Problem

Consider a house that has a 10-m × 20-m base and a 4-m-high wall.

All four walls of the house have an *R* of 2.31 $m^2\ {}^0C/W$. The two 10-m × 4-m walls have no windows. The third wall has five windows made of 0.5-cm thick glass (*k = 0.78 W/m · C), 1.2 m×1.8 m in size. The* fourth wall has the same size and number of windows, but they are double-paned with a 1.5-cm-thick stagnant air space (*k=0.026 W/m·*^{0}*C) enclosed between two 0.5 cm-thick* glass layers.

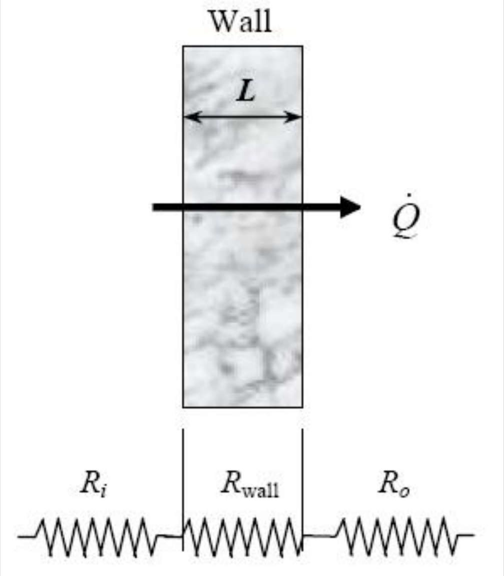
The thermostat in the house is set at 24 ^0C and the average temperature outside at that location is 8 ^0C during the seven-month long heating season. Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 7 and 18 $W/m^2\ {}^0C$, respectively, **determine** the average rate of heat transfer through each wall.

If the house is electrically heated and the price of electricity is $0.08/kWh, determine the amount of money this household will save per heating season by converting the single-pane windows to double-pane windows.

Solution

- ***Assumptions:*** *1 Steady heat transfer , 2 Heat transfer is one-dimensional, 3 Thermal conductivities of the glass and air are constant. 4 Heat transfer by radiation is disregarded.*
- ***Properties:*** *k = 0.026 W/m·°C for air, and 0.78 W/m·°C for glass.*
- ***Analysis:***

Walls without windows:



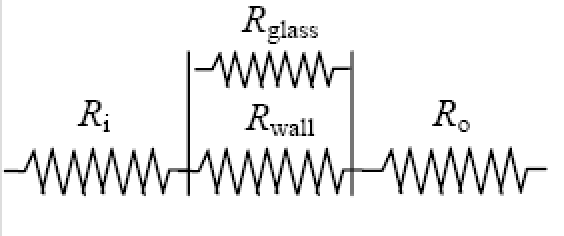
$$R_i = \frac{1}{h_i A} = \frac{1}{7\times(10\times 4)} = 0.003571\ ^0\mathrm{C/W}$$

$$\mathrm{R}_{\mathrm{wall}} = \frac{L_{wall}}{kA} = \frac{L_{wall}/k}{A} = \frac{2.31}{10\times 4} = 0.05775\ ^0\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{18\times(10\times 4)} = 0.001389\ ^0\mathrm{C/W}$$

$$\mathrm{R}_{\mathrm{total}} = R_i + R_{wall} + R_o = 0.003571 + 0.05775 + 0.001389 = 0.06271\ ^0\mathrm{C/W}$$

$$\dot{\mathrm{Q}} = \frac{\mathrm{T}_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{24-8}{0.06271} = 255.1\,\mathrm{W}$$



- **Wall with single pane windows:**

$$R_i = \frac{1}{h_i A} = \frac{1}{7\times(20\times 4)} = 0.001786\ {}^0\mathrm{C/W}$$

$$\mathrm{R}_{\mathrm{wall}} = \frac{L_{wall}}{kA} = \frac{L_{wall}/k}{A} = \frac{2.31}{(20\times 4)-5(12\times 1.8)} = 0.033382\ {}^0\mathrm{C/W}$$

$$R_{glass} = \frac{L_{glass}}{kA} = \frac{0.005}{0.78\times(1.2\times 1.8)} = 0.002968\ {}^0\mathrm{C/W}$$

