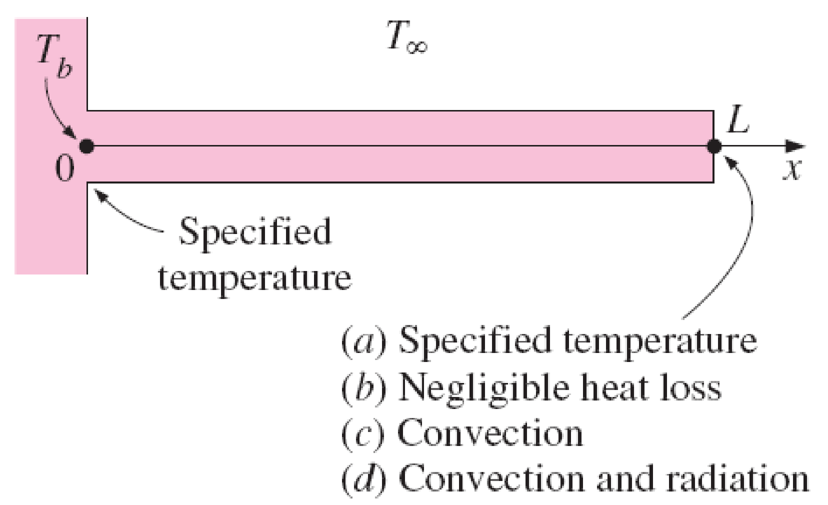
2 Negligible Heat Loss from the Fin Tip (Adiabatic fin tip, $Q_{\text{fin tip}} = 0$)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic assumption is for heat transfer from the fin tip to be negligible since the surface area of the fin tip is usually a negligible fraction of the total fin area.

Boundary condition at fin tip

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$



The variation of temperature along the fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

Heat transfer from the entire fin

$$\dot{Q}_{\text{adiabatic tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL$$

3 Specified Temperature ($T_{fin,tip} = T_L$)

In this case the temperature at the end of the fin (the fin tip) is fixed at a specified temperature T_L.

This case could be considered as a generalization of the case of *Infinitely Long Fin* where the fin tip temperature was fixed at T_∞.

Boundary condition at fin tip: $\theta(L) = \theta_L = T_L - T_\infty$

Specified fin tip temperature:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L-x)}{\sinh mL}$$

Specified fin tip temperature:

$$\dot{Q}_{\text{specified temp.}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$$

4 Convection from Fin Tip

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that may also include the effects of radiation. Consider the case of convection only at the tip. The condition at the fin tip can be obtained from an energy balance at the fin tip.

$(\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}})$

Boundary condition at fin tip: $$-kA_c \frac{dT}{dx}\bigg|_{x=L} = hA_c[T(L) - T_\infty]$$

Convection from fin tip: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x) + (h/mk) \sinh m(L - x)}{\cosh mL + (h/mk) \sinh mL}$$

Convection from fin tip:

$$\dot{Q}_{\text{convection}} = -kA_c \frac{dT}{dx}\bigg|_{x=0}$$

$$= \sqrt{hpkA_c}(T_b - T_\infty)\frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$$

4 Convection from Fin Tip - cont

Replace the *fin length L* in the relation for the *insulated tip (Fig. 39)* by a **corrected length** defined as

$$L_c = L + \frac{A_c}{p}$$

t *is* the thickness of the rectangular fins.
D *is* the diameter of the cylindrical fins.

