10.24. Friction of a V-thread

We have seen Art. 10.18 that the normal reaction in case of a square threaded screw is

$$R_N = W \cos \alpha, \text{ where } \alpha = \text{Helix angle.}$$

But in case of V-thread (or acme or trapezoidal threads), the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load W, as shown in Fig. 10.14.

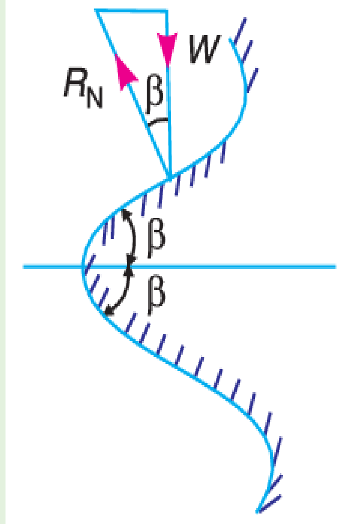


Fig. 10.14. V-thread.

Let 2β = Angle of the V-thread, and

β = Semi-angle of the V-thread.

$$\therefore \quad R_N = \frac{W}{\cos \beta}$$

and frictional force, $F = \mu . R_N = \mu \times \frac{W}{\cos \beta} = \mu_1 . W$

where $\frac{\mu}{\cos \beta} = \mu_1$, known as virtual coefficient of friction.

Notes : 1. When coefficient of friction, $\mu_1 = \dfrac{\mu}{\cos\beta}$ is considered, then the V-thread is equivalent to a square thread.

2. All the equations of square threaded screw also hold good for V-threads. In case of V-threads, μ_1 (*i.e.* $\tan\phi_1$) may be substituted in place of μ (*i.e.* $\tan\phi$). Thus for V-threads,

$$P = W\tan(\alpha \pm \phi_1)$$

where ϕ_1 = Virtual friction angle, such that $\tan\phi_1 = \mu_1$.

Example 10.13. *Two co-axial rods are connected by a turn buckle which consists of a box nut, the one screw being right handed and the other left handed on a pitch diameter of 22 mm, the pitch of thread being 3 mm. The included angle of the thread is 60º. Assuming that the rods do not turn, calculate the torque required on the nut to produce a pull of 40 kN, given that the coefficient of friction is 0.15.*

Solution. Given : $d = 22$ mm ; $p = 3$ mm ; $2\beta = 60°$ or $\beta = 30°$, $W = 40\text{ kN} = 40 \times 10^3$ N ; $\mu = 0.15$

We know that $$\tan\alpha = \frac{p}{\pi d} = \frac{3}{\pi \times 22} = 0.0434$$

and virtual coefficient of friction

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.15}{\cos 30°} = 0.173$$

We know that the force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha.\tan\phi_1}\right]$$

$$= 40 \times 10^3\left[\frac{0.0434 + 0.173}{1 - 0.0434 \times 0.173}\right] = 8720\text{ N}$$

and torque on one rod, $T = P \times d/2 = 8720 \times 22/2 = 95\,920$ N-mm $= 95.92$ N-m

Since the turn buckle has right and left hand threads and the torque on each rod is $T = 95.92$ N-m, therefore the torque required on the nut,

$$T_1 = 2T = 2 \times 95.92 = 191.84\text{ N-m}$$ **Ans.**

$$= 10 \times 10^3 \left[\frac{0.064 + 0.113}{1 - 0.064 \times 0.113} \right] = 1783 \text{ N}$$

We know that total torque transmitted,

$$T = P \times \frac{d}{2} + \mu_2 . W . R = 1783 \times \frac{25}{2} + 0.16 \times 10 \times 10^3 \times 25 \text{ N-mm}$$

$$= 62\ 300 \text{ N-mm} = 62.3 \text{ N-m} \quad \ldots (i)$$

Let P_1 = Force required at the end of a spanner.