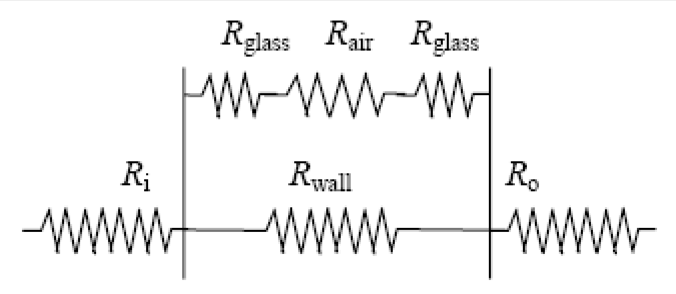
$$\frac{1}{\mathrm{R}_{\mathrm{eq}}} = \frac{1}{R_{wall}} + 5\frac{1}{R_{glass}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \Rightarrow \mathrm{R}_{\mathrm{eq}} = 0.000583\ {}^0\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{18\times(20\times 4)} = 0.000694\ {}^0\mathrm{C/W}$$

$$\mathrm{R}_{\mathrm{total}} = R_i + R_{eq} + R_o = 0.001786 + 0.000583 + 0.000694 = 0.003063\ {}^0\mathrm{C/W}$$

$$\dot{\mathrm{Q}} = \frac{\mathrm{T}_{\infty 1} - \mathrm{T}_{\infty 2}}{\mathrm{R}_{\mathrm{total}}} = \frac{24-8}{0.003063} = 5224\ \mathrm{W}$$

- **4th wall with double pane windows:**



$$R_i = \frac{1}{h_i A} = \frac{1}{7 \times (20 \times 4)} = 0.001786\ ^0\text{C/W}$$

$$\text{R}_{\text{wall}} = \frac{L_{wall}}{kA} = \frac{L_{wall}/k}{A} = \frac{2.31}{(20 \times 4) - 5(12 \times 1.8)} = 0.033382\ ^0\text{C/W}$$

$$R_{glass} = \frac{L_{glass}}{kA} = \frac{0.005}{0.78 \times (1.2 \times 1.8)} = 0.002968\ ^0\text{C/W}$$

$$\text{R}_{\text{air}} = \frac{L_{air}}{k_{air} A} = \frac{0.015}{0.026(1.2 \times 1.8)} = 0.267094\ ^0\text{C/W}$$

$$\text{R}_{\text{window}} = 2R_{glass} + R_{air} = 2 \times 0.002968 + 0.267094 = 0.27303\ ^0\text{C/W}$$

$$\frac{1}{\text{R}_{\text{eq}}} = \frac{1}{R_{wall}} + 5\frac{1}{R_{glass}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \Rightarrow \text{R}_{\text{eq}} = 0.020717\ ^0\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{18\times(20\times 4)} = 0.000694 \ \ {}^0\mathrm{C/W}$$

$$\mathrm{R_{total}} = R_i + R_{eq} + R_o = 0.001786 + 0.020717 + 0.000694 = 0.023197 \ {}^0\mathrm{C/W}$$

$$\dot{\mathrm{Q}} = \frac{\mathrm{T_{\propto 1} - T_{\propto 2}}}{\mathrm{R_{total}}} = \frac{24-8}{0.023197} = 690 \ \mathrm{W}$$

- **The rate of heat transfer saved if the single pane windows are converted to double pane windows**

$$\dot{Q}_{save} = \dot{Q}_{\mathrm{singlepane}} - \dot{Q}_{\mathrm{doublepane}} = 5224 - 690 = 4534 \ \mathrm{W}$$