$$L_{c,\,\text{rectangular fin}} = L + \frac{t}{2}$$

$$L_{c,\,\text{cylindrical fin}} = L + \frac{D}{4}$$

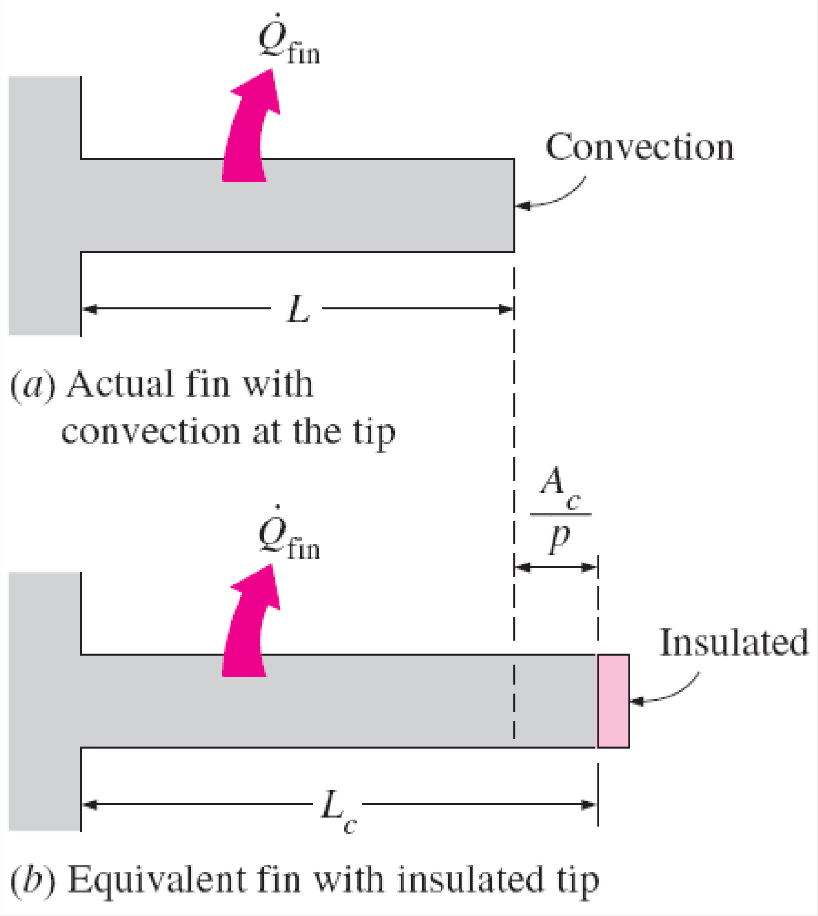


Fig 39: Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

Fin Efficiency

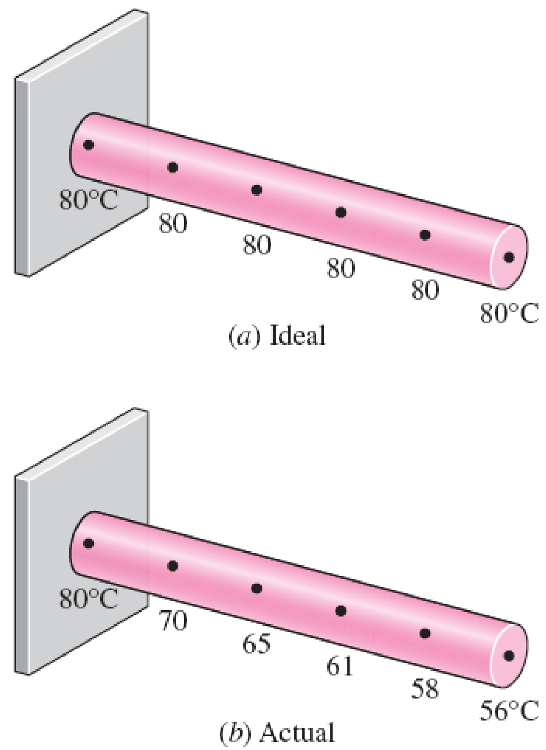


FIGURE 3–42
Ideal and actual temperature distribution along a fin.

Zero thermal resistance or *infinite thermal conductivity* ($T_{\text{fin}} = T_b$)

