Axial Deformation

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by

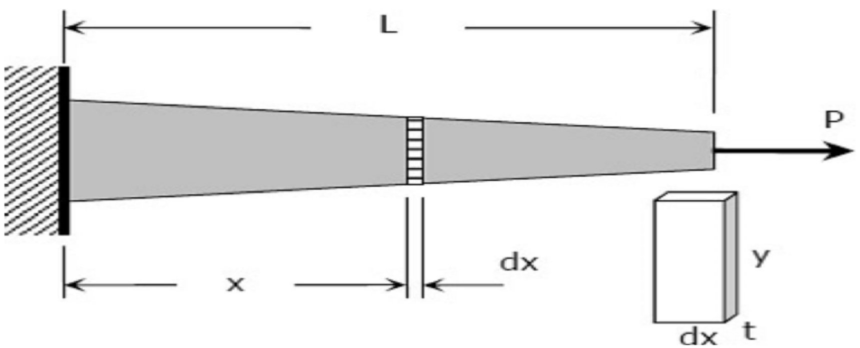
$$\sigma = E\varepsilon$$

since $\sigma = P/A$ and $\varepsilon = \delta/L$, then $\dfrac{P}{A} = E\ \dfrac{\delta}{L}$

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit.

If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.



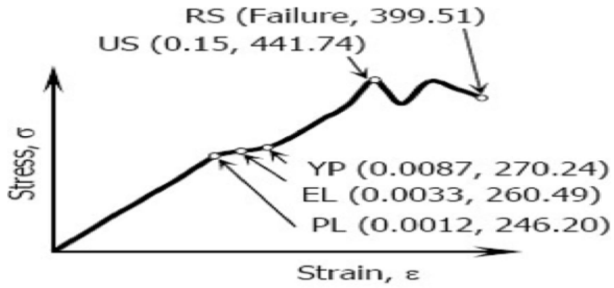
$$\delta = \frac{P}{E}\int_0^L \frac{dx}{L}$$

where $A = ty$ and y and t, if variable, must be expressed in terms of x.

For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is

$$\delta = \frac{\rho g L^2}{2E} = \frac{MgL}{2AE}$$

18 800	0.030	0.0006	122.13
25 100	0.040	0.0008	163.05
31 300	0.050	0.001	203.33
37 900	0.060	0.0012	246.20
40 100	0.163	0.0033	260.49
41 600	0.433	0.0087	270.24
46 200	1.250	0.025	300.12
52 400	2.500	0.05	340.40
58 500	4.500	0.09	380.02
68 000	7.500	0.15	441.74
59 000	12.500	0.25	383.27
67 800	15.500	0.31	440.44
65 000	20.000	0.4	422.25
61 500	Failure		399.51

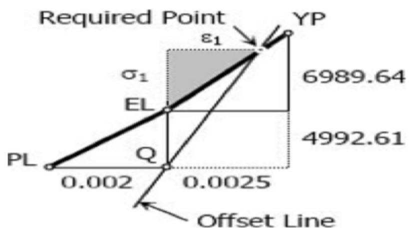


Stress-Strain Diagram
(not drawn to scale)

PL = Proportional Limit
EL = Elastic Limit
YP = Yield Point
US = Ultimate Strength
RS = Rupture Strength

From stress-strain diagram:

a. Proportional Limit = **246.20 MPa**
b. Modulus of Elasticity
   E = slope of stress-strain diagram within proportional limit
   $$E = \frac{246.20}{0.0012} = 205\,166.67\text{ MPa}$$
   $$E = 205.2\text{ GPa}$$
c. Yield Point = **270.24 MPa**



Slope of 0.2% offset = E = 10,529,861.82 psi

Test for location:
slope = rise / run

$$10,529,861.82 = \frac{6989.64 + 4992.61}{\text{run}}$$

run = 0.00113793 < 0.0025, therefore, the required point is just before YP.

Slope of EL to YP

$$\frac{\sigma_1}{\varepsilon_1} = \frac{6989.64}{0.0025}$$

$$\frac{\sigma_1}{\varepsilon_1} = 2\,795\,856$$

$$\varepsilon_1 \quad \frac{\sigma_1}{2\,795\,856}$$

For the required point:

$$E = \frac{4992.61 + \sigma_1}{\varepsilon_1}$$

$$10\,529\,861.82 = \frac{4992.61 + \sigma_1}{\frac{\sigma_1}{2\,795\,856}}$$

$$3.7662\sigma_1 = 4992.61 + \sigma_1$$

$$\sigma_1 = 1804.84\ \text{psi}$$

Yield Strength at 0.2% Offset
= EL+ σ_1
= 62906.85 + 1804.84
= **64,711.69 psi**

e. Ultimate Strength = **73,890.58 psi**
f. Rupture Strength = **67,899.45 psi**

Solution to Problem 205 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer

Given:
Length of bar = L
Cross-sectional area = A

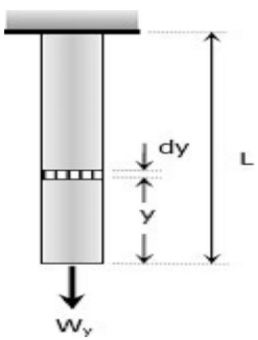
Unit mass = ρ
The bar is suspended vertically from one end