- **The amount of energy saved:**

$$\dot{Q}_{save} = \dot{Q}_{save}\Delta t = \left(\frac{4534}{1000}\mathrm{kW}\right)\times(7\times 30\times 24 \ \mathrm{hr}) = 22851 \ \mathrm{kWhr}$$

- **Money saved:**

$$\mathrm{Money\ savings} = \mathrm{Energy\ \ saved} \times \mathrm{unit\ cost\ of\ energy}$$
$$= 22851\times 0.08 = \$1818$$

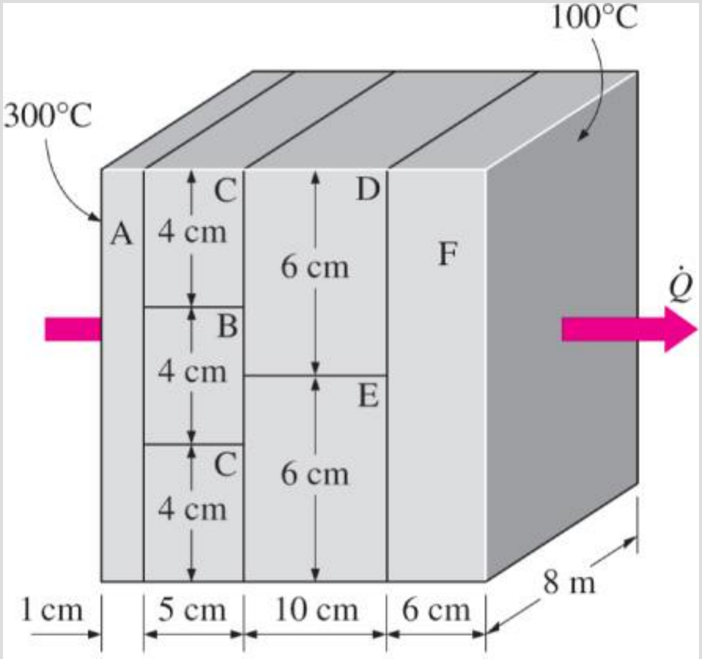
Problem

Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure.

The thermal conductivities of various materials used, in W/m·^{0}C, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, *and* $k_E = 35$. *The* left and right surfaces of the wall are maintained at uniform temperatures of 300^{0}C and 100^{0}C, respectively.

Assuming heat transfer through the wall to be one-dimensional, determine (*a) the rate of heat transfer through the wall; (b) the temperature* at the point where the sections *B, D, and E meet; and* (*c) the temperature drop across the section F.*

Disregard any contact resistances at the interfaces.

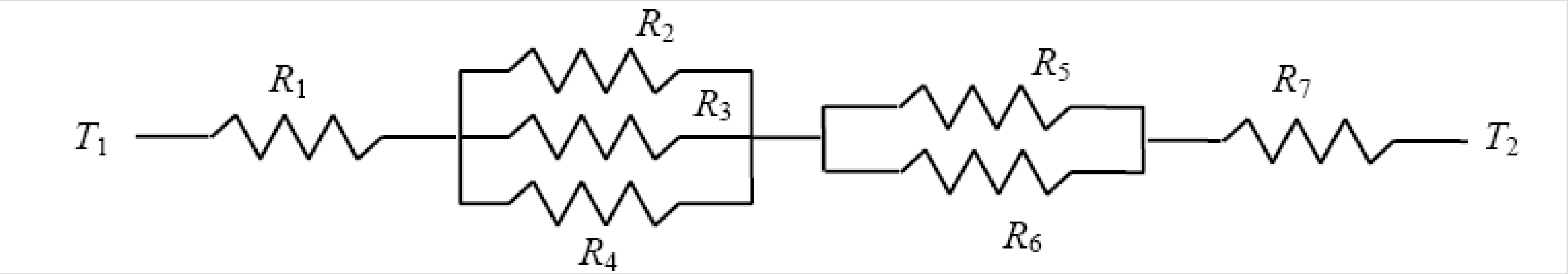


Solution

- ***Assumptions:*** *1. steady state, 2. one-dimensional, 3. Thermal conductivities are constant. 4 Thermal contact resistances at the interfaces are disregarded.*
- ***Properties:*** $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, $k_E = 35$ *W/m·°C.*
- ***Analysis :***

(a) The representative surface area is A=012×1=012 m² .