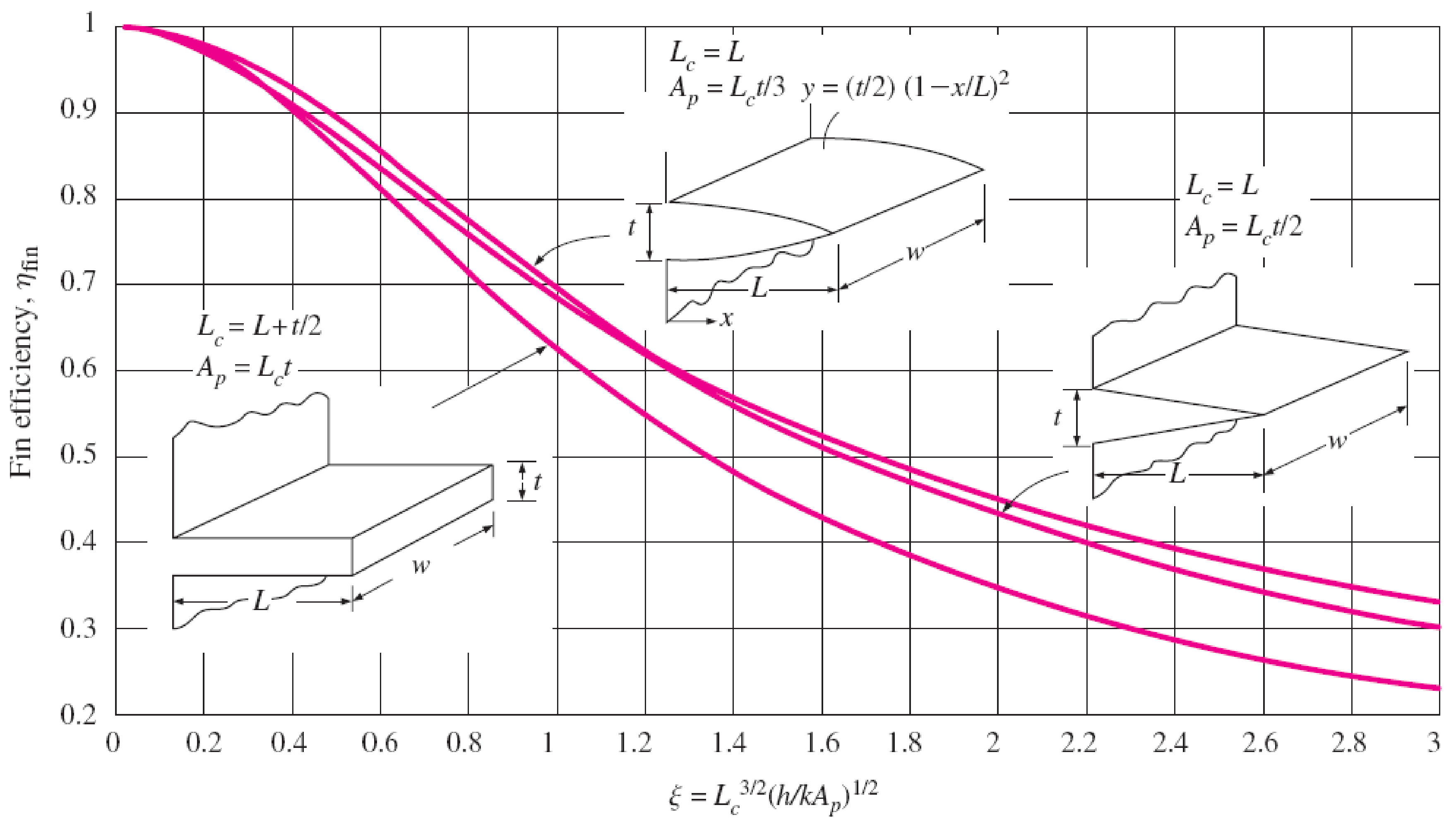
$$\boxed{\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}\,(T_b - T_\infty)}$$

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}}$$

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}}$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$$

$$\eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{\tanh mL}{mL}$$



Efficiency of straight fins of rectangular, triangular, and parabolic profiles.

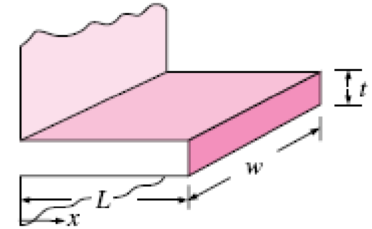
Triangular and parabolic are more efficient than rectangular, contain less material and more suitable for applications requiring less weight and less space