$\therefore$ Torque required at the end of a spanner,

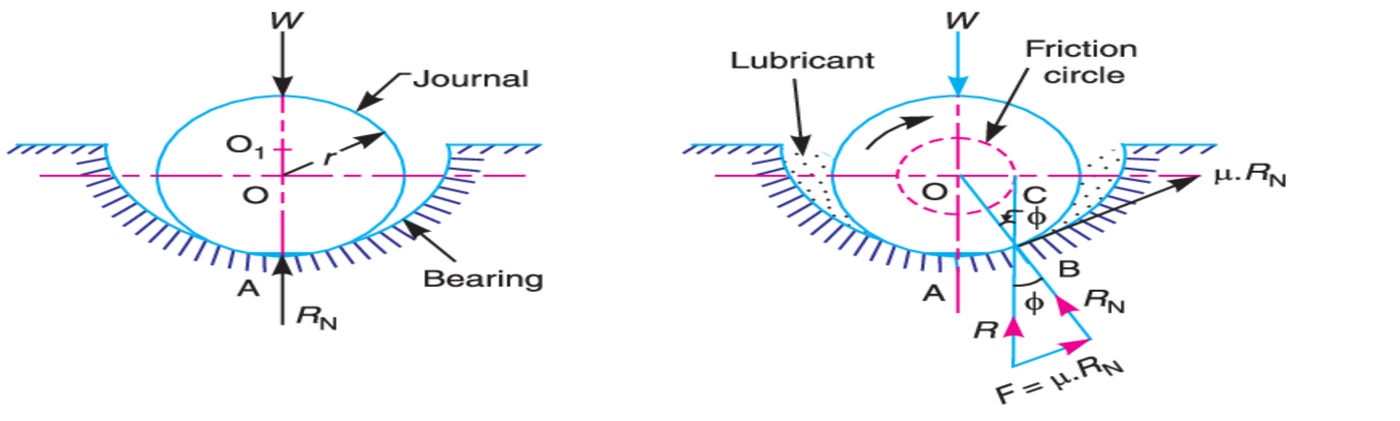
$$T = P_1 \times l = P_1 \times 0.5 = 0.5\, P_1 \text{ N-m} \quad \ldots (ii)$$

Equating equations ***(i)*** and ***(ii)***,

$$P_1 = 62.3/0.5 = 124.6 \text{ N}$$ **Ans.**

10.25. Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig. 10.15 (*a*). The fixed outer element of a turning pair is called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.*** The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.



(*a*) (*b*)

Fig. 10.15. Friction in journal bearing.

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig. 10.15 (*a*). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction R_N acts vertically upwards at point A. This point A is known as ***seat*** or ***point of pressure.***

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig. 10.15 (*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction R does not act vertically upward, but acts at another point of pressure B. This is due to the fact that when shaft rotates, a frictional force $F = \mu R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point A to point B.

In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let ϕ = Angle between R (resultant of F and R_N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$$R = W, \text{ and } T = W \times OC = W \times OB \sin\phi = W.r \sin\phi$$

Since ϕ is very small, therefore substituting $\sin\phi = \tan\phi$

$$\therefore \quad T = W.r \tan\phi = \mu.W.r \qquad \ldots(\because \mu = \tan\phi)$$

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \times 2\pi N/60 \text{ watts}$$

where N = Speed of the shaft in r.p.m.

Notes : **1.** If a circle is drawn with centre O and radius $OC = r \sin\phi$, then this circle is called the ***friction circle*** of a bearing.

2. The force R exerted by one element of a turning pair on the other element acts along a tangent to the friction circle.

