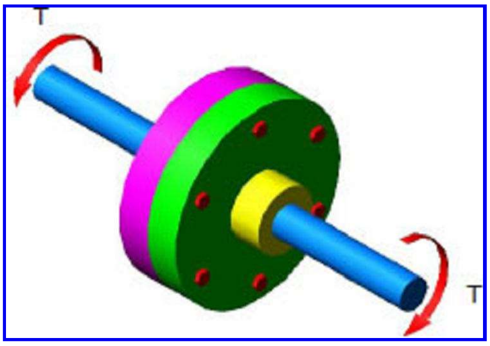
$$\tau_{max} = \frac{16T}{\pi D^3}$$

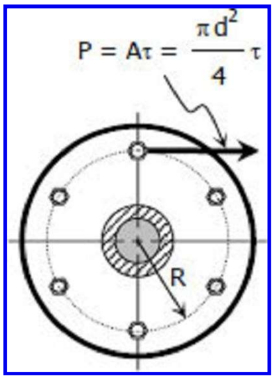
$$\tau_{of\ 6.5'\ shaft} = \frac{16(817.32)(12)}{\pi(2^3)} = 6243.86\,\text{psi} \quad \textbf{\textit{answer}}$$

$$\tau_{of\ 3.25'\ shaft} = \frac{16(817.32)(12)}{\pi(1.5^3)} = 14\,800.27\,\text{psi} \quad \textbf{\textit{answer}}$$

Flanged bolt couplings

In shaft connection called flanged bolt couplings (see figure), the torque is transmitted by the shearing force P created in he bolts that is assumed to be uniformly distributed. For any number of bolts n, the torque capacity of the coupling is

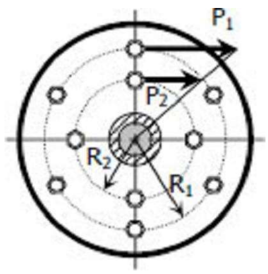




$$T = PRn = \frac{\pi d^2}{4}\tau Rn$$

If a coupling has two concentric rows of bolts, the torque capacity is

$$T = P_1R_1n_1 + P_2R_2n_2$$



where the subscript 1 refer to bolts on the outer circle an subscript 2 refer to bolts on the inner circle. See figure.

For rigid flanges, the shear deformations in the bolts are proportional to their radial distances from the shaft axis. The shearing strains are related by

$$\frac{\gamma_1}{R_1} = \frac{\gamma_2}{R_2}$$

Using Hooke’s law for shear, $G = \tau/\gamma$, we have

$$\frac{\tau_1}{G_1R_1} = \frac{\tau_2}{G_2R_2} \text{ or } \frac{P_1/A_1}{G_1R_1} = \frac{P_2/A_2}{G_2R_2}$$

If the bolts on the two circles have the same area, $A_1 = A_2$, and if the bolts are made of the same material, $G_1 = G_2$, the relation between P_1 and P_2 reduces to

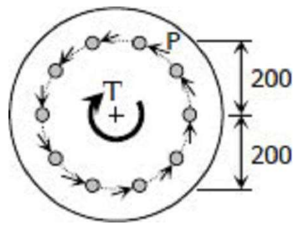
$$\frac{P_1}{R_1} = \frac{P_2}{R_2}$$

Solution to Problem 326 | Flanged bolt couplings

A flanged bolt coupling consists of ten 20-mm-diameter bolts spaced evenly around a bolt circle 400 mm in diameter. Determine the torque capacity of the coupling if the allowable shearing stress in the bolts is 40 MPa.

Solution 326

$$T = PRn = A\tau Rn = \tfrac{1}{4}\pi d^2 \tau Rn$$
$$T = \tfrac{1}{4}\pi(20^2)(40)(200)(10)$$
$$T = 8\,000\,000\pi \text{ N}\cdot\text{mm}$$
$$T = 8\pi \text{ kN}\cdot\text{m} = 25.13 \text{ kN}\cdot\text{m} \quad \textbf{\textit{answer}}$$



Solution to Problem 327 | Flanged bolt couplings

Problem 327

A flanged bolt coupling consists of ten steel ½-in.-diameter bolts spaced evenly around a bolt circle 14 in. in diameter. Determine the torque capacity of the coupling if the allowable shearing stress in the bolts is 6000 psi.

Solution 327

$$T = PRn = A\tau Rn = \tfrac{1}{4}\pi d^2 \tau Rn$$
$$T = \tfrac{1}{4}\pi(1/2)^2(6000)(7)(10)$$
$$T = 26250\pi \text{ lb}\cdot\text{in}$$
$$T = 2187.5\pi \text{ lb}\cdot\text{ft} = 6872.23 \text{ lb}\cdot\text{ft} \quad \textbf{\textit{answer}}$$

Solution to Problem 328 | Flanged bolt couplings

A flanged bolt coupling consists of eight 10-mm-diameter steel bolts on a bolt circle 400 mm in diameter, and six 10-mm-diameter steel bolts on a concentric bolt circle 300 mm in diameter, as shown in Fig. 3-7. What torque can be applied without exceeding a shearing stress of 60 MPa in the bolts?

Solution 328

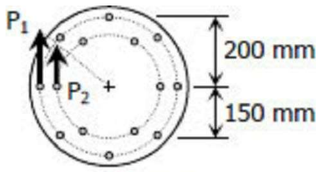


Figure 3-7

For one bolt in the outer circle:

$$P_1 = A\tau = \frac{\pi(10^2)}{4}(60)$$
$$P_1 = 1500\pi \text{ N}$$

For one bolt in the inner circle:

$$\frac{P_1}{R_1} = \frac{P_2}{R_2}$$
$$\frac{1500\pi}{200} = \frac{P_2}{150}$$
$$P_2 = 1125\pi \text{ N}$$

$$T = P_1 R_1 n_1 + P_2 R_2 n_2$$
$$T = 1500\pi(200)(8) + 1125\pi(150)(6)$$
$$T = 3\,412,500\pi \text{ N}\cdot\text{mm}$$
$$T = 3.4125\pi \text{ kN}\cdot\text{m} = 10.72 \text{ kN}\cdot\text{m} \quad \textit{answer}$$

Solution to Problem 329 | Flanged bolt couplings

Problem 329

A torque of 700 lb-ft is to be carried by a flanged bolt coupling that consists of eight ½-in.-diameter steel bolts on a circle of diameter 12 in. and six ½-in.-diameter steel bolts on a circle of diameter 9 in. Determine the shearing stress in the bolts.

Solution 329

$$\frac{P_1}{R_1} = \frac{P_2}{R_2}$$
$$\frac{A\tau_1}{6} = \frac{A\tau_2}{4.5}$$
$$\tau_2 = 0.75\tau_1$$

$$T = P_1R_1n_1 + P_2R_2n_2$$
$$700(12) = \tfrac{1}{4}\pi(1/2)^2\tau_1(6)(8) + \tfrac{1}{4}\pi(1/2)^2\tau_2(4.5)(6)$$
$$8400 = 3\pi\tau_1 + 1.6875\pi(0.75\tau_1)$$
$$8400 = 13.4\tau_1$$
$\tau_1 = 626.87$ psi $\longrightarrow$ bolts in the outer circle ***answer***

$\tau_2 = 0.75(626.87) = 470.15$ psi $\longrightarrow$ bolts in the inner circle ***answer***

Solution to Problem 330 | Flanged bolt couplings

Determine the number of 10-mm-diameter steel bolts that must be used on the 400-mm bolt circle of the coupling described in Prob. 328 to increase the torque capacity to 14 kN·m

Solution 330

$$T = P_1R_1n_1 + P_2R_2n_2$$
$$14(1000^2) = 1500\pi(200)n_1 + 1125\pi(150)(6)$$
$n_1 = 11.48$ **say 12 bolts *answer***

Solution to Problem 331 | Flanged bolt couplings

A flanged bolt coupling consists of six ½-in. steel bolts evenly spaced around a bolt circle 12 in. in diameter, and four ¾-in. aluminum bolts on a concentric bolt circle 8 in. in diameter. What torque can be applied without exceeding 9000 psi in the steel or 6000 psi in the aluminum? Assume $G_{st} = 12 \times 10^6$ psi and $G_{al} = 4 \times 10^6$ psi.