(b) The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01}{2\times 0.12} = 0.04\ ^0\text{C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05}{20\times 0.04} = 0.06\ ^0\text{C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05}{8\times 0.04} = 0.16\ ^0\text{C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1}{15\times 0.06} = 0.11\ {}^0\mathrm{C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1}{35\times 0.06} = 0.05\ {}^0\mathrm{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06}{2\times 0.12} = 0.25\ {}^0\mathrm{C/W}$$

$$\frac{1}{R_{equ,\ mid1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \Rightarrow R_{equ,\ mid1} = 0.025^0\,\mathrm{C/W}$$

$$\frac{1}{R_{equ,\ mid2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \Rightarrow R_{equ,\ mid2} = 0.034^0\,\mathrm{C/W}$$

$$R_{total} = R_1 + R_{equ,\ mid1} + R_{equ,\ mid2} + R_7 = 0.04 + 0.025 + 0.034 + 025 = 0.349^0\,\mathrm{C/W}$$

- The heat transfer rate through the wall per section of area, A=0.12 m²

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{300 - 100}{0.349} = 572 \text{ W}$$

- The steady rate of heat transfer through entire wall

$$\dot{Q}_{total} = 572 \frac{5 \times 8}{0.12} = 1.91 \times 10^5 \text{ W}$$

- Total thermal resistance between left surface and the point where the sections B,D and E meet

$$R_{total,1} = R_1 + R_{equ,mid1} = 0.04 + 0.025 = 0.065 \ {}^0\text{C/W}$$

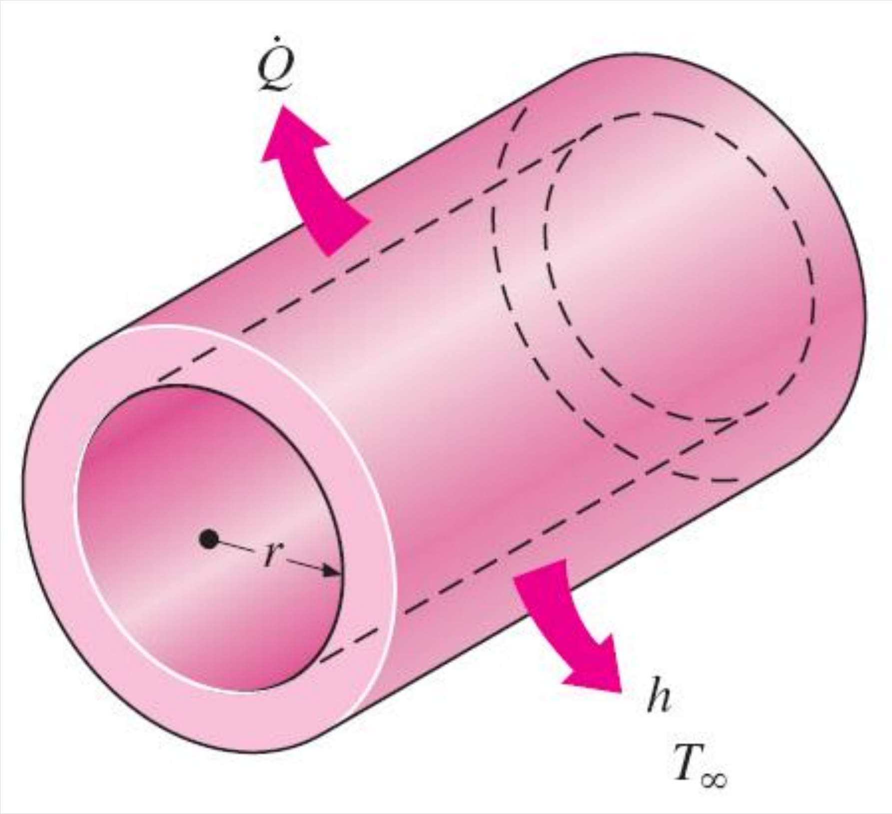
Then the temperature at the point B, D and F meet

$$\dot{Q} = \frac{T_1 - T}{R_{total,1}} \Rightarrow T = T_1 - \dot{Q} \times R_{total,1} = 300 - 572 \times 0.065 = 263 \ {}^0\text{C}$$