Example 10.15. *A 60 mm diameter shaft running in a bearing carries a load of 2000 N. If the coefficient of friction between the shaft and bearing is 0.03, find the power transmitted when it runs at 1440 r.p.m.*

Solution. Given : d = 60 mm or r = 30 mm = 0.03 m ; W = 2000 N ; μ = 0.03 ; N = 1440 r.p.m. or $\omega = 2\pi \times 1440/60 = 150.8$ rad/s

We know that torque transmitted,

$$T = \mu.W.r = 0.03 \times 2000 \times 0.03 = 1.8 \text{ N-m}$$

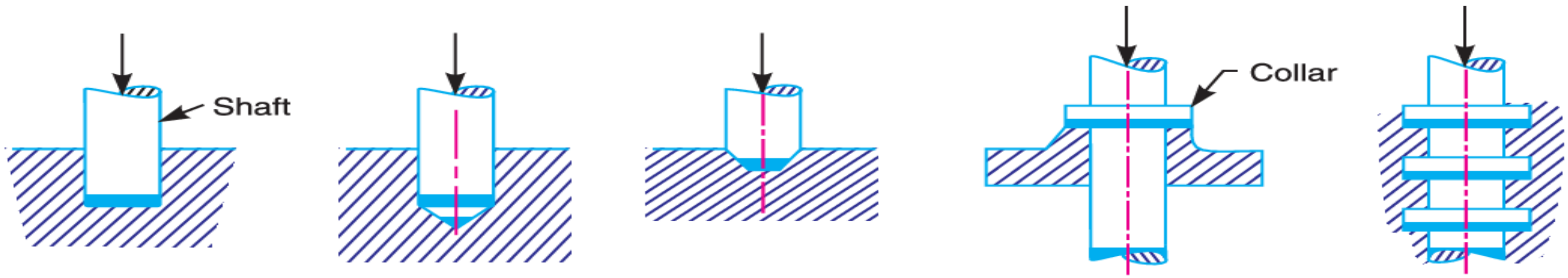
$\therefore$ Power transmitted, $P = T.\omega = 1.8 \times 150.8 = 271.4$ W **Ans.**

10.26. Friction of Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as ***pivots.*** The pivot may have a flat surface or conical surface as shown in Fig. 10.16 (*a*) and (*b*) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig. 10.16 (*c*).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Fig. 10.16 (*d*) or several collars along the length of a shaft, as shown in Fig. 10.16 (*e*) in order to reduce the intensity of pressure.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

Fig. 10.16. Pivot and collar bearings.

In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

10.31. Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.

2. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.

3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

The friction clutches of the following types are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),

2. Cone clutches, and

3. Centrifugal clutches.

We shall now discuss, these clutches, in detail, in the following pages. It may be noted that the disc and cone clutches are based on the same theory as the pivot and collar bearings.