Efficiency and surface areas of common fin configurations

Straight rectangular fins

$$m = \sqrt{2h/kt}$$
$$L_c = L + t/2$$
$$A_{\text{fin}} = 2wL_c$$

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c}$$

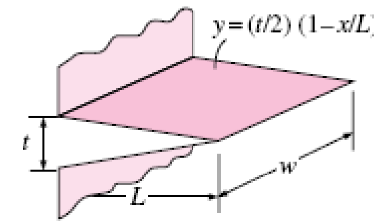


Straight triangular fins

$$m = \sqrt{2h/kt}$$

$$A_{\text{fin}} = 2w\sqrt{L^2 + (t/2)^2}$$

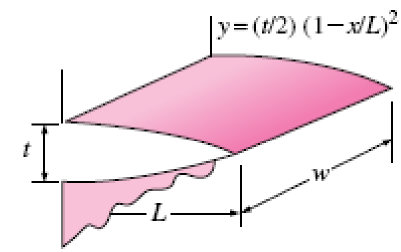
$$\eta_{\text{fin}} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



Straight parabolic fins

$$m = \sqrt{2h/kt}$$
$$A_{\text{fin}} = wL[C_1 + (L/t)\ln(t/L + C_1)]$$
$$C_1 = \sqrt{1 + (t/L)^2}$$

$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$

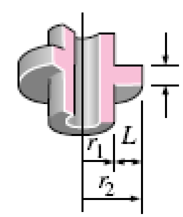


Circular fins of rectangular profile

$$m = \sqrt{2h/kt}$$
$$r_{2c} = r_2 + t/2$$
$$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$$

$$\eta_{\text{fin}} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

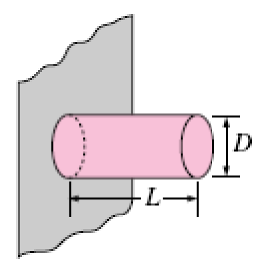
$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



Pin fins of rectangular profile

$$m = \sqrt{4h/kD}$$
$$L_c = L + D/4$$
$$A_{\text{fin}} = \pi DL_c$$

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c}$$

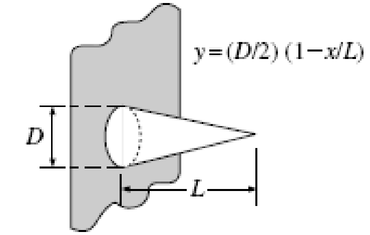


Pin fins of triangular profile

$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$$

$$\eta_{\text{fin}} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$



Pin fins of parabolic profile

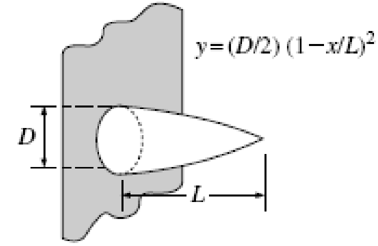
$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi L^3}{8D}\left[C_3C_4 - \frac{L}{2D}\ln(2DC_4/L + C_3)\right]$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = \sqrt{1 + (D/L)^2}$$

$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$

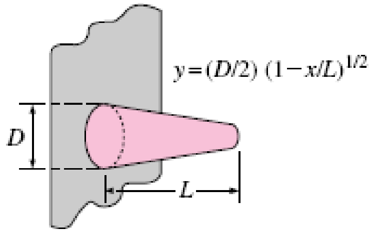


Pin fins of parabolic profile (blunt tip)

$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$



- The fin efficiency decreases with increasing fin length because of decrease in fin temperature with length.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically.
- The efficiency of most fins used in practice is above 90 percent.