- **The temperature drop across the section F is**

$$\dot{Q} = \frac{\Delta T}{R_7} \Rightarrow \Delta T = \dot{Q} \times R_7 = 572 \times 0.25 = 143\ ^0C$$

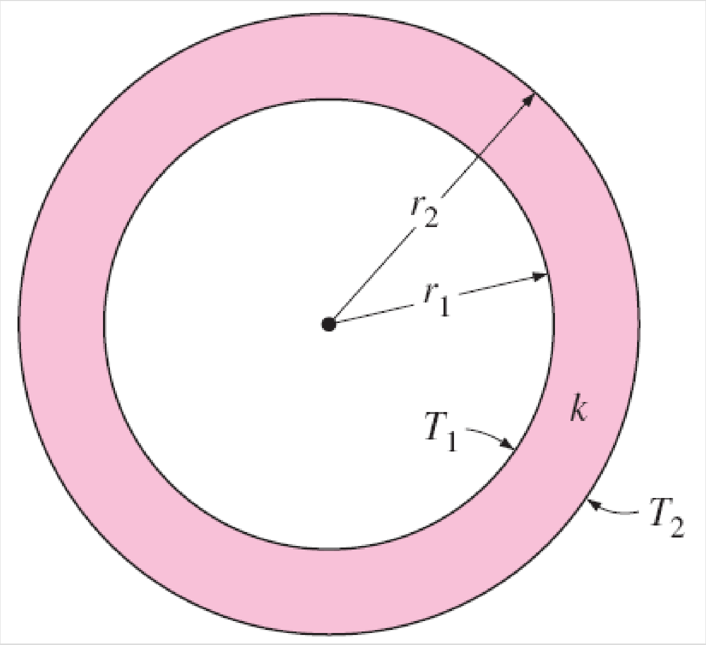
HEAT CONDUCTION IN CYLINDERS AND SPHERES



Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

Heat transfer through the pipe can be modeled as *steady* and *one-dimensional.*

The temperature of the pipe depends on one direction only (the radial *r*-direction) and can be expressed as $T = T(r)$.



A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

$$A = 2\pi rL$$

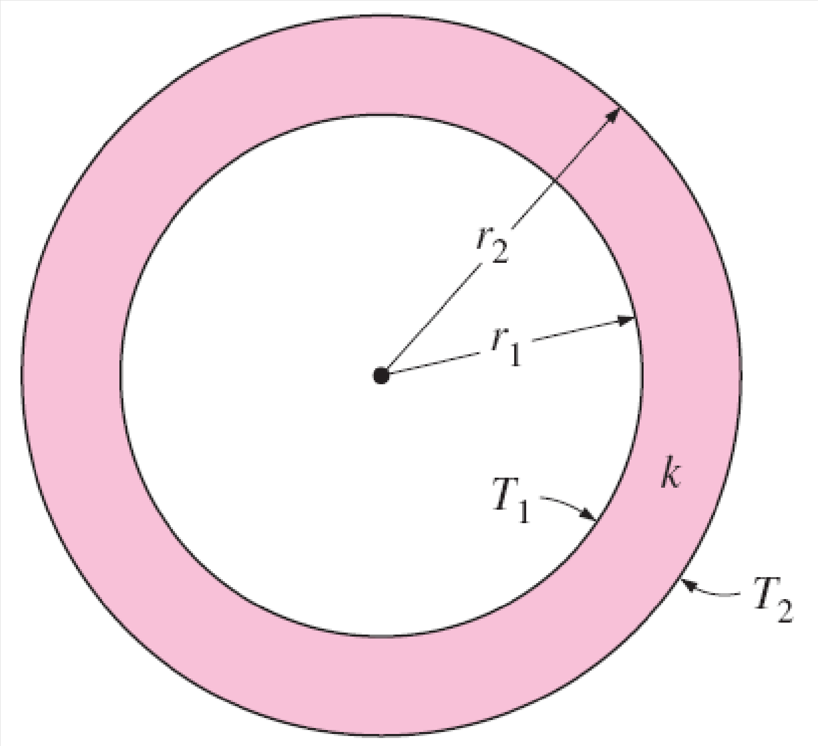
$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)}$$