Required:
Show that the total elongation $\delta = \rho g L^2 / 2E$.
If total mass is M, show that $\delta = MgL/2AE$

Solution 205

$$\delta = \frac{PL}{AE}$$

From the figure:
$\delta = d\delta$
$P = W_y = (\rho A y)g$
$L = dy$

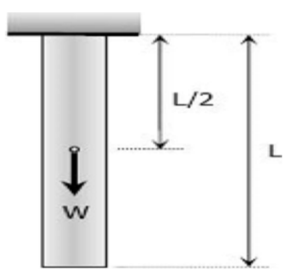


$$d\delta = \frac{(\rho A y)g\,dy}{AE}$$
$$\delta = \frac{\rho g}{E}\int_0^L y\,dy = \frac{\rho g}{E}\left[\frac{y^2}{2}\right]_0^L$$
$$\delta = \frac{\rho g}{2E}\,[\,L^2 - 0^2\,]$$
$$\delta = \rho g L^2/2E \textit{ ok!}$$

Given the total mass M
$\rho = M/V = M/AL$

$$\delta = \frac{\rho g L^2}{2E} = \frac{\frac{M}{AL}\cdot gL^2}{2E}$$
$$\delta = \frac{MgL}{2AE} \textit{ ok!}$$

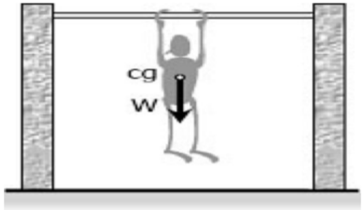
Another Solution:



$$\delta = \frac{PL}{AE}$$

Where:
$P = W = (\rho A L)g$
$L = L/2$

$$\delta = \frac{[(\rho A L)g](L/2)}{AE}$$
$$\delta = \rho g L^2/2E \textit{ ok!}$$



For you to feel the situation, position yourself in pull-up exercise with your hands on the bar and your body hang freely above the ground. Notice that your arms suffer all your weight and your lower body fells no stress (center of weight is approximately just below the chest). If your body is the bar, the elongation will occur at the upper half of it.

Solution to Problem 206 Axial Deformation

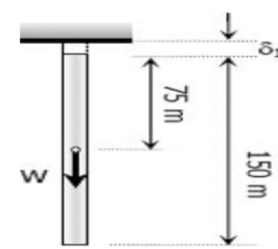
Strength of Materials 4th Edition by Pytel and Singer

Given:
Cross-sectional area = 300 mm^2
Length = 150 m
tensile load at the lower end = 20 kN
Unit mass of steel = 7850 kg/m^3
E = 200×10^3 MN/m^2

Required: Total elongation of the rod

Solution 206

Elongation due to its own weight:



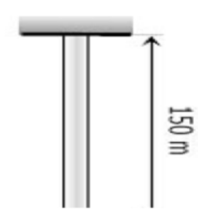
$$\delta_1 = \frac{PL}{AE}$$

Where:
P = W = 7850(1/1000)3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_1 = \frac{3\,465.3825(75\,000)}{300(200\,000)}$$

δ_1 = 4.33 mm

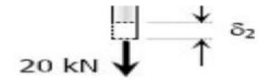
Elongation due to applied load:



$$\delta_2 = \frac{PL}{AE}$$

Where:
P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2

E = 200 000 MPa



$$\delta_2 = \frac{20\,000(150\,000)}{300(200\,000)}$$

$\delta_2 = 50$ mm

Total elongation:

$\delta = \delta_1 + \delta_2$

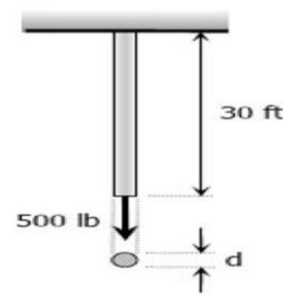
$\delta = 4.33 + 50 = 54.33 \text{ mm} \longrightarrow$ ***answer***

Solution to Problem 207 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer

Given:
Length of steel wire = 30 ft
Load = 500 lb
Maximum allowable stress = 20 ksi
Maximum allowable elongation = 0.20 inch
$E = 29 \times 10^6$ psi