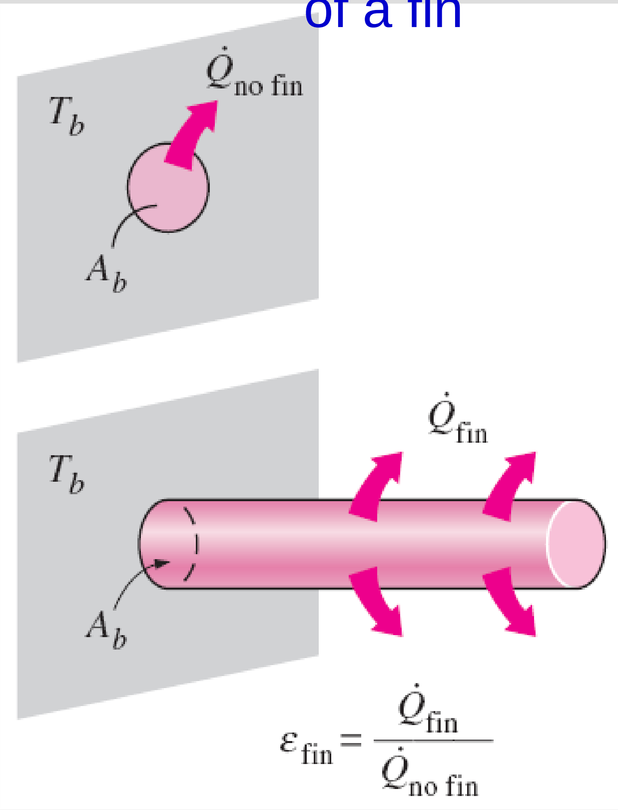
Fin Effectiveness

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

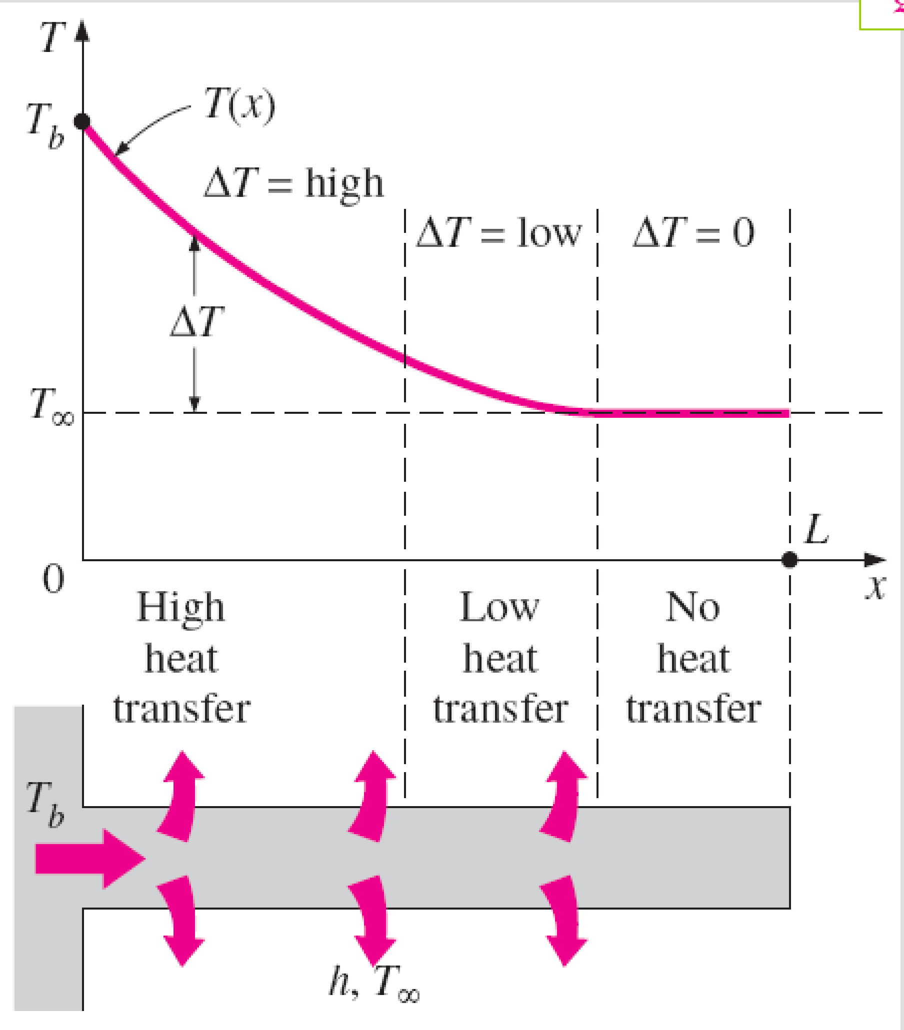
Effectiveness of a fin



- The *thermal conductivity* ***k*** of the fin material should be as high as possible. Use aluminum, copper, iron.
- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible. Use slender or thin pin fins.
- *Low convection heat transfer coefficient* ***h****.* Place fins on the gas (air) side.

Proper Length of a Fin

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh mL$$



Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.