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)}$$

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

is *the conduction resistance* of the cylinder layer.

SPHERES

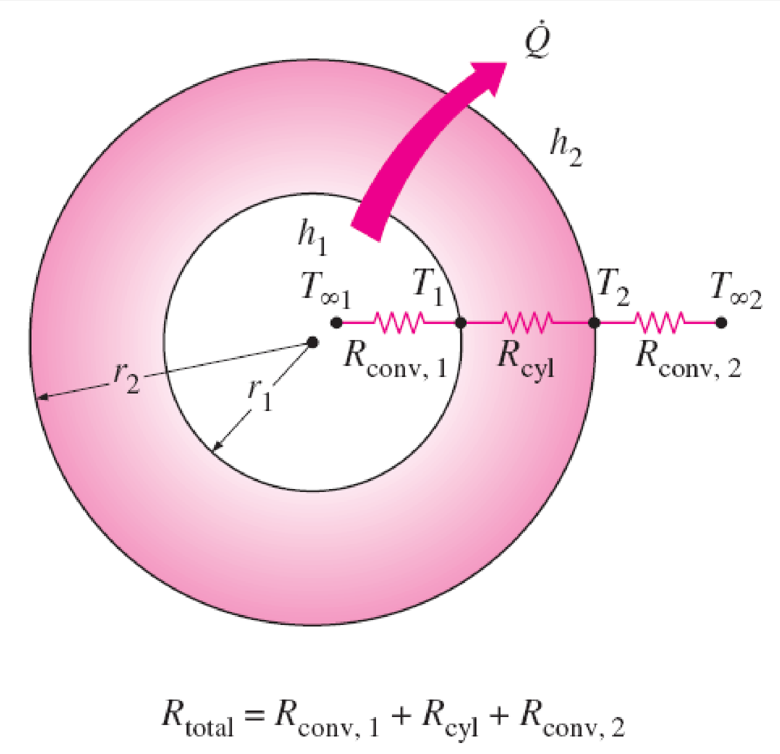


A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

is the conduction resistance of the spherical layer.



The thermal resistance network for a cylindrical (or spherical) in case of convection from in and out sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

where

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}} \\ &= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2} \end{aligned}$$