Required: Diameter of the wire



Solution 207

Based on maximum allowable stress:

$$\sigma = \frac{P}{A}$$

$$20\,000 = \frac{500}{\frac{1}{4}\pi d^2}$$

$d = 0.1784$ in

Based on maximum allowable deformation:

$$\delta = \frac{PL}{AE}$$

$$0.20 = \frac{500(30 \times 12)}{\frac{1}{4}\pi d^2(29 \times 10^6)}$$

$d = 0.1988$ in

Use the bigger diameter, **d = 0.1988 inch**

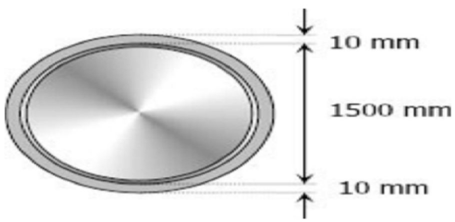
Solution to Problem 208 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer
Problem 208 page 40

Given:
Thickness of steel tire = 100 mm
Width of steel tire = 80 mm
Inside diameter of steel tire = 1500.0 mm
Diameter of steel wheel = 1500.5 mm
Coefficient of static friction = 0.30
E = 200 GPa

Required: Torque to twist the tire relative to the wheel

Solution 208

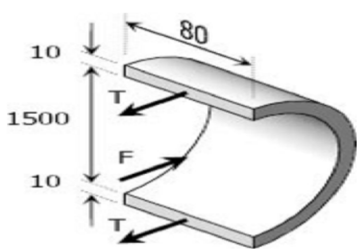


$$\delta = \frac{PL}{AE}$$

Where:
$\delta = \pi\,(1500.5 - 1500) = 0.5\pi$ mm
P = T
$L = 1500\pi$ mm
$A = 10(80) = 800\ \text{mm}^2$
E = 200 000 MPa

$$0.5\pi = \frac{T(1500\pi)}{800(200\,000)}$$

$$T = 53\,333.33\ \text{N}$$



$$F = 2T$$

$p(1500)(80) = 2(53\,333.33)$
$p = 0.8889\text{ MPa} \longrightarrow$ internal pressure

Total normal force, N:
N = p × contact area between tire and wheel
$N = 0.8889 \times \pi(1500.5)(80)$
N = 335 214.92 N

Friction resistance, f:
$f = \mu N = 0.30(335\,214.92)$
f = 100 564.48 N = 100.56 kN
Torque = f × ½(diameter of wheel)
Torque = 100.56 × 0.75025
Torque = **75.44 kN · m**

Solution to Problem 209 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer

Given:
Cross-section area = 0.5 in^2
E = 10×10^6 psi
The figure below:

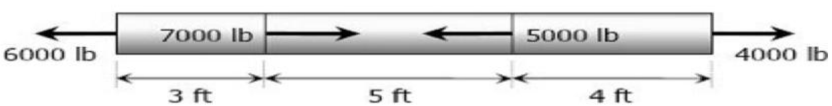
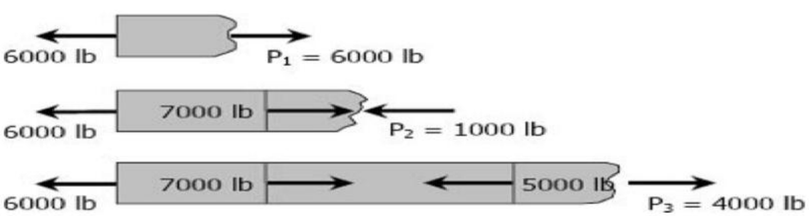


Figure P-209 and P-210

Required: Total change in length

Solution 209



P_1 = 6000 lb tension
P_2 = 1000 lb compression
P_3 = 4000 lb tension

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

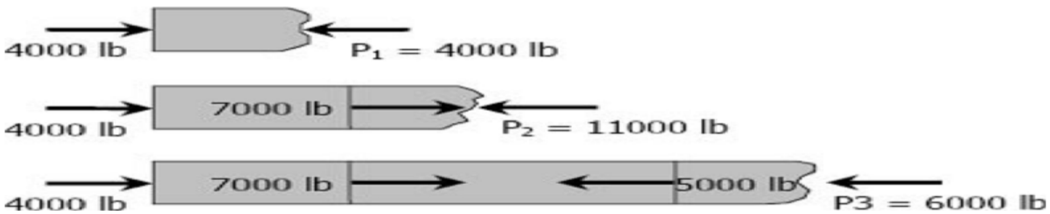
$$\delta = \frac{6000(3 \times 12)}{0.5(10 \times 10^6)} - \frac{1000(5 \times 12)}{0.5(10 \times 10^6)} + \frac{4000(4 \times 12)}{0.5(10 \times 10^6)}$$

$\delta = 0.0696$ in (lengthening) $\longrightarrow$ ***answer***

Solution to Problem 210 Axial Deformation

Solve Prob. 209 if the points of application of the 6000-lb and the 4000-lb forces are interchanged.