The variation of heat transfer from a fin relative to that from an infinitely long fin

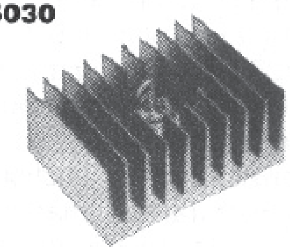
mL	$\dfrac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

mL = 5 $\rightarrow$ an infinitely long fin
mL = 1 offers a good compromise between heat transfer performance and the fin size.

- **Heat sinks:** Specially designed finned surfaces which are commonly used in the cooling of electronic equipment, and involve one-of-a-kind complex geometries.
- The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances R.*
- A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long.

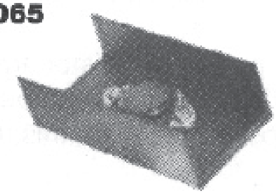
HS 5030



R = 0.9°C/W (vertical)
R = 1.2°C/W (horizontal)

Dimensions: 76 mm × 105 mm × 44 mm
Surface area: 677 cm^2

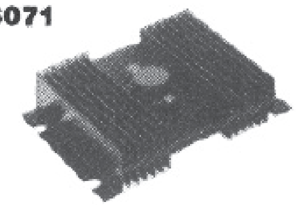
HS 6065



R = 5°C/W

Dimensions: 76 mm × 38 mm × 24 mm
Surface area: 387 cm^2

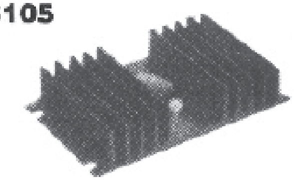
HS 6071



R = 1.4°C/W (vertical)
R = 1.8°C/W (horizontal)

Dimensions: 76 mm × 92 mm × 26 mm
Surface area: 968 cm^2

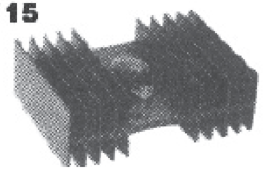
HS 6105



R = 1.8°C/W (vertical)
R = 2.1°C/W (horizontal)

Dimensions: 76 mm × 127 mm × 91 mm
Surface area: 677 cm^2

HS 6115



R = 1.1°C/W (vertical)
R = 1.3°C/W (horizontal)

Dimensions: 76 mm × 102 mm × 25 mm
Surface area: 929 cm^2

EXAMPLE 3–12 **Heat Transfer from Fins of Variable Cross-Section**

Aluminum pin fins of parabolic profile with blunt tips are attached on a plane wall with surface temperature of 200°C (Fig. 3–49). Each fin has a length of 20 mm and a base diameter of 5 mm. The fins are exposed to an ambient air condition of 25°C and the convection heat transfer coefficient is 50 W/m²·K. If the thermal conductivity of the fins is 240 W/m²·K, determine the efficiency, heat transfer rate, and effectiveness of each fin.

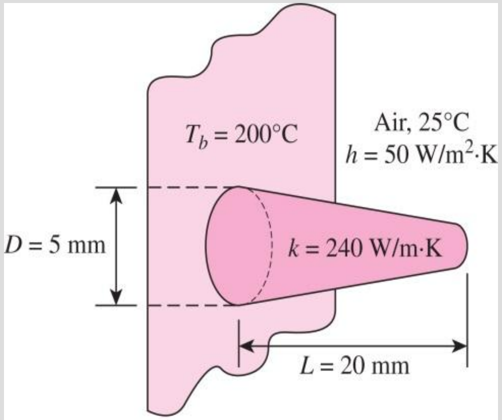


FIGURE 3–49
Schematic for Example 3–12.

SOLUTION The efficiency, heat transfer rate, and effectiveness of a pin fin of parabolic profile with blunt tips are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 240 W/m²·K.

Analysis From Table 3–3, for pin fins of parabolic profile (blunt tip), we have

$$mL = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4(50\,\text{W/m}^2\cdot\text{K})}{(240\,\text{W/m}\cdot\text{K})(0.005\text{ m})}}(0.020\text{m}) = 0.2582$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2}\left\{\left[16\left(\frac{L}{D}\right)^2 + 1\right]^{3/2} - 1\right\} = \frac{\pi\ \ (0.005\text{ m})^4}{96(0.020\text{ m})^2}\left\{\left[16\left(\frac{0.020\text{ m}}{0.005\text{ m}}\right)^2 + 1\right]^{3/2} - 1\right\}$$

$$= 2.106 \times 10^{-4}\text{ m}^2$$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)} = \frac{3}{2(0.2582)}\frac{I_1[4(0.2582)/3]}{I_0[4(0.2582)/3]} = 5.8095\frac{I_1[0.3443]}{I_0[0.3443]}$$

The values of the Bessel functions corresponding to $x = 0.3443$ are determined from Table 3–4 to be $I_0 = 1.0350$ and $I_1 = 0.1716$. Substituting, the fin efficiency is determined to be

$$\eta_{\text{fin}} = 5.8095\frac{0.1716}{1.0350} = \mathbf{0.9632}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty)$$
$$= (0.9632)(50\,\text{W/m}^2\cdot\text{K})(2.106 \times 10^{-4}\text{ m}^2)(200-25)°\text{C} = \mathbf{1.77\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(\pi D^2/4)(T_b - T_\infty)}$$
$$= \frac{1.77\text{ W}}{(50\text{W/m}^2\cdot\text{K})[\pi(0.005\text{ m})^2/4](200 - 25)°\text{C}}$$

EXAMPLE 3–12 Effect of Fins on Heat Transfer from Steam Pipes

Steam in a heating system flows through tubes whose outer diameter is $D_1 = 3$ cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins ($k = 180$ W/m · °C) of outer diameter $D_2 = 6$ cm and constant thickness $t = 2$ mm are attached to the tube, as shown in Fig. 3–48. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25$°C, with a combined heat transfer coefficient of $h = 60$ W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

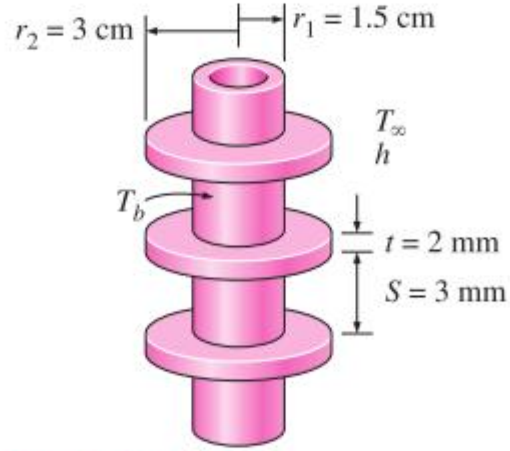


FIGURE 3–48

SOLUTION Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be $k = 180$ W/m · °C.

Analysis In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03 \text{ m})(1 \text{ m}) = 0.0942 \text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty)$$
$$= (60 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.0942 \text{ m}^2)(120 - 25)^\circ\text{C}$$
$$= 537 \text{ W}$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$\frac{r_2 + \frac{1}{2}t}{r_1} = \frac{(0.03 + \frac{1}{2} \times 0.002) \text{ m}}{0.015 \text{ m}} = 2.07$$

$$(L + \tfrac{1}{2}t)\sqrt{\frac{h}{kt}} = (0.015 + \tfrac{1}{2} \times 0.002) \text{ m} \times \sqrt{\frac{60 \text{ W/m}^2 \cdot {}^\circ\text{C}}{(180 \text{ W/m} \cdot {}^\circ\text{C})(0.002}}$$

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$
$$= 2\pi[(0.03 \text{ m})^2 - (0.015 \text{ m})^2] + 2\pi(0.03 \text{ m})(0.002$$
$$= 0.00462 \text{ m}^2$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.95(60 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.00462 \text{ m}^2)(120 - 25)^\circ\text{C}$$
$$= 25.0 \text{ W}$$

Heat transfer from the unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 S = \pi(0.03 \text{ m})(0.003 \text{ m}) = 0.000283 \text{ m}^2$$

$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty)$$
$$= (60 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.000283 \text{ m}^2)(120 - 25)^\circ\text{C}$$
$$= 1.60 \text{ W}$$

Noting that there are 200 fins and thus 200 interfin spacings per m
of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6) \text{ W} = 5320 \text{ V}$$

Therefore, the increase in heat transfer from the tube per meter of i
a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \mathbf{4783 \text{ W}} \qquad \text{(per m tu}$$