10.32. Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

Single disc clutch

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

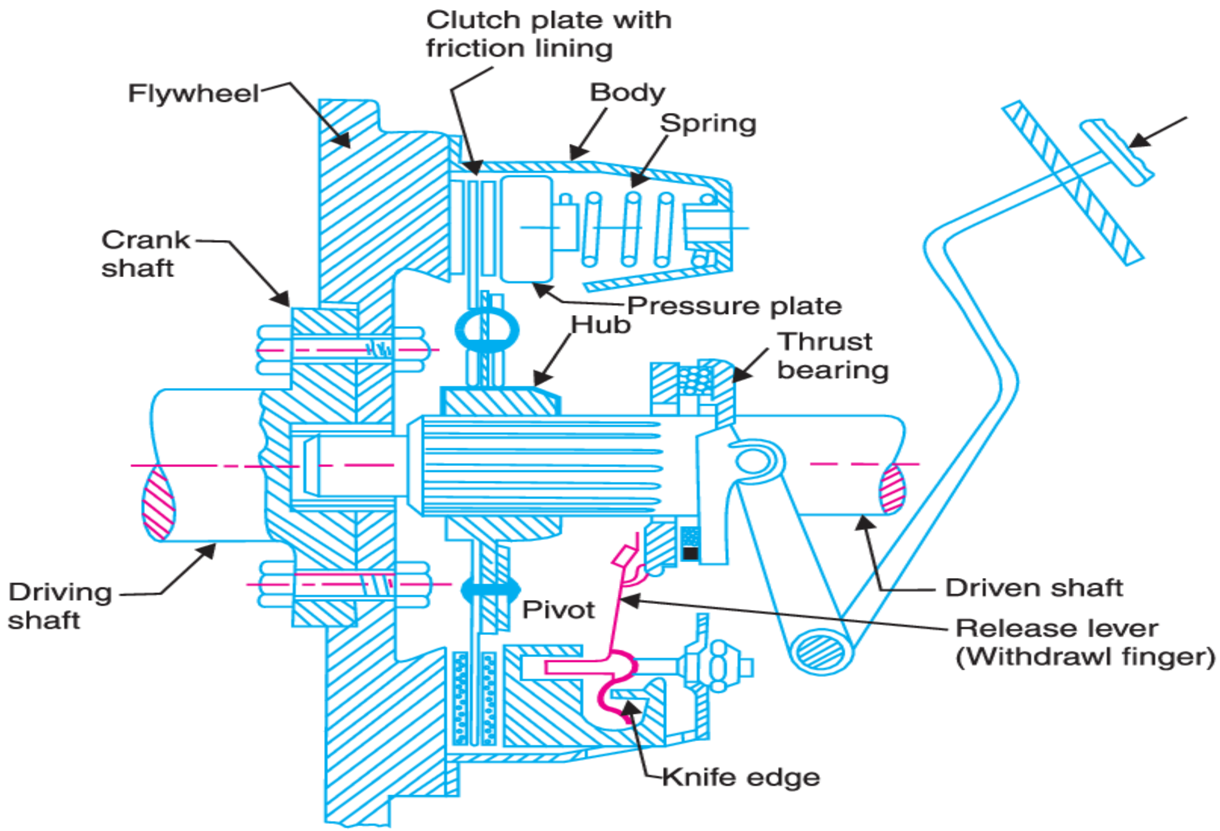


Fig. 10.21. Single disc or plate clutch.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

Now consider two friction surfaces, maintained in contact by an axial thrust W, as shown in Fig. 10.22 (*a*).

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 10.22 (*b*).

We know that area of contact surface or friction surface,

$$= 2\,\pi\, r.dr$$

$\therefore$ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\,\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu.p \times 2\,\pi\, r.dr$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\,\pi\, r.dr \times r = 2\,\pi \times \mu\, .p.r^2\, dr$$

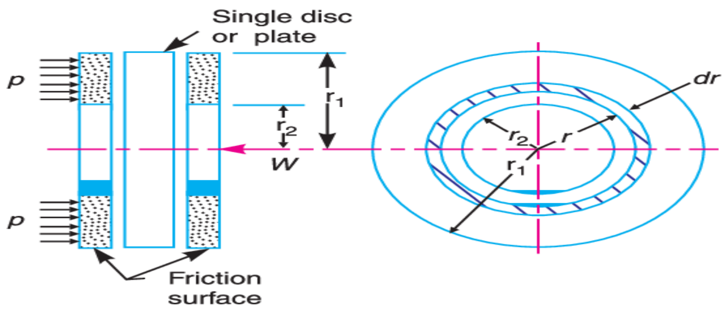


Fig. 10.22. Forces on a single disc or plate clutch.

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu.p.r^2\, dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu . p . r^2 . dr = 2\pi\mu p \left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu . W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu . W . R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering uniform wear*

In Fig. 10.22, let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant)} \quad \text{or} \quad p = C/r \qquad ...(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu . p r^2 . dr = 2\pi\mu \times \frac{C}{r} \times r^2 . dr = 2\pi\mu . C . r . dr$$

$$...(\because \quad p = C/r)$$

∴ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu . C . r . dr = 2\pi\mu . C \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu . C \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu . C [(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu . W (r_1 + r_2) = \mu . W . R$$

where R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

Notes : 1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction or contact surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore, a single disc clutch has two pairs of surfaces in contact, *i.e.* $n = 2$.

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation (*i*) may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C/r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation (*i*) may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C/r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

10.33. Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 10.23, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion

Dual Disc Clutches.

(except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$

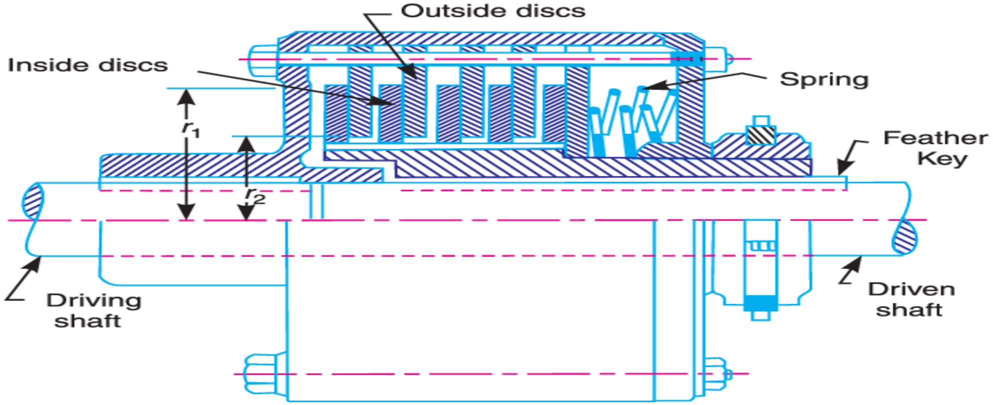


Fig. 10.23. Multiple disc clutch.

Example 10.22. *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

$$\therefore \quad p_{max} = 4 \times 10^3 / 15\,710 = 0.2546 \text{ N/mm}^2$$ **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\pi C (r_1 - r_2) = 2\pi \times 100\, p_{min} (100 - 50) = 31\,420\, p_{min}$$

$$\therefore \quad p_{min} = 4 \times 10^3/31\,420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 10.23. *A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm². If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.*

Solution. Given : $d_1 = 300$ mm or $r_1 = 150$ mm ; $d_2 = 200$ mm or $r_2 = 100$ mm ; $p = 0.1$ N/mm^2 ; $\mu = 0.3$; $N = 2500$ r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 10 (150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

$$\text{...}(\because n = 2, \text{for both sides of plate effective})$$

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW} \quad \textbf{Ans.}$$

10.34. Cone Clutch

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch.

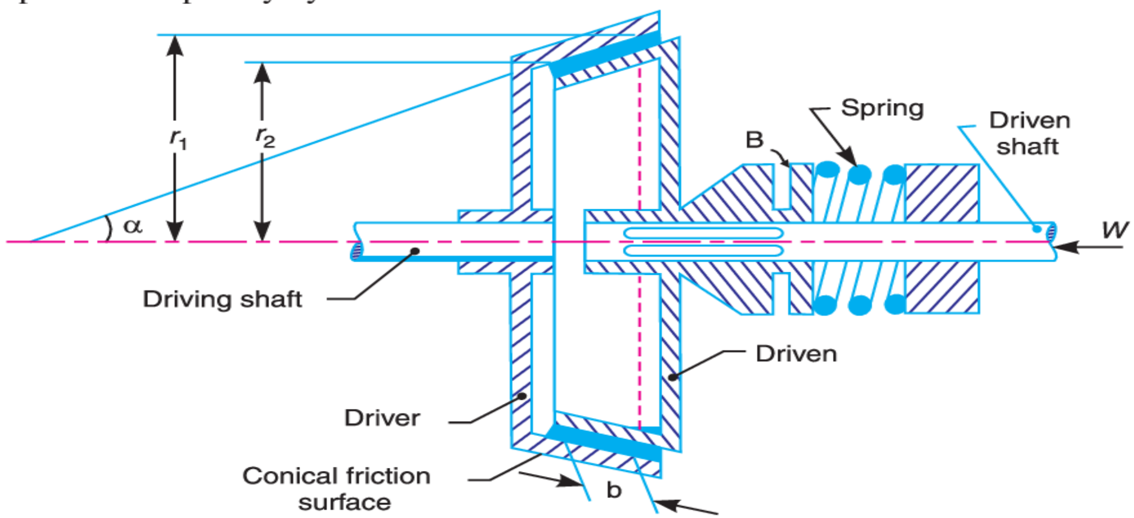


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at B, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

Consider a pair of friction surface as shown in Fig. 10.25 (*a*). Since the area of contact of a pair of friction surface is a frustrum of a cone, therefore the torque transmitted by the cone clutch may be determined in the similar manner as discussed for conical pivot bearings in Art. 10.28.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

r_1 and r_2 = Outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surface $= \frac{r_1 + r_2}{2}$,

α = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).

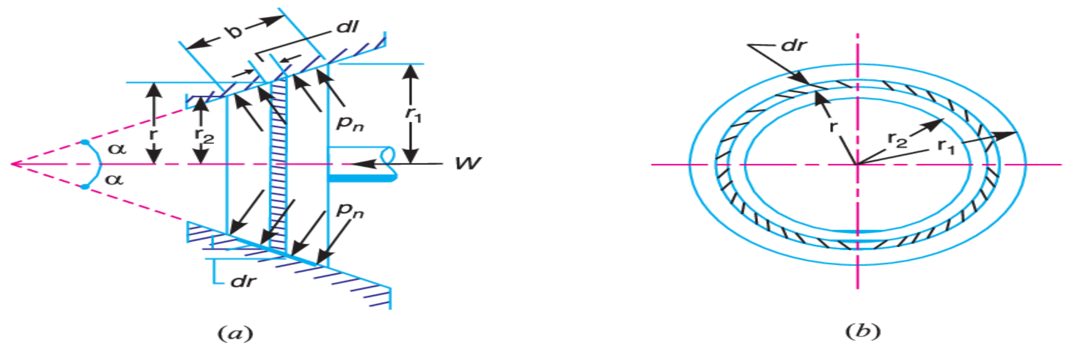


Fig. 10.25. Friction surfaces as a frustrum of a cone.

Consider a small ring of radius r and thickness dr, as shown in Fig. 10.25 (b). Let dl is length of ring of the friction surface, such that

$$dl = dr.\text{cosec}\ \alpha$$

∴ Area of the ring,

$$A = 2\pi\ r.dl = 2\pi r.dr\ \text{cosec}\ \alpha$$

We shall consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\ \pi\ r.dr.\text{cosec}\ \alpha$$

and the axial load acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\ r.dr.\ \text{cosec}\ \alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

∴ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi\, p_n.r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$\therefore \quad p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \ldots(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\,\pi\, r.dr.\text{cosec}\,\alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\,\pi\, r.dr.\ \text{cosec}\,\alpha.r = 2\,\pi\,\mu.p_n.\text{cosec}\,\alpha.r^2\,dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec}\,\alpha.r^2.dr = 2\pi\mu\,p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu\,p_n.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation ***(i)***, we get

$$T = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad ..(ii)$$

2. *Considering uniform wear*

In Fig. 10.25, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore \qquad p_r.r = C \text{ (a constant)} \quad \text{or} \quad p_r = C/r$$

We know that the normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\,\alpha$$

and the axial load acting on the ring ,

$$\delta W = \delta W_n \times \sin\alpha = p_r.2\,\pi\, r.dr.\text{cosec}\,\alpha\ .\sin\alpha = p_r \times 2\,\pi\, r.dr$$

$$= \frac{C}{r} \times 2\,\pi\, r.dr = 2\,\pi\, C.dr \qquad ...(\because\ p_r = C/r)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\, C.dr = 2\pi C\,[r]_{r_2}^{r_1} = 2\pi\, C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\,\pi\,(r_1 - r_2)} \qquad ...(iii)$$

We know that frictional force acting on the ring,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\,\pi\, r \times dr\ \text{cosec}\,\alpha$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\,\pi\, r.dr.\text{cosec}\,\alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\,\pi\, r^2.dr.\text{cosec}\,\alpha = 2\,\pi\mu.C\ \text{cosec}\,\alpha \times r\ dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\ \alpha.r\,dr = 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation ***(i)***, we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

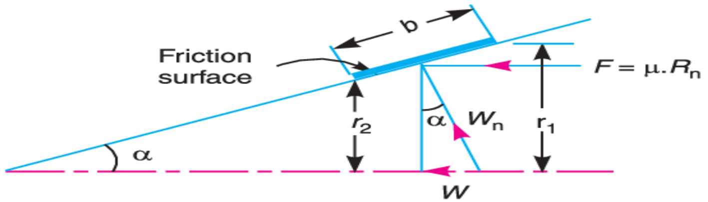
$$= \mu.W\ \text{cosec}\ \alpha\left(\frac{r_1 + r_2}{2}\right) = \mu.W.R\ \text{cosec}\ \alpha \qquad \text{...(iv)}$$

where $R = \frac{r_1 + r_2}{2}$ = Mean radius of friction surface

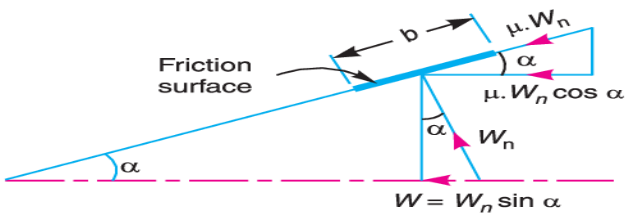
Since the normal force acting on the friction surface, $W_n = W/\sin\alpha$, therefore the equation ***(iv)*** may be written as

$$T = \mu.W_n.R \qquad \text{...(v)}$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 10.26.



(*a*) For steady operation of the clutch.



(*b*) During engagement of the clutch.

Fig. 10.26. Forces on a friction surface.

From Fig. 10.26 (*a*), we find that

$$r_1 - r_2 = b\sin\alpha;\ \text{and}\ R = \frac{r_1 + r_2}{2}\quad \text{or}\quad r_1 + r_2 = 2R$$

Example 10.32. *An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5º and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm². Determine : **1.** the axial spring force necessary to engage to clutch, and **2.** the face width required.*

Solution. Given : $P = 45$ kW $= 45 \times 10^3$ W ; $N = 1000$ r.p.m. or $\omega = 2\pi \times 1000/60 = 104.7$ rad/s ; $\alpha = 12.5°$; $D = 500$ mm or $R = 250$ mm $= 0.25$ m ; $\mu = 0.2$; $p_n = 0.1$ N/mm²

1. Axial spring force necessary to engage the clutch

First of all, let us find the torque (T) developed by the clutch and the normal load (W_n) acting on the friction surface.

We know that power developed by the clutch (P),

$$45 \times 10^3 = T.\omega = T \times 104.7 \quad \text{or} \quad T = 45 \times 10^3/104.7 = 430 \text{ N-m}$$

We also know that the torque developed by the clutch (T),

$$430 = \mu.W_n.R = 0.2 \times W_n \times 0.25 = 0.05\, W_n$$

$$\therefore \quad W_n = 430/0.05 = 8600 \text{ N}$$

and axial spring force necessary to engage the clutch,

$$W_e = W_n (\sin\alpha + \mu\cos\alpha)$$

$$= 8600 (\sin 12.5° + 0.2 \cos 12.5°) = 3540 \text{ N} \quad \textbf{Ans.}$$

2. Face width required

Let b = Face width required.

We know that normal load acting on the friction surface (W_n),

$$8600 = p_n \times 2\,\pi\,R.b = 0.1 \times 2\pi \times 250 \times b = 157\, b$$

$$\therefore \quad b = 8600/157 = 54.7 \text{ mm} \quad \textbf{Ans.}$$

10.35. Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held

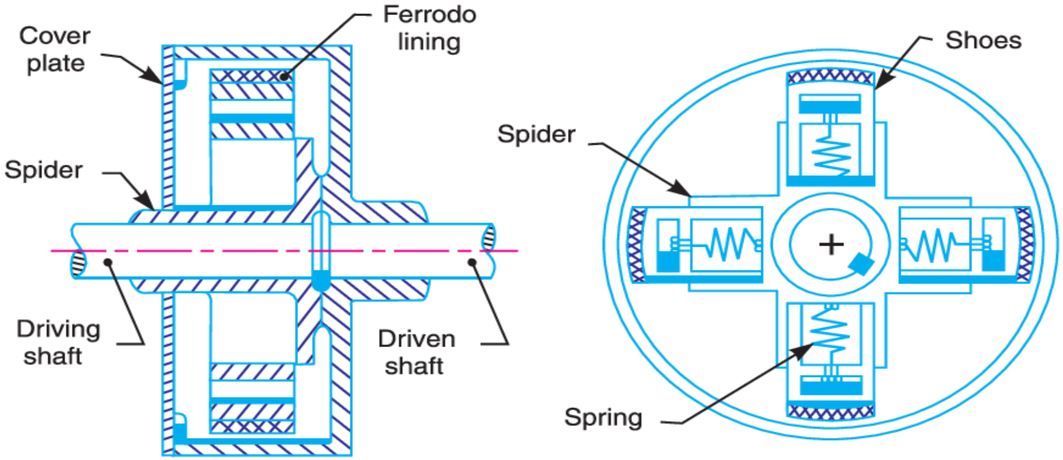


Fig. 10.28. Centrifugal clutch.

against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder

Centrifugal clutch.

and enables more torque to be transmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted :

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig. 10.29.

Let m = Mass of each shoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s = $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

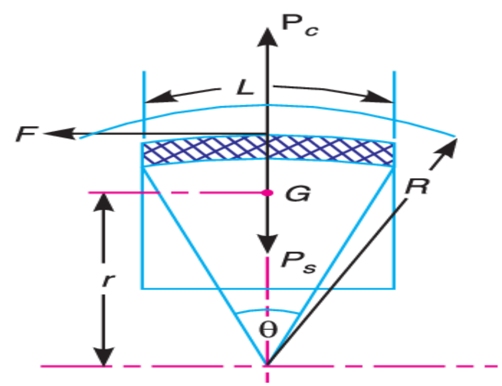


Fig. 10.29. Forces on a shoe of centrifugal clutch.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$$P_s = m\,(\omega_1)^2\,r$$

∴ The net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

∴ Frictional torque acting on each shoe,

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

Let l = Contact length of the shoes,

b = Width of the shoes,

* The radial clearance between the shoe and the rim being very small as compared to r, therefore it is neglected. If, however, the radial clearance is given, then the operating radius of the mass centre of the shoe from the axis of the clutch,

$$r_1 = r + c, \text{ where } c = \text{Radial clearance.}$$

Then $P_c = m.\omega^2.r_1$, and $P_s = m\ (\omega_1)^2\ r_1$

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that $\theta = l/R$ rad or $l = \theta.R$

∴ Area of contact of the shoe,

$$A = l.b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

Example 10.35. *A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine : **1.** Mass of the shoes, and **2.** Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60º and the pressure exerted on the shoes is 0.1 N/mm².*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. or $\omega = 2\pi \times 900/60 = 94.26$ rad/s ; $n = 4$; $R = 150$ mm $= 0.15$ m ; $r = 120$ mm $= 0.12$ m ; $\mu = 0.25$

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$$15 \times 10^3 = T.\omega = T \times 94.26 \quad \text{or} \quad T = 15 \times 10^3/94.26 = 159 \text{ N-m}$$

1. *Mass of the shoes*

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed ω_1,

$$P_s = m\,(\omega_1)^2\,r = m\,(70.7)^2 \times 0.12 = 600\,m \text{ N}$$

∴ Frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s) = 0.25\,(1066\,m - 600\,m) = 116.5\,m \text{ N}$$

We know that the torque transmitted (T),

$$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m \quad \text{or} \quad m = 2.27 \text{ kg}$$ **Ans.**

2. *Size of the shoes*

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,