for a *cylindrical* layer, and

for a *spherical* layer

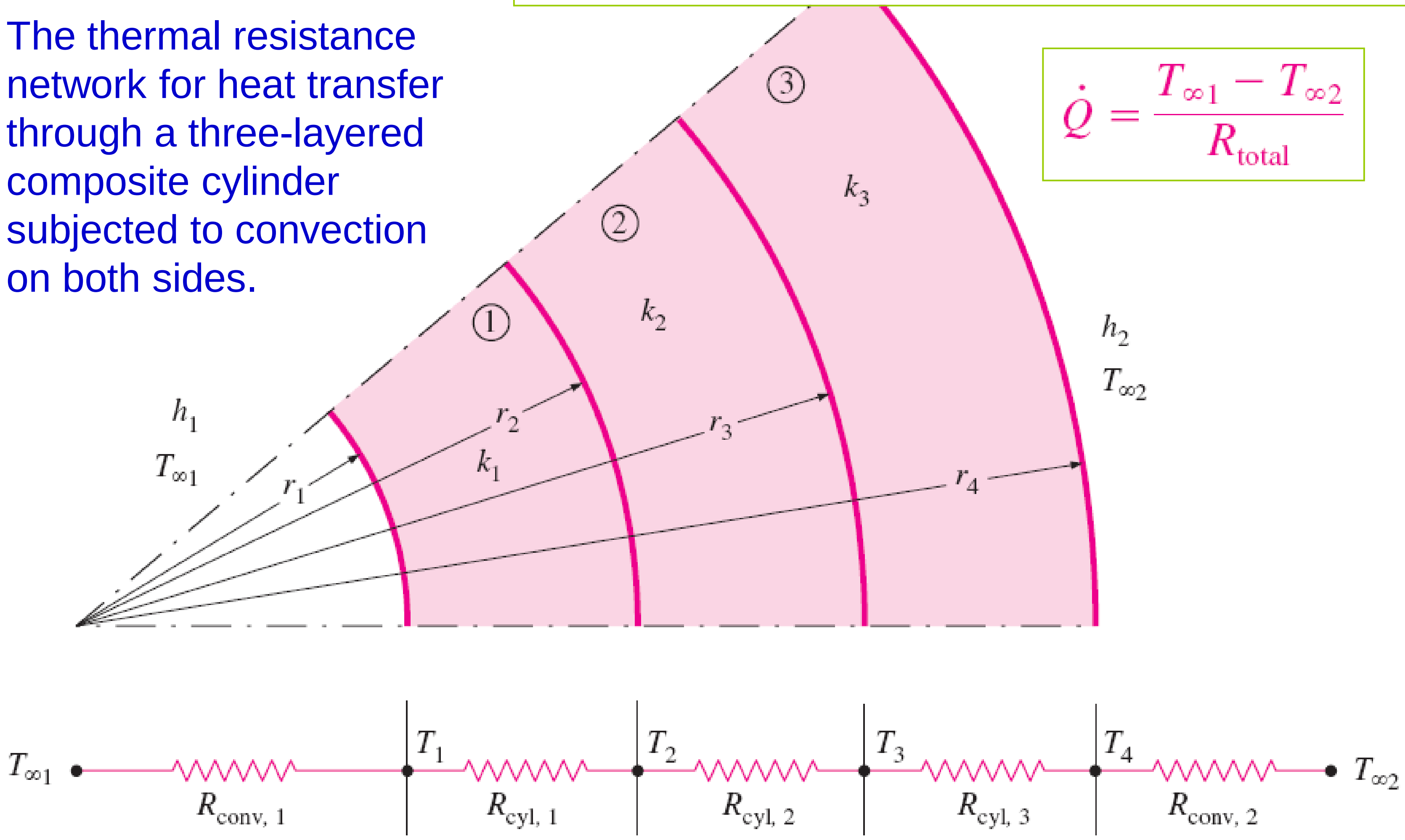
$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{sph}} + R_{\text{conv, 2}} \\ &= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \end{aligned}$$

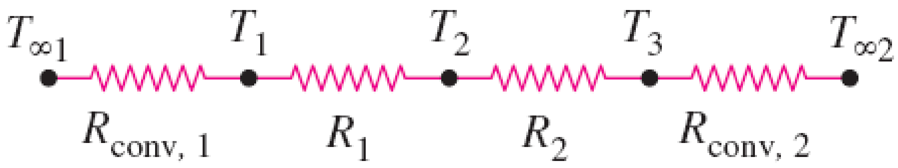
Multilayered Cylinders and Spheres

$$
\begin{aligned}
R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{cyl, 1}} + R_{\text{cyl, 2}} + R_{\text{cyl, 3}} + R_{\text{conv, 2}} \\
&= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}
\end{aligned}
$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.





$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$$

$$= \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$$

$$= \frac{T_1 - T_3}{R_1 + R_2}$$

$$= \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}}$$

$$= \cdots$$

Once heat transfer rate Q has been calculated, the interface temperature T_2 can be determined from any of the following two relations:

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{cyl, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv, 2}}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$