Solution 210



$P_1 = 4000$ lb compression
$P_2 = 11000$ lb compression
$P_3 = 6000$ lb compression

$$\delta = \frac{PL}{AE}$$

$$\delta = -\delta_1 - \delta_2 - \delta_3$$

$$\delta = -\frac{4\,000(3 \times 12)}{0.5(10 \times 10^6)} - \frac{11\,000(5 \times 12)}{0.5(10 \times 10^6)} - \frac{6\,000(4 \times 12)}{0.5(10 \times 10^6)}$$

$\delta = -0.19248$ in $= 0.19248$ in (shortening) $\longrightarrow$ ***answer***

Solution to Problem 211 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer
Problem 211 page 40

Given:
Maximum overall deformation = 3.0 mm
Maximum allowable stress for steel = 140 MPa
Maximum allowable stress for bronze = 120 MPa
Maximum allowable stress for aluminum = 80 MPa
E_{st} = 200 GPa
E_{al} = 70 GPa
E_{br} = 83 GPa
The figure below:

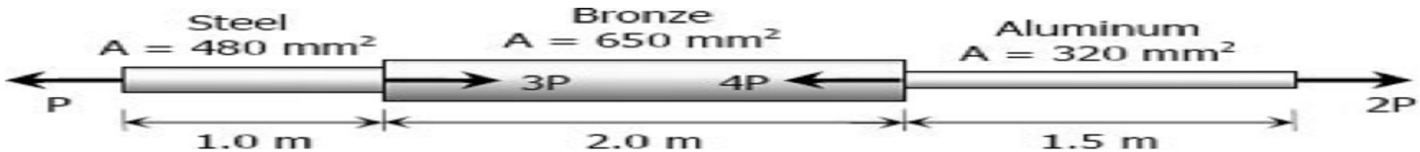
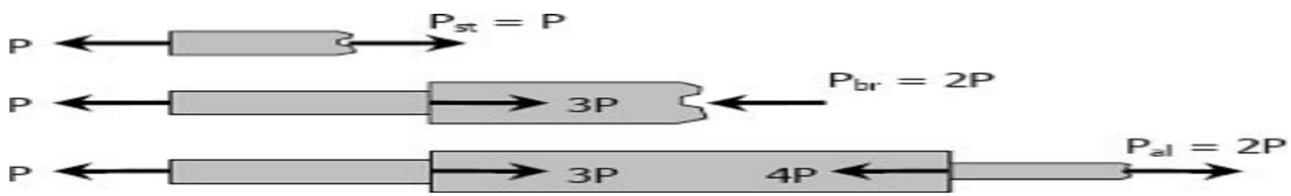


Figure P -211

Required: The largest value of P

Solution 211



Based on allowable stresses:

Steel:

$$P_{st} = \sigma_{st} A_{st}$$
$$P = 140(480) = 67\,200 \text{ N}$$
$$P = 67.2 \text{ kN}$$

Bronze:

$$P_{br} = \sigma_{br} A_{br}$$
$$2P = 120(650) = 78\,000 \text{ N}$$
$$P = 39\,000 \text{ N} = 39 \text{ kN}$$

Aluminum:

$$P_{al} = \sigma_{al} A_{al}$$
$$2P = 80(320) = 25\,600 \text{ N}$$
$$P = 12\,800 \text{ N} = 12.8 \text{ kN}$$

Based on allowable deformation:
(steel and aluminum lengthens, bronze shortens)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$
$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70\,000)} + \frac{2P(1500)}{320(83\,000)}$$
$$3 = \left(\frac{1}{96\,000} - \frac{1}{11\,375} + \frac{3}{26\,560}\right) P$$
$$P = 84\,610.99 \text{ N} = 84.61 \text{ kN}$$