Solution 331

$$T = (PRn)_{st} + (PRn)_{al}$$
$$T = (A\tau Rn)_{st} + (A\tau Rn)_{al}$$
$$T = \frac{1}{4}\pi(1/2)^2\tau_{st}(6)(6) + \frac{1}{4}\pi(3/4)^2\tau_{al}(4)(4)$$
$$T = 2.25\pi\tau_{st} + 2.25\pi\tau_{al}$$
$$T = 2.25\pi(\tau_{st} + \tau_{al}) \;\rightarrow\; \textbf{Equation (1)}$$

$$\left(\frac{\tau}{GR}\right)_{st} = \left(\frac{\tau}{GR}\right)_{al}$$
$$\frac{\tau_{st}}{(12\times10^6)(6)} = \frac{\tau_{al}}{(4\times10^6)(4)}$$
$$\tau_{st} = \frac{9}{2}\tau_{al} \;\rightarrow\; \textbf{Equation (2a)}$$
$$\tau_{al} = \frac{2}{9}\tau_{at} \;\rightarrow\; \textbf{Equation (2b)}$$

Equations (1) and (2a)

$$T = 2.25\pi(\tfrac{9}{2}\tau_{al} + \tau_{al}) = 12.375\pi\tau_{al}$$
$$T = 12.375\pi(6000) = 74\,250\pi \text{ lb}\cdot\text{in}$$
$$T = 233.26 \text{ kip}\cdot\text{in}$$

Equations (1) and (2b)

$$T = 2.25\pi(\tau_{st} + \tfrac{2}{9}\tau_{st}) = 2.75\pi\tau_{st}$$
$$T = 2.25\pi(9000) = 24\,750\pi \text{ lb}\cdot\text{in}$$
$$T = 77.75 \text{ kip}\cdot\text{in}$$

Use **T = 77.75 kip·in** ***answer***

Solution to Problem 332 | Flanged bolt couplings

In a rivet group subjected to a twisting couple T, show that the torsion formula τ = Tρ/J can be used to find the shearing stress τ at the center of any rivet. Let $J = \Sigma A\rho^2$, where A is the area of a rivet at the radial distance ρ from the centroid of the rivet group.

Solution 332

The shearing stress on each rivet is P/A

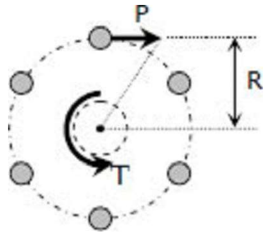
$\tau = T\rho / J$

Where:

$T = PRn$

$\rho = R$

$J = \Sigma A\rho^2 = AR^2 n$



$$\tau = \frac{PRn(R)}{AR^2 n}$$

$$\tau = \frac{P}{A} \quad \textbf{\textit{ok!}}$$

This shows that $\tau = T\rho / J$ can be used to find the shearing stress at the center of any rivet.

Solution to Problem 333 | Flanged bolt couplings

A plate is fastened to a fixed member by four 20-mm-diameter rivets arranged as shown in Fig. P-333. Compute the maximum and minimum shearing stress developed.

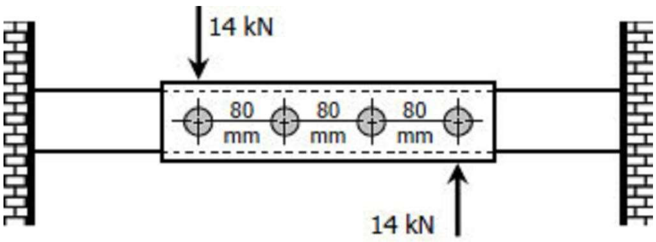
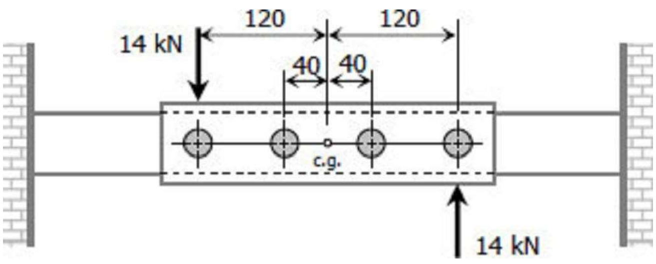


Figure P-333

Solution 333

$$\tau = \frac{T\rho}{J}$$



Where:

$$T = 14(1000)(120) = 1\,680\,000 \text{ N} \cdot \text{mm}$$

$$J = \Sigma A\rho^2 = \tfrac{1}{4}\pi(20)^2 \,[\, 2(40^2) + 2(120^2) \,]$$

$$J = 3\,200\,000\pi \text{ mm}^4$$

Maximum shearing stress (ρ = 120 mm):

$$\tau_{max} = \frac{1\,680\,000(120)}{3\,200\,000\pi}$$

$$\tau_{max} = 20.05 \text{ MPa} \quad \textit{answer}$$

Minimum shearing stress (ρ = 40 mm):

$$\tau_{min} = \frac{1\,680\,000(40)}{3\,200\,000\pi}$$

$$\tau_{min} = 6.68 \text{ MPa} \quad \textit{answer}$$

Solution to Problem 334 | Flanged bolt couplings

Six 7/8-in-diameter rivets fasten the plate in Fig. P-334 to the fixed member. Using the results of Prob. 332, determine the average shearing stress caused in each rivet by the 14 kip loads. What additional loads P can be applied before the shearing stress in any rivet exceeds 8000 psi?

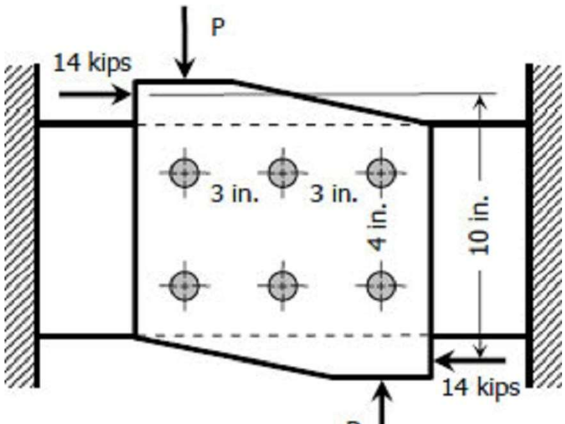
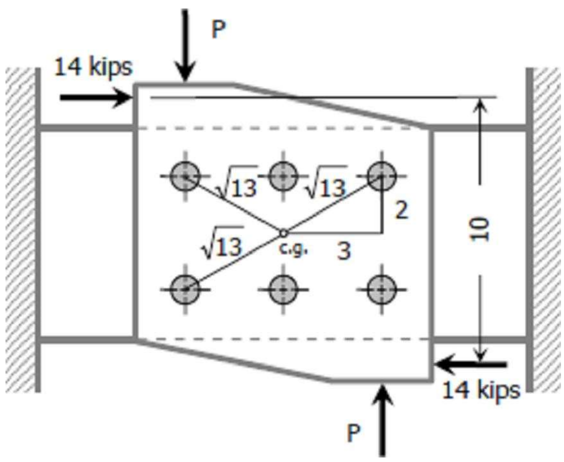


Figure P-334

Solution 334

Without the loads P:

$$\tau = \frac{T\rho}{J}$$



Where:

$$T = 14(10) = 140\,\text{kip}\cdot\text{in}$$

$$\rho = \sqrt{13}\,\text{in}$$

$$J = \Sigma A\rho^2 = \tfrac{1}{4}\pi(\tfrac{7}{8})^2\,[\,4(\sqrt{13}) + 2(2)^2\,] = 36.08\,\text{in}^4$$

$$\tau_{maximum} = \frac{140\sqrt{14}}{36.08} = 14.0\,\text{ksi}$$ ***answer***

$$\tau_{minimum} = \frac{140(2)}{36.08} = 7.76\,\text{ksi}$$ ***answer***

With the loads P, two cases will arise:

1st case (P < 14 kips)

$T = 10(14) – 6P = (140 – 6P)\ \text{kip} \cdot \text{in}$

$\tau = \dfrac{T\rho}{J}$

$8000 = \dfrac{(140 - 6P)(1000)(\sqrt{13})}{36.08}$

$80.05 = 140 – 6P$

$P = 10.0\ \text{kips}$ ***answer***

2nd case (P > 14 kips)

$T = 6P – 10(14) = (6P – 140)\ \text{kip} \cdot \text{in}$

$\tau = \dfrac{T\rho}{J}$

$8000 = \dfrac{(6P - 140)(1000)(\sqrt{13})}{36.08}$

$80.05 = 6P - 140$

$P = 36.68\ \text{kips}$ ***answer***

Solution to Problem 335 | Flanged bolt couplings

The plate shown in Fig. P-335 is fastened to the fixed member by five 10-mm-diameter rivets. Compute the value of the loads P so that the average shearing stress in any rivet does not exceed 70 MPa. (Hint: Use the results of Prob. 332.)

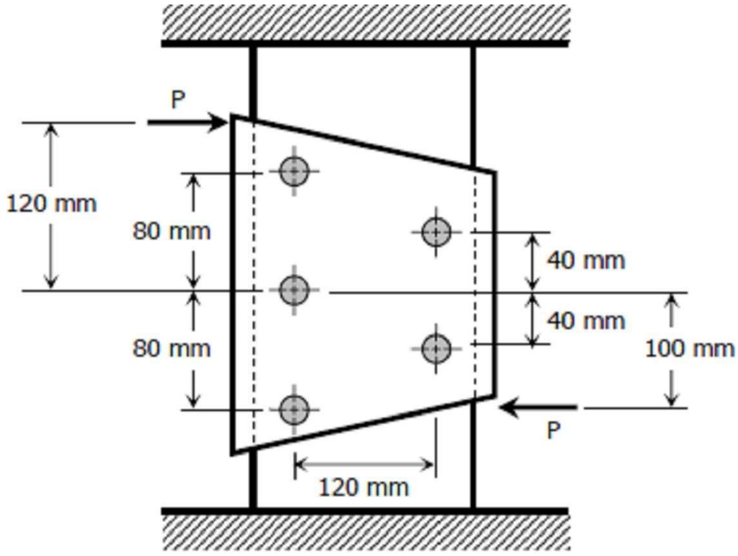
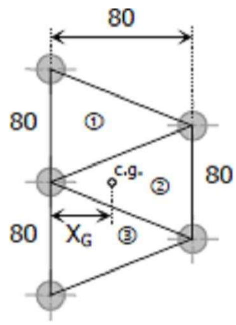


Figure P-335

Solution 335



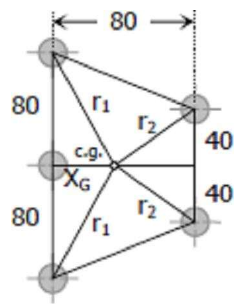
Solving for location of centroid of rivets:

$$AX_G = \Sigma ax$$

Where

$$A = \tfrac{1}{2}(80 + 160)(80) = 9600\text{ mm}^2$$
$$a_1 = a_2 = a_3 = \tfrac{1}{2}(80)(80) = 3200\text{ mm}^2$$
$$x_1 = x_3 = \tfrac{1}{3}(80) = 80/3\text{ mm}$$
$$x_2 = \tfrac{2}{3}(80) = 160/3\text{ mm}$$

$$9600X_G = 3200(80/3) + 3200(160/3) + 3200(80/3)$$
$$X_G = 320/9\text{ mm}$$



$$r_1 = \sqrt{(320/9)^2 + 80^2} = 87.54\text{ mm}$$
$$r_2 = \sqrt{(80 - 320/9)^2 + 40^2} = 59.79\text{ mm}$$

$$J = \Sigma A\rho^2 = \tfrac{1}{4}\pi(10^2)(2r_1^2 + 2r_2^2 + X_G^2)$$
$$J = \tfrac{1}{4}\pi(10^2)\,[\,2(87.54)^2 + 2(59.79)^2 + (320/9)^2\,]$$
$$J = 1\,864\,565.79\text{ mm}^4$$
$$T = (120 + 100)P = 220P$$

The critical rivets are at distance r_1 from centroid: