
- Friction Clutches
- Single Disc or Plate Clutch
- Multiple Disc Clutches

## Friction of a V-thread

We have seen Art. 10.18 that the normal reaction in case of a square threaded screw is

$$R_N = W \cos \alpha, \text{ where } \alpha = \text{Helix angle.}$$

But in case of V-thread (or acme or trapezoidal threads), the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load $W$, as shown in Fig. 10.14.

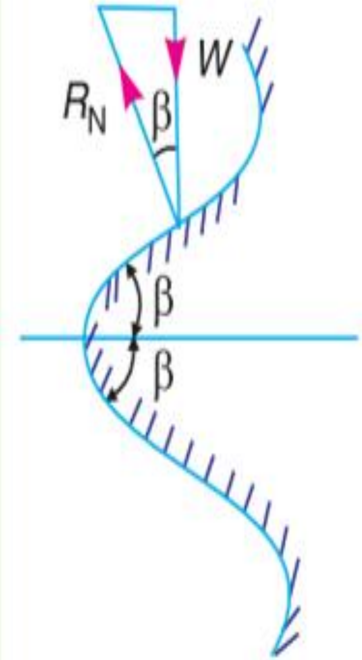


**Fig. 10.14.** V-thread.

Let $2\beta$ = Angle of the V-thread, and

$\beta$ = Semi-angle of the V-thread.

$$\therefore \quad R_N = \frac{W}{\cos \beta}$$

and frictional force, $F = \mu.R_N = \mu \times \frac{W}{\cos \beta} = \mu_1.W$

where $\frac{\mu}{\cos \beta} = \mu_1$, known as virtual coefficient of friction.

**Notes : 1.** When coefficient of friction, $\mu_1 = \dfrac{\mu}{\cos\beta}$ is considered, then the V-thread is equivalent to a square thread.

**2.** All the equations of square threaded screw also hold good for V-threads. In case of V-threads, $\mu_1$ (*i.e.* $\tan\phi_1$) may be substituted in place of $\mu$ (*i.e.* $\tan\phi$). Thus for V-threads,

$$P = W\tan(\alpha \pm \phi_1)$$

where $\phi_1$ = Virtual friction angle, such that $\tan\phi_1 = \mu_1$.

**Example 10.13.** *Two co-axial rods are connected by a turn buckle which consists of a box nut, the one screw being right handed and the other left handed on a pitch diameter of 22 mm, the pitch of thread being 3 mm. The included angle of the thread is 60°. Assuming that the rods do not turn, calculate the torque required on the nut to produce a pull of 40 kN, given that the coefficient of friction is 0.15.*

**Solution.** Given : $d = 22$ mm ; $p = 3$ mm ; $2\beta = 60°$ or $\beta = 30°$, $W = 40$ kN $= 40 \times 10^3$ N ; $\mu = 0.15$

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{3}{\pi \times 22} = 0.0434$

and virtual coefficient of friction

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.15}{\cos 30°} = 0.173$$

We know that the force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha.\tan\phi_1}\right]$$

$$= 40 \times 10^3\left[\frac{0.0434 + 0.173}{1 - 0.0434 \times 0.173}\right] = 8720 \text{ N}$$

and torque on one rod, $T = P \times d/2 = 8720 \times 22/2 = 95\,920$ N-mm $= 95.92$ N-m

Since the turn buckle has right and left hand threads and the torque on each rod is $T = 95.92$ N-m, therefore the torque required on the nut,

$$T_1 = 2T = 2 \times 95.92 = 191.84 \text{ N-m}$$ **Ans.**

## Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that

**1.** The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.

**2.** The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.

**3.** The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

The friction clutches of the following types are important from the subject point of view :

**1.** Disc or plate clutches (single disc or multiple disc clutch),

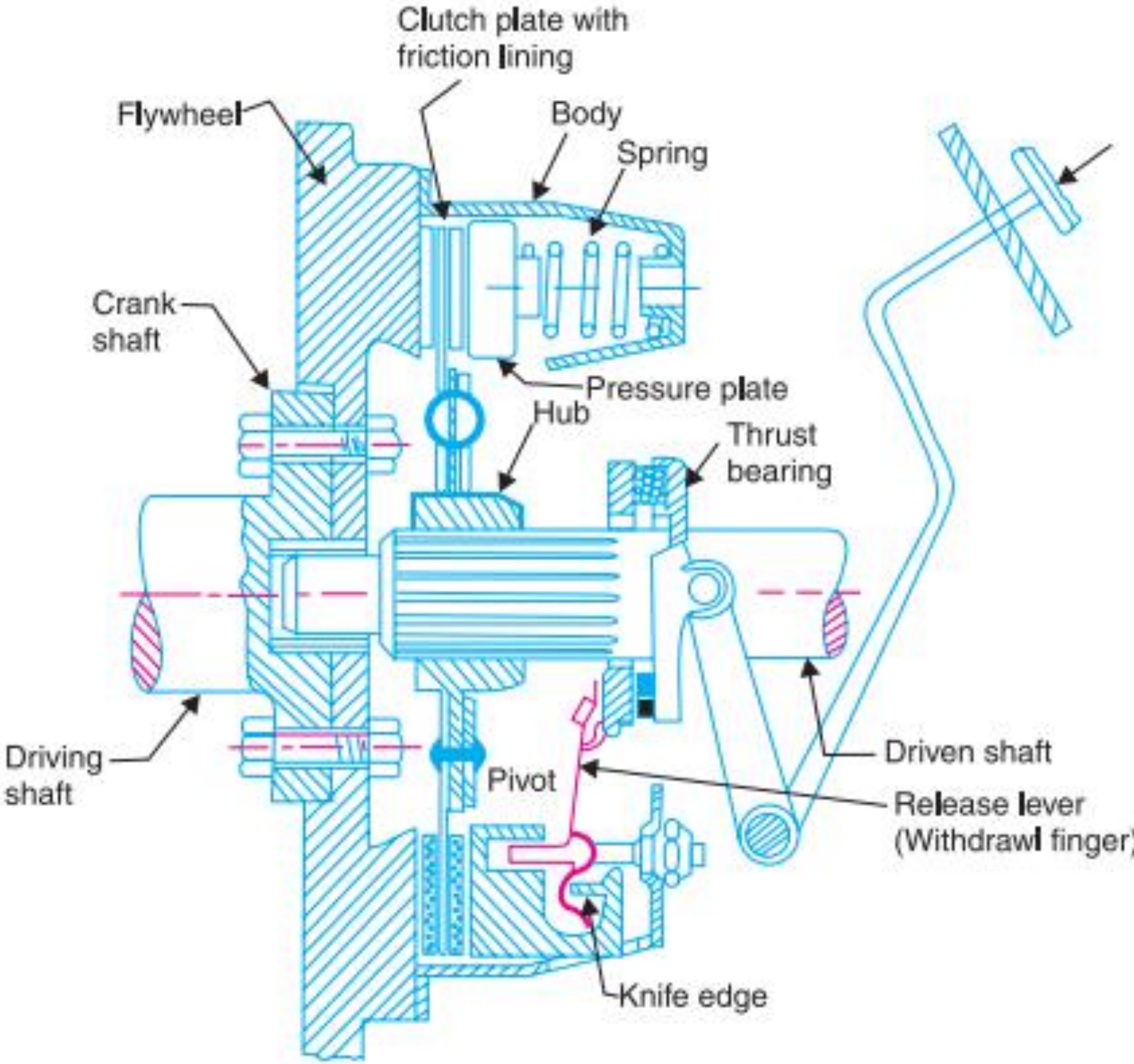
**2.** Cone clutches, and

**3.** Centrifugal clutches.

We shall now discuss, these clutches, in detail, in the following pages. It may be noted that the disc and cone clutches are based on the same theory as the pivot and collar bearings.

## Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine



Clutch plate with
friction lining
Flywheel
Body
Spring
Crank
shaft
Pressure plate
Hub
Thrust
bearing
Driving
shaft
Pivot
Driven shaft
Release lever
(Withdrawl finger)
Knife edge

Let $T$ = Torque transmitted by the clutch,

$p$ = Intensity of axial pressure with which the contact surfaces are held together,

$r_1$ and $r_2$ = External and internal radii of friction faces, and

$\mu$ = Coefficient of friction.

Consider an elementary ring of radius $r$ and thickness $dr$ as shown in Fig. 10.22 (*b*).

We know that area of contact surface or friction surface,

$$= 2\,\pi\, r.dr$$

∴ Normal or axial force on the ring,

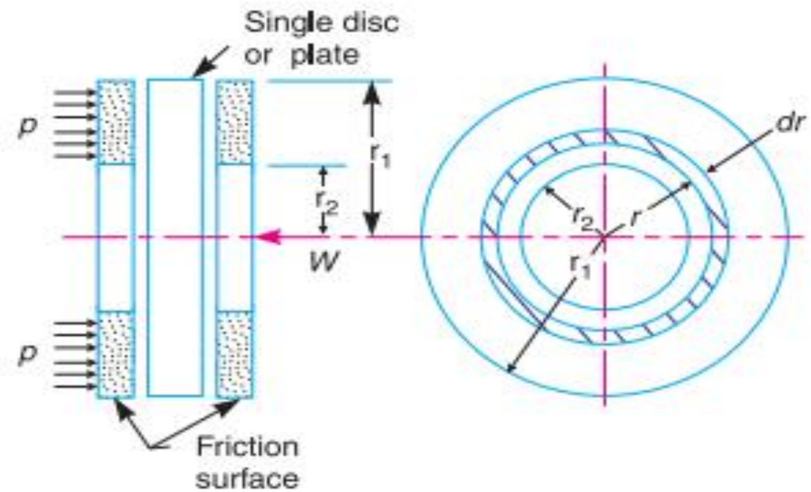
$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\,\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius $r$,

$$F_r = \mu.\delta W = \mu.p \times 2\,\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\,\pi\, r.dr \times r = 2\,\pi \times \mu\,.p.r^2\, dr$$




(*a*) (*b*)

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

## 1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \text{...}(i)$$

where $W$ = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius $r$ and thickness $dr$ is

$$T_r = 2\,\pi\,\mu.p.r^2\,dr$$

Integrating this equation within the limits from $r_2$ to $r_1$ for the total frictional torque.

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\,\pi\mu\,p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\,p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of $p$ from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where $R$ = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

## 2. *Considering uniform wear*

In Fig. 10.22, let $p$ be the normal intensity of pressure at a distance $r$ from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant)} \quad \text{or} \quad p = C/r \qquad \ldots(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\,r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

$$\ldots(\because\ p = C/r)$$

$\therefore$ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

$$= \frac{1}{2} \times \mu.W(r_1 + r_2) = \mu.W.R$$

where $R$ = Mean radius of the friction surface $= \frac{r_1 + r_2}{2}$

**Notes : 1.** In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$T = n.\mu.W.R$$

where $n$ = Number of pairs of friction or contact surfaces, and

$R$ = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$

**2.** For a single disc or plate clutch, normally both sides of the disc are effective. Therefore, a single disc clutch has two pairs of surfaces in contact, *i.e.* $n = 2$.

**3.** Since the intensity of pressure is maximum at the inner radius ($r_2$) of the friction or contact surface, therefore equation *(i)* may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C/r_2$$

**4.** Since the intensity of pressure is minimum at the outer radius ($r_1$) of the friction or contact surface, therefore equation *(i)* may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C/r_1$$

**5.** The average pressure ($p_{av}$) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

**6.** In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate.

**7.** The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

## Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 10.23, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion

Dual Disc Clutches.

(except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let $n_1$ = Number of discs on the driving shaft, and

$n_2$ = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

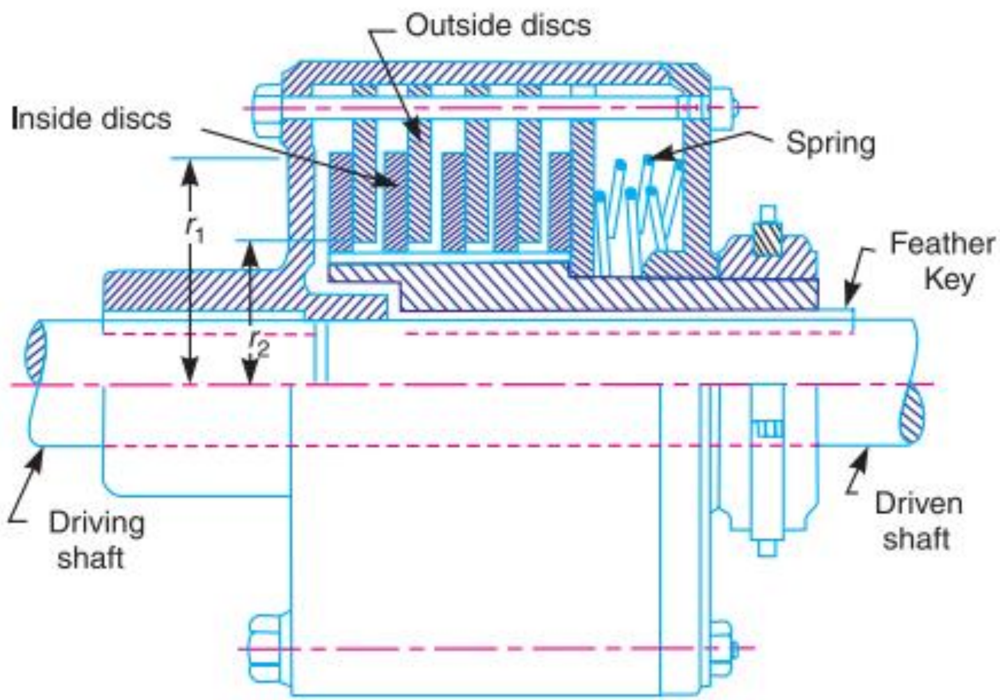
and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where $R$ = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$



**Example 10.22.** *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

**Solution.** Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

***Maximum pressure***

Let $p_{max}$ = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius ($r_2$), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface ($W$),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

$$\therefore \quad p_{max} = 4 \times 10^3/15\,710 = 0.2546 \text{ N/mm}^2$$ **Ans.**

***Minimum pressure***

Let $p_{min}$ = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius ($r_1$), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface ($W$),

$$4 \times 10^3 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times 100\,p_{min}\,(100 - 50) = 31\,420\,p_{min}$$

$$\therefore \quad p_{min} = 4 \times 10^3/31\,420 = 0.1273 \text{ N/mm}^2$$ **Ans.**

***Average pressure***

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$ **Ans.**

**Example 10.23.** *A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm². If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.*

**Solution.** Given : $d_1 = 300$ mm or $r_1 = 150$ mm ; $d_2 = 200$ mm or $r_2 = 100$ mm ; $p = 0.1$ N/mm$^2$ ; $\mu = 0.3$ ; $N = 2500$ r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure ($p$) is maximum at the inner radius ($r_2$), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 10 (150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

...($\because n = 2$, for both sides of plate effective)

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW}$$ **Ans.**

**_Thank You_**

**_For Your_**

**_Attention_**