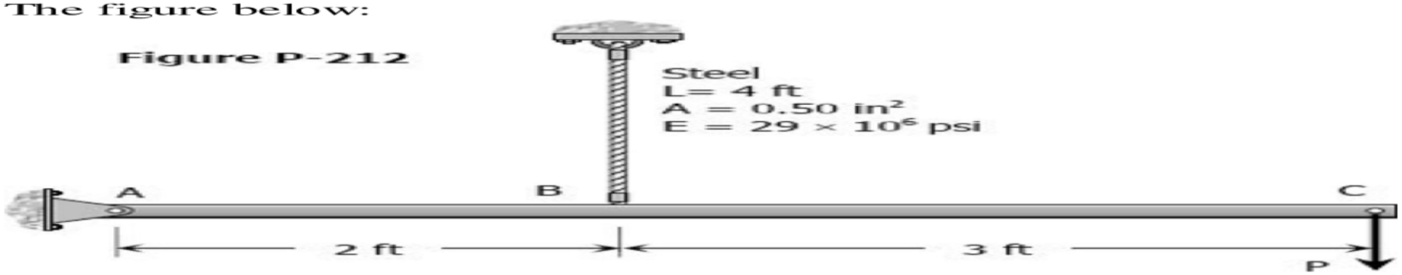
Use the smallest value of P, **P = 12.8 kN**

Solution to Problem 212 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer

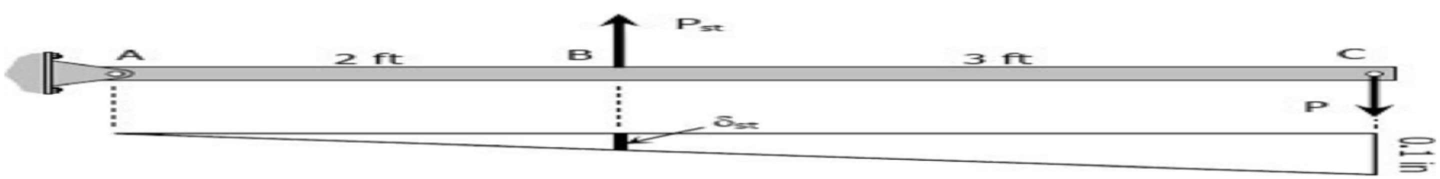
Given:
Maximum stress in steel rod = 30 ksi
Maximum vertical movement at C = 0.10 inch
The figure below:

Figure P-212



Required: The largest load P that can be applied at C

Solution 212



Based on maximum stress of steel rod:

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4\sigma_{at}A_{st}$

$P = 0.4\,[\,30(0.50)\,]$

$P = 6\text{ kips}$

Based on movement at C:

$\dfrac{\delta_{st}}{2} = \dfrac{0.1}{5}$

$\delta_{st} = 0.04\text{ in}$

$\dfrac{P_{st}L}{AE} = 0.04$

$\dfrac{P_{st}(4 \times 12)}{0.50(29 \times 10^6)} = 0.04$

$P_{st} = 12083.33\text{ lb}$

$\Sigma M_A = 0$

For steel:

$\Sigma M_A = 0$

$6P_{st} = 3.5(50)$

$P_{st} = 29.17\text{ kN}$

$\delta = \dfrac{PL}{AE}$

$\delta_{st} = \dfrac{29.17(4)1000^2}{300(200\,000)}$

$\delta_{st} = 1.94\text{ mm}$

Movement diagram:

$\dfrac{y}{3.5} = \dfrac{1.94 - 1.78}{6}$

$y = 0.09\text{ mm}$

$\delta_B =$ vertical movement of P

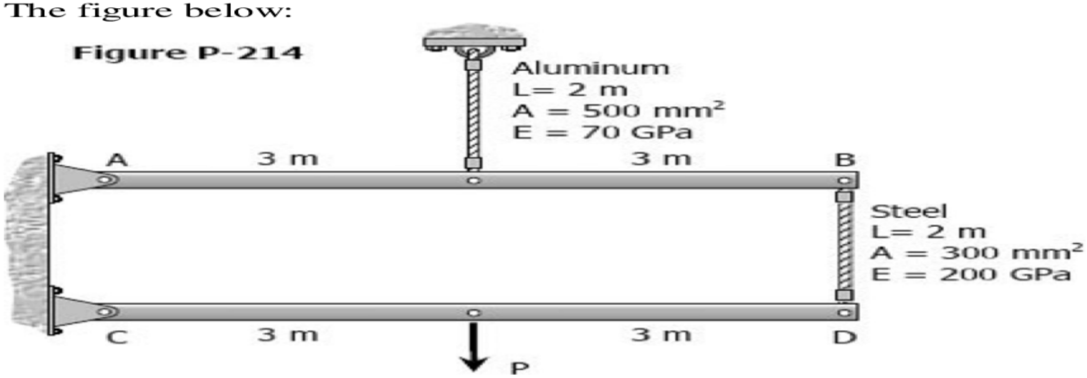
$\delta_B = 1.78 + y = 1.78 + 0.09$

$\delta_B = 1.87\text{ mm} \longrightarrow$ ***answer***

Solution to Problem 214 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer

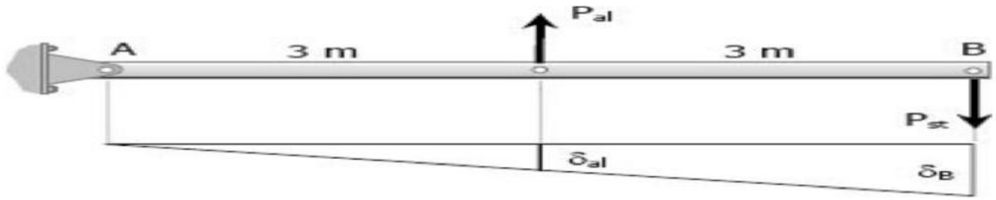
Given:
Maximum vertical movement of P = 5 mm
The figure below:



Required: The maximum force P that can be applied neglecting the weight of all members.

Solution 41

Member AB:



FBD and movement diagram of bar AB

$\Sigma M_A = 0$
$3P_{al} = 6P_{st}$
$P_{al} = 2P_{st}$

By ratio and proportion:

$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$

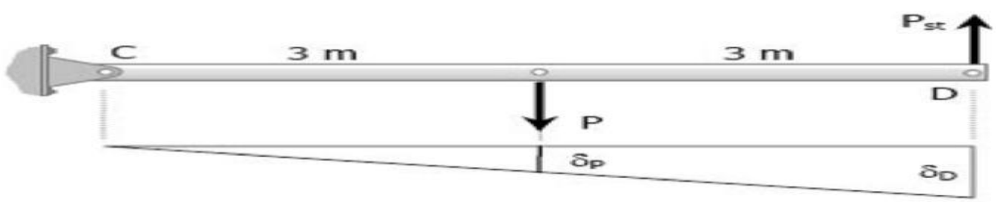
$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$

$\delta_B = 2\left[\frac{P_{al}\,(2000)}{500(70\,000)}\right]$

$\delta_B = \frac{1}{8750}P_{al} = \frac{1}{8750}(2P_{st})$

$\delta_B = \frac{1}{4375}P_{st} \longrightarrow$ movement of B

Member CD:



FBD and movement diagram of bar CD

Movement of D:

$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \frac{1}{4375}P_{st}$

$\delta_D = \frac{P_{st}\,(2000)}{300(200\,000)} + \frac{1}{4375}P_{st}$

$\delta_D = \frac{11}{42\,000}P_{st}$

$\Sigma M_C = 0$

$$\delta_{st} = \delta_{al}$$
$$\left[\frac{PL}{AE}\right]_{st} = \left[\frac{PL}{AE}\right]_{al}$$
$$\frac{\frac{1}{3}W(6 \times 12)}{A_{st}(29 \times 10^6)} = \frac{\frac{2}{3}W(4 \times 12)}{A_{al}(10 \times 10^6)}$$
$$\frac{A_{al}}{A_{st}} = \frac{\frac{2}{3}W(4 \times 12)(29 \times 10^6)}{\frac{1}{3}W(6 \times 12)(10 \times 10^6)}$$
$$\frac{A_{al}}{A_{st}} = 3.867 \quad \longrightarrow \textit{answer}$$

Solution to Problem 216 Axial Deformation

Strength of Materials 4th Edition by Pytel and Singer

Given:
Vertical load P = 6000 lb
Cross-sectional area of each rod = 0.60 in^2
E = 10 × 10^6 psi
α = 30°
θ = 30°
The figure below:

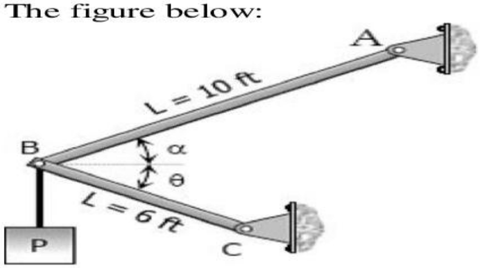
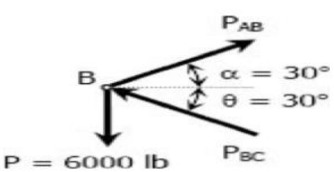


Figure P-216 and P-217

Required: Elongation of each rod and the horizontal and vertical displacements of point B

Solution 216



$$\Sigma F_H = 0$$
$$P_{AB} \cos 30° = P_{BC} \cos 30°$$

$P_{AB} = P_{BC}$

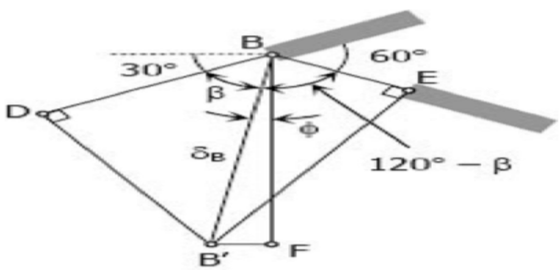
$\Sigma F_V = 0$
$P_{AB} \sin 30° + P_{BC} \sin 30° = 6000$
$P_{AB}(0.5) + P_{AB}(0.5) = 6000$
$P_{AB} = 6000$ lb tension
$P_{BC} = 6000$ lb compression

$$\delta = \frac{PL}{AE}$$

$$\delta_{AB} = \frac{6000(10 \times 12)}{0.6(10 \times 10^6)} = 0.12 \text{ inch lengthening} \longrightarrow \textit{answer}$$

$$\delta_{BC} = \frac{6000(6 \times 12)}{0.6(10 \times 10^6)} = 0.072 \text{ inch shortening} \longrightarrow \textit{answer}$$

$DB = \delta_{AB} = 0.12$ inch
$BE = \delta_{BE} = 0.072$ inch
$\delta_B = BB' =$ displacement of B
$B' =$ final position of B after elongation



Movement of B

Triangle BDB':

$$\cos \beta = \frac{0.12}{\delta_B}$$

$$\delta_B = \frac{0.12}{\cos \beta}$$

Triangle BEB':

$$\cos(120° - \beta) = \frac{0.072}{\delta_B}$$

$$\delta_B = \frac{0.072}{\cos(120° - \beta)}$$

$$\delta_B = \delta_B$$

$$\frac{0.12}{\cos\beta} = \frac{0.072}{\cos(120° - \beta)}$$

$$\frac{\cos 120° \cos\beta + \sin 120° \sin\beta}{\cos\beta} = 0.6$$

$$-0.5 + \sin 120° \tan\beta = 0.6$$

$$\tan\beta = \frac{1.1}{\sin 120°}$$

$$\beta = 51.79°$$

$$\phi = 90 - (30° + \beta) = 90° - (30° + 51.79°)$$

$$\phi = 8.21°$$

$$\delta_B = \frac{0.12}{\cos 51.79°}$$

$$\delta_B = 0.194 \text{ inch}$$

Triangle BFB':

$$\delta_h = B'F = \delta_B \sin\phi = 0.194 \sin 8.21°$$

$$\delta_h = 0.0277 \text{ inch}$$

$$\delta_h = 0.0023 \text{ ft} \longrightarrow \text{horizontal displacement of B}$$

$$\delta_v = BF = \delta_B \cos\phi = 0.194 \cos 8.21°$$

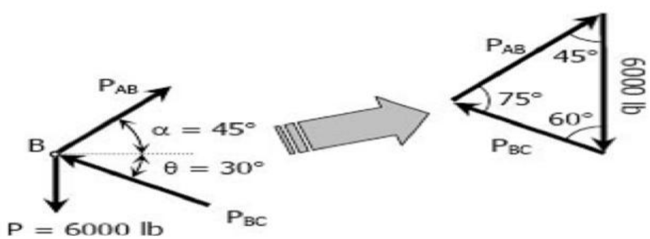
$$\delta_v = 0.192 \text{ inch}$$

$$\delta_v = 0.016 \text{ ft} \longrightarrow \text{vertical displacement of B}$$

Solution to Problem 217 Axial Deformation

Solve Prob. 216 if rod AB is of steel, with E = 29 × 10^6 psi. Assume α = 45° and θ = 30°; all other data remain unchanged.

Solution 217



By Sine Law

$$\frac{P_{AB}}{\sin 60°} = \frac{6000}{\sin 75°}$$

$$P_{AB} = 5379.45 \text{ lb} \text{ (Tension)}$$

$$\frac{P_{BC}}{\sin 45°} = \frac{6000}{\sin 75°}$$

$P_{BC} = 4392.30$ lb (Compression)

$$\delta = \frac{PL}{AE}$$

$$\delta_{AB} = \frac{5379.45(10 \times 12)}{0.6(29 \times 10^6)} == 0.0371 \text{ inch (lengthening)}$$

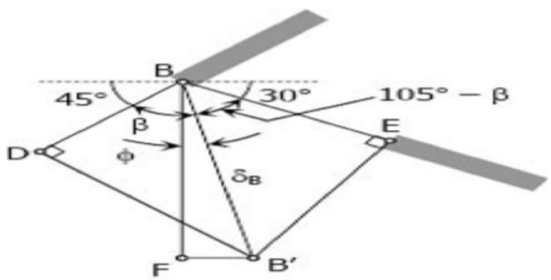
$$\delta_{BC} = \frac{4392.30(6 \times 12)}{0.6(10 \times 10^6)} = 0.0527 \text{ inch (shortening)}$$

$DB = \delta_{AB} = 0.0371$ inch

$BE = \delta_{BE} = 0.0527$ inch

$\delta_B = BB' =$ displacement of B

B' = final position of B after deformation



Movement of B

Triangle BDB':

$$\cos\beta = \frac{0.0371}{\delta_B}$$

$$\delta_B = \frac{0.0371}{\cos\beta}$$

Triangle BEB':

$$\cos(105° - \beta) = \frac{0.0527}{\delta_B}$$

$$\delta_B = \frac{0.0527}{\cos(105° - \beta)}$$

$$\delta_B = \delta_B$$

$$\frac{0.0371}{\cos\beta} = \frac{0.0527}{\cos(105° - \beta)}$$

$$\frac{\cos 105° \ \cos\beta + \sin 105° \ \sin\beta}{\cos\beta} = 1.4205$$

$$-0.2588 + 0.9659 \tan\beta = 1.4205$$
$$\tan\beta = \frac{1.4205 + 0.2588}{0.9659}$$
$$\tan\beta = 1.7386$$
$$\beta = 60.1^\circ$$

$$\delta_B = \frac{0.0371}{\cos 60.1^\circ}$$
$$\delta_B = 0.0744 \text{ inch}$$

$$\phi = (45^\circ + \beta) - 90^\circ$$
$$\phi = (45^\circ + 60.1^\circ) - 90^\circ$$
$$\phi = 15.1^\circ$$

Triangle BFB':
$$\delta_h = FB' = \delta_B \sin\phi = 0.0744 \sin 15.1^\circ$$
$$\delta_h = 0.0194 \text{ inch}$$
$$\delta_h = 0.00162 \text{ ft} \;\longrightarrow \text{horizontal displacement of } B$$

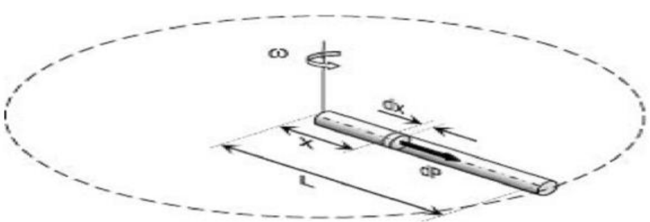
$$\delta_v = BF = \delta_B \cos\phi = 0.0744 \cos 15.1^\circ$$
$$\delta_v = 0.07183 \text{ inch}$$
$$\delta_v = 0.00598 \text{ ft} \;\longrightarrow \text{vertical displacement of } B$$

Solution to Problem 218 Axial Deformation

A uniform slender rod of length L and cross sectional area A is rotating in a horizontal plane about a vertical axis through one end. If the unit mass of the rod is ρ, and it is rotating at a constant angular velocity of ω rad/sec, show that the total elongation of the rod is $\rho\omega^2 L^3/3E$.

Solution 218

$$\delta = \frac{PL}{AE}$$



from the frigure:
$$d\delta = \frac{dP\,x}{AE}$$

Where:
dP = centrifugal force of differential mass
$dP = dM\,\omega^2\,x = (\rho A\,dx)\omega^2\,x$
$dP = \rho A\omega^2\,x\,dx$

$$d\delta = \frac{(\rho A\omega^2 x\,dx)x}{AE}$$
$$\delta = \frac{\rho\omega^2}{E}\int_0^L x^2\,dx = \frac{\rho\omega^2}{E}\left[\frac{x^3}{3}\right]_0^L$$
$$\delta = \frac{\rho\omega^2}{E}\,[\,L^3 - 0^3\,]$$
$$\delta = \rho\omega^2L^3/3E \quad \textbf{ok!}$$

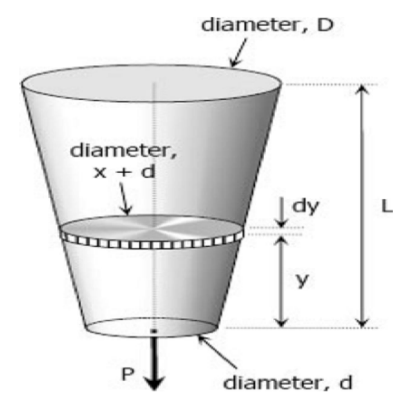
Solution to Problem 219 Axial Deformation

A round bar of length L, which tapers uniformly from a diameter D at one end to a smaller diameter d at the other, is suspended vertically from the large end. If w is the weight per unit volume, find the elongation of ω the rod caused by its own weight. Use this result to determine the elongation of a cone suspended from its base.

Solution 219

$$\delta = \frac{PL}{AE}$$

For the differential strip shown:
$\delta = d\delta$
P = weight carried by the strip = weight of segment y
L = dy
A = area of the strip



For weight of segment y (Frustum of a cone):
$$P = wV_y$$

From section along the axis: