Course Outcome 2 (CO2)

Students should be able to understand and evaluate one-dimensional heat flow and in different geometries

Lesson Outcomes from CO2 (Part 2)

- To **derive** the equation for temperature distribution in various geometries
- Thermal Resistance concept – to **derive** expression for various geometries
- To **evaluate** the heat transfer using thermal resistance in various geometries
- To **evaluate** the critical radius of insulation
- To **evaluate** heat transfer from the rectangular fins

STEADY HEAT CONDUCTION IN PLANE WALLS

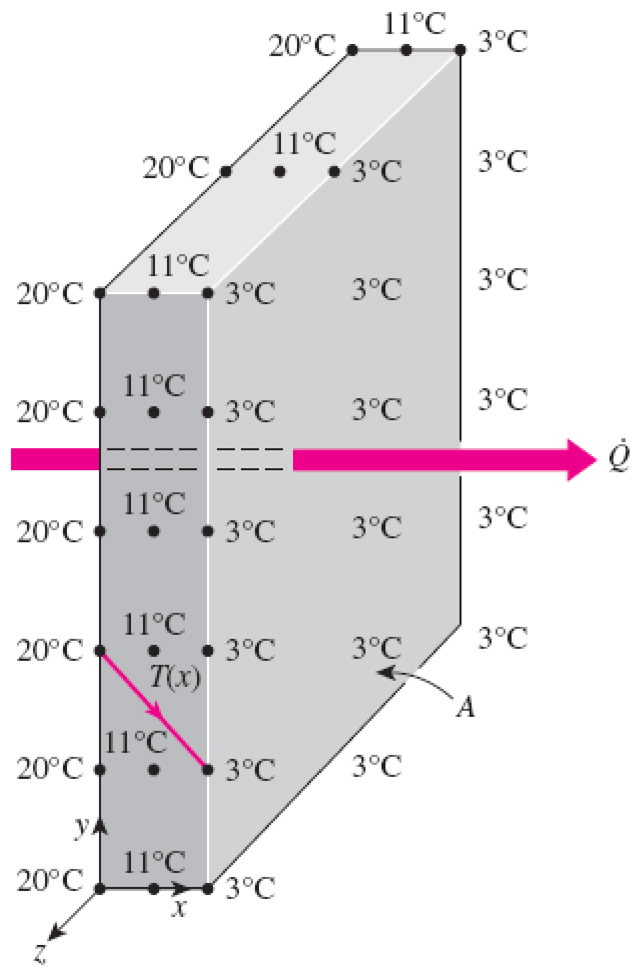


FIGURE 3–1
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

Heat transfer through the wall can be modeled as *steady* and *one-dimensional.*

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt}$$

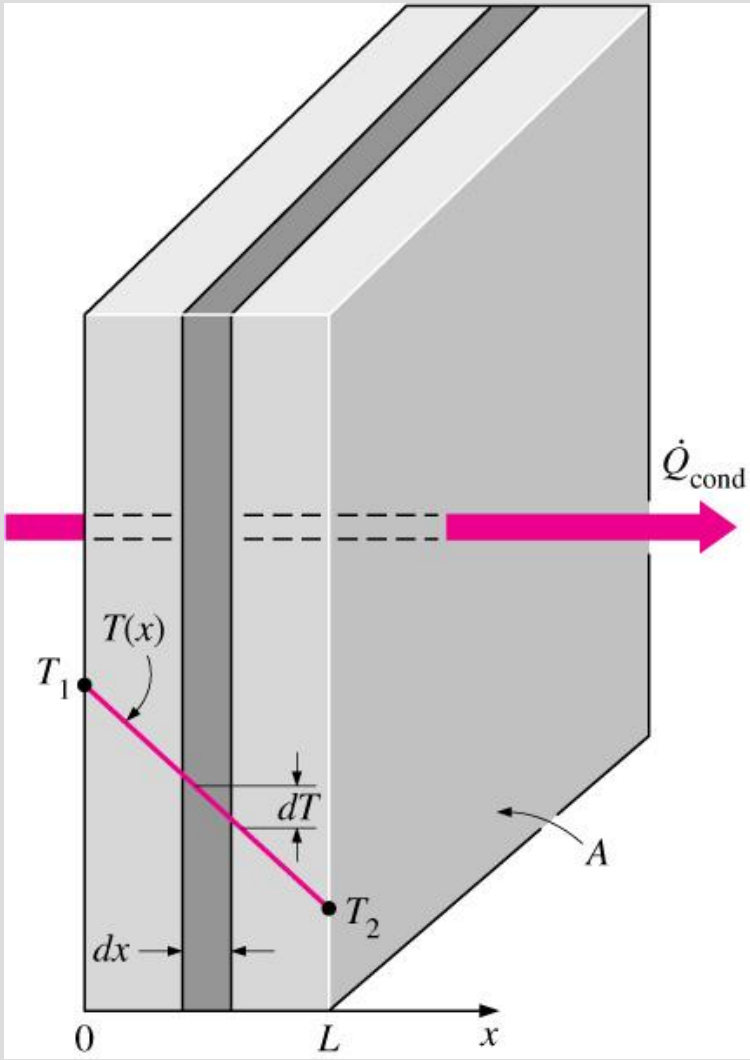
$$dE_{\text{wall}}/dt = 0$$

for *steady* operation

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Fourier’s law of heat conduction



Under steady conditions, the temperature distribution in a plane wall is a straight line: ***dT*/*dx* = const**.

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

Once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 by T, and L by x.

Concept of Thermal Resistance

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L}$$

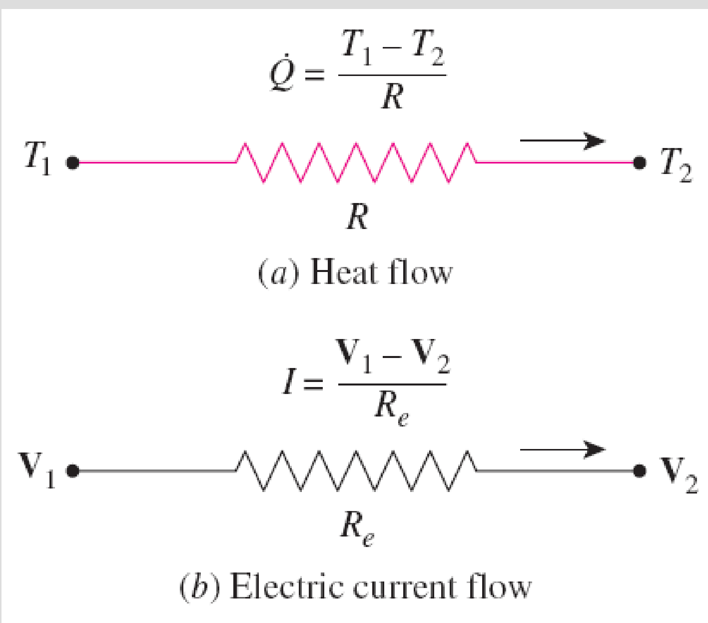
Heat conduction through wall

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

Increasing of R value will decrease the Q value and vice versa

$$R_{\text{wall}} = \frac{L}{kA} \qquad (^\circ\text{C/W})$$

Conduction resistance of the wall: *Thermal resistance* of the wall against heat conduction.



Ohm's Law $V = IR$

Voltage = Current x Resistance

rate of heat transfer $\rightarrow$ electric current
thermal resistance $\rightarrow$ electrical resistance
temperature difference $\rightarrow$ voltage difference

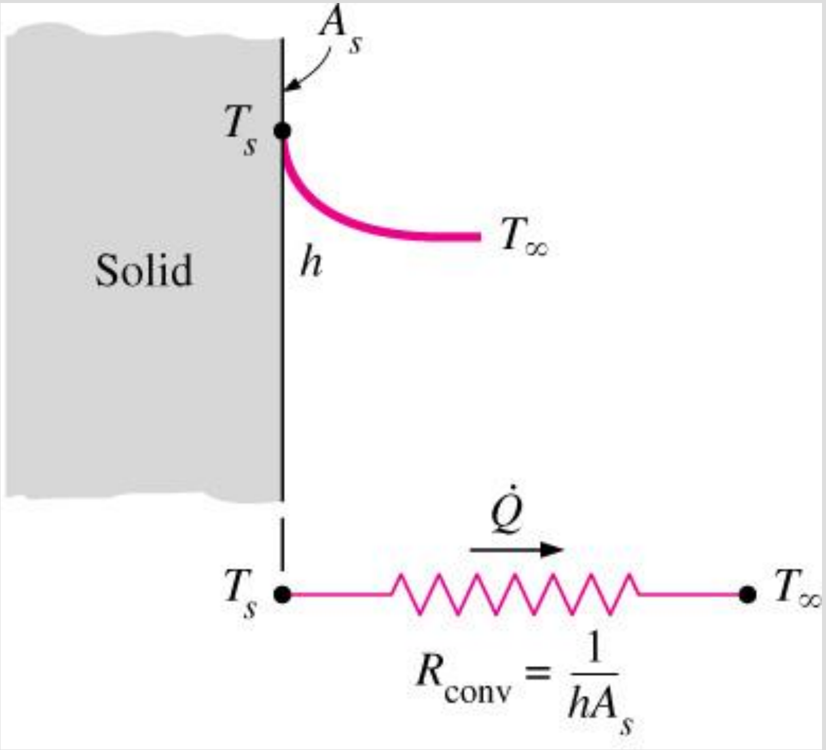
Newton's law of cooling for convection

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \qquad \text{(W)}$$

$$R_{\text{conv}} = \frac{1}{hA_s} \qquad (^\circ\text{C/W})$$

Convection resistance of the surface: *Thermal resistance* of the surface against heat convection.



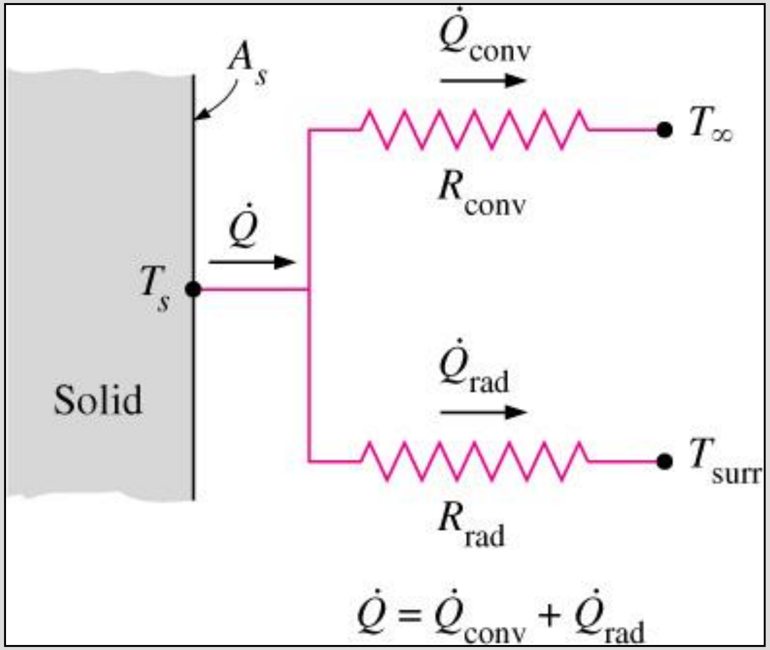
Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T$.

That is, the surface offers *no resistance to convection*.

Combined Heat Transfer Coefficient

- A surface exposed to the surrounding air might involves convection and radiation simultaneously.
- Total heat transfer at the surface is determined by adding (subtracting if opposite direction) the radiation and convection components
- The convection and radiation resistances are parallel to each other.
- When $T_{surr} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by $h_{combined} = h_{conv} + h_{rad}$



Schematic for convection and radiation resistances at a surface.

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s(T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \qquad (\text{K/W})$$

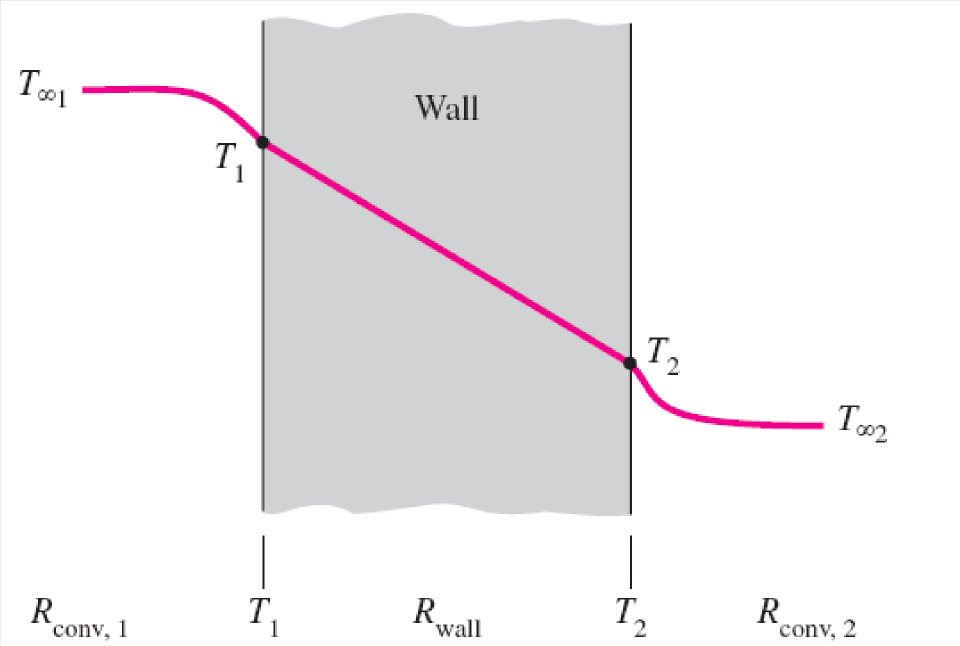
Radiation resistance of the surface: *Thermal resistance* of a surface against radiation.

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \frac{\varepsilon\sigma A_s\left(T_s^4 - T_{surr}^4\right)}{A_s\left(T_s - T_{surr}\right)} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)$$

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \qquad (\text{W/m}^2 \cdot \text{K})$$

Radiation heat transfer coefficient

Consider steady one-dimensional heat transfer through a plane wall that is exposed to convection on both sides.



$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

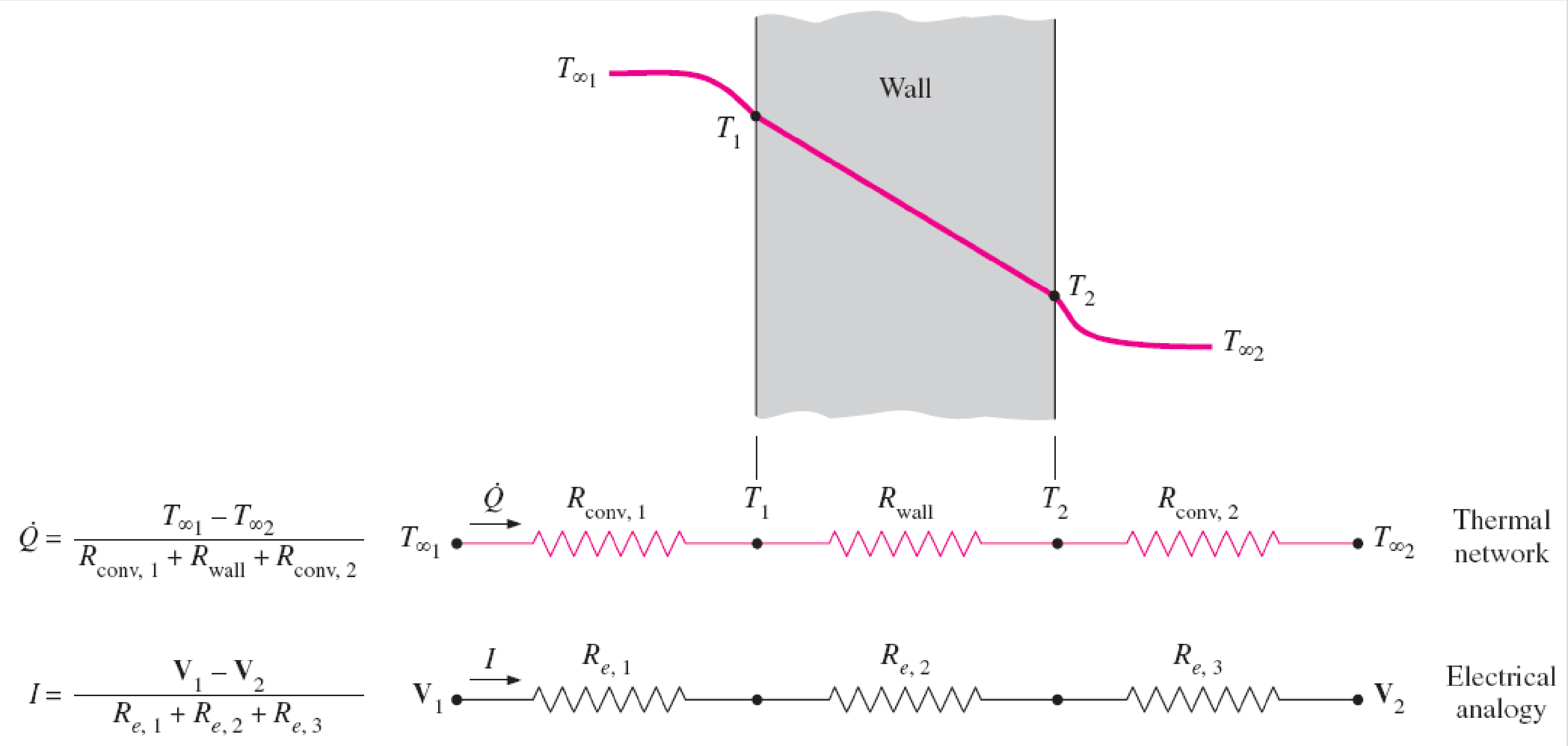
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

$\Rightarrow$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (^\circ\text{C/W})$$



The thermal resistance network in electrical analogy.

Temperature drop

The temperature drop is proportional to thermal resistance of the layer

from $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

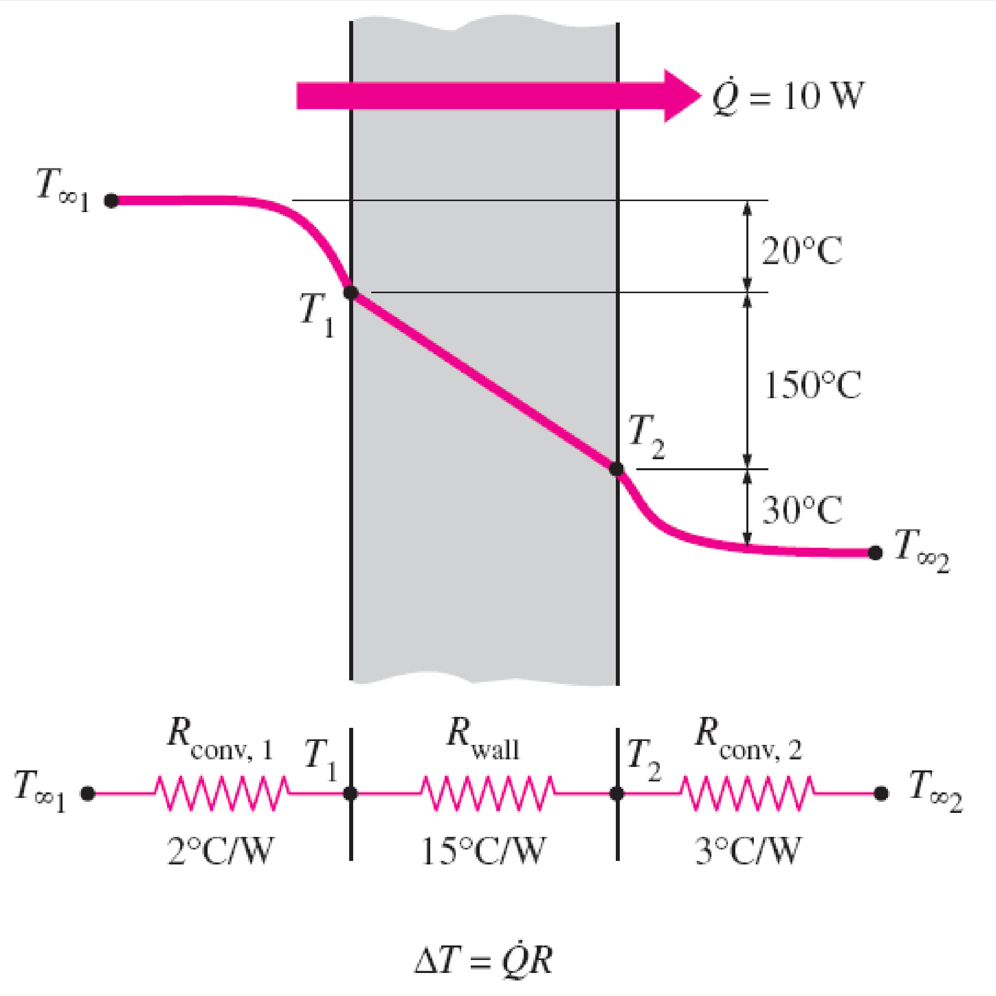
rearrange to

$$\Delta T = \dot{Q}R \qquad (°C)$$

$$\dot{Q} = UA\,\Delta T \qquad (W)$$

$$UA = \frac{1}{R_{total}} \qquad (°C/K)$$

U is the overall heat transfer coefficient



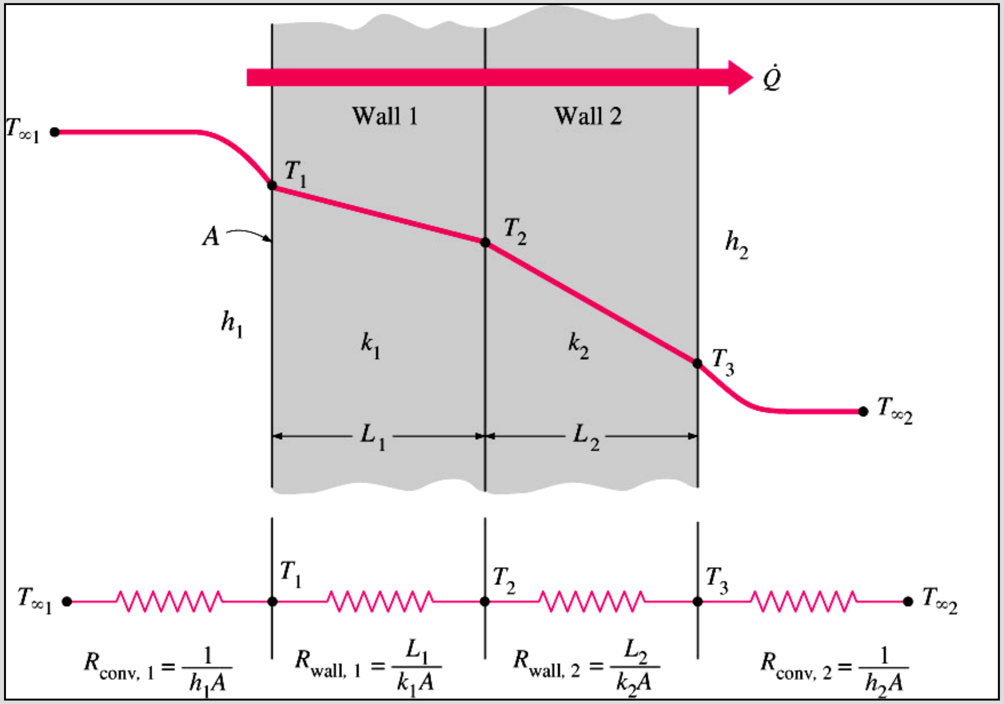
The temperature drop across a layer is proportional to its thermal resistance.

Multilayer Plane Walls

- Often walls are made of several layers of different materials. The thermal resistance concept can still be used for these composite walls.
- This is done by developing a total thermal resistance for the wall.
- The rate of steady heat transfer through this two-layer composite wall can be expressed by:

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

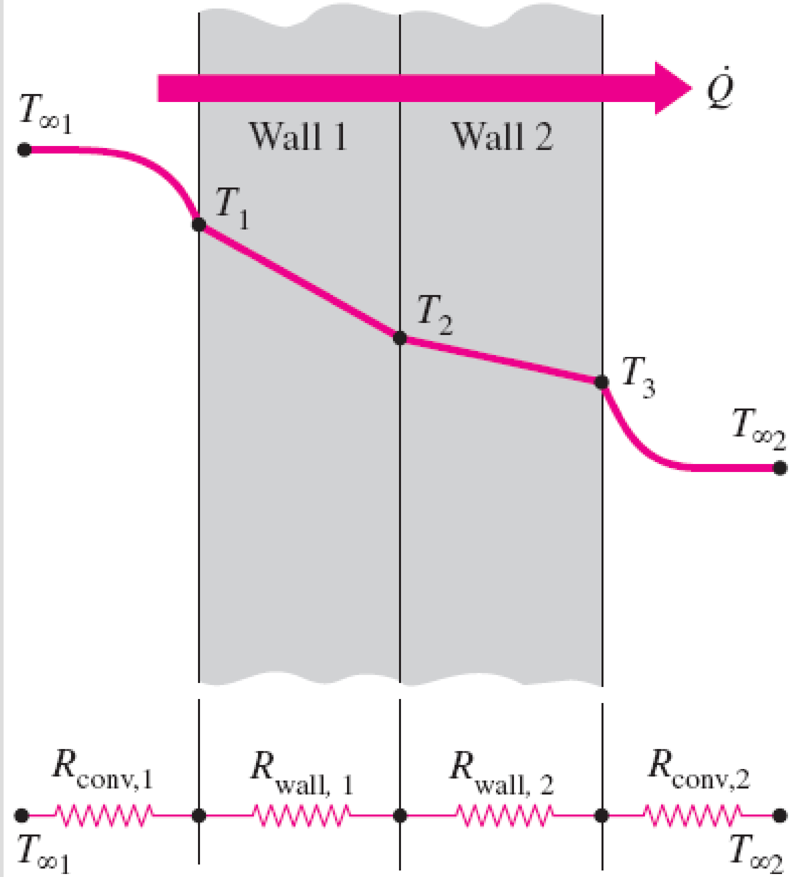
$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}}$$



$$R_{\text{total}} = R_{\text{conv},\,1} + R_{\text{wall},\,1} + R_{\text{wall},\,2} + R_{\text{conv},\,2}$$
$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

Example of heat transfer to wall 1

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{wall, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv,1}}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv,1}} + R_{\text{wall, 1}}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv,2}}}$

EXAMPLE 3–3 Heat Loss through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 10-mm-wide stagnant air space ($k = 0.026$ W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m²·K and $h_2 = 40$ W/m²·K, which includes the effects of radiation.

SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem involves two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 3–13. Noting that the area of the window is again $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00427°\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.3205°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

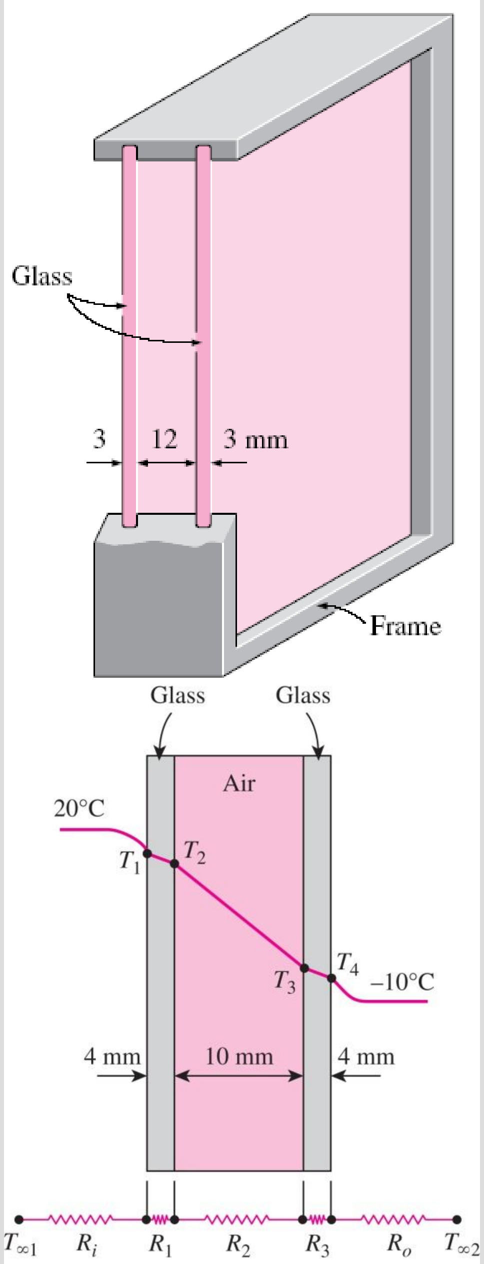


FIGURE 3–13
Schematic for Example 3–3.

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass, 1}} + R_{\text{air}} + R_{\text{glass, 2}} + R_{\text{conv, 2}} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332°\text{C/ W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.4332°\text{C/ W}} = \mathbf{69.2\ W}$$

which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv, 1}} = 20°\text{C} - (69.2\ \text{W})(0.08333°\text{C/ W}) = \mathbf{14.2°C}$$

which is considerably higher than the −2.2°C obtained in the previous example. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce the heat gain in summer, and thus reduce the air-conditioning costs.

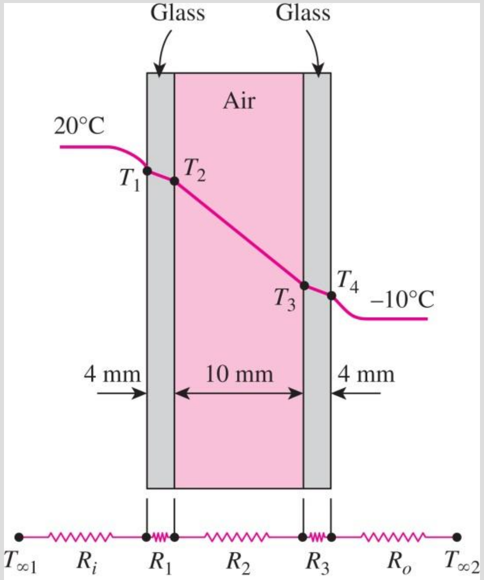
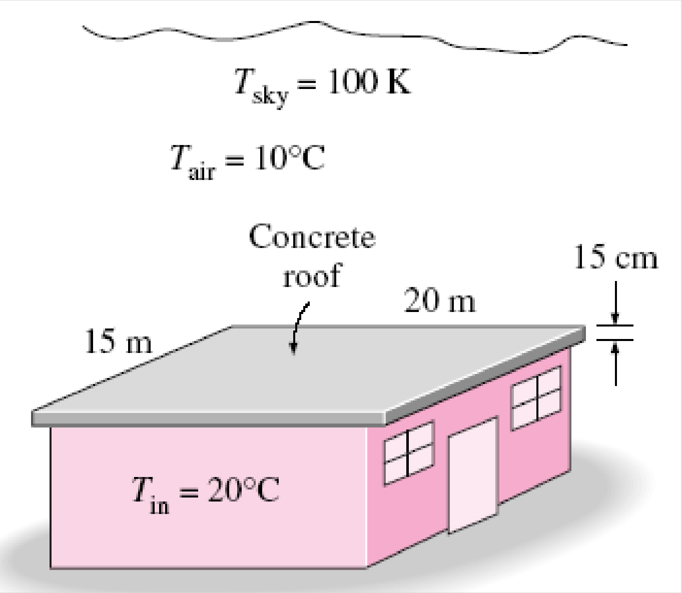


FIGURE 3–13
Schematic for Example 3–3.

Problem

The roof of a house consists of a 15-cm-thick concrete slab ($k = 2$ W/m·^{0}C) that is 15 m wide and 20 m long. The convection heat transfer coefficients on the inner and outer surfaces of the roof are 5 and 12 W/m^2 ^{0}C, respectively.

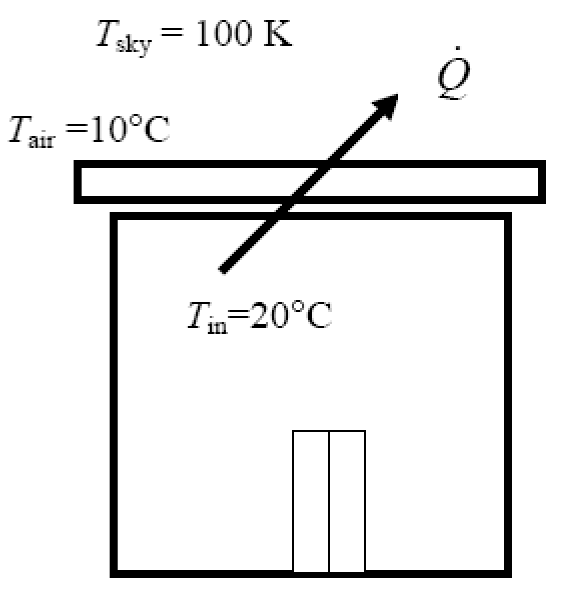
On a clear winter night, the ambient air is reported to be at 10 ^{0}C, while the night sky temperature is 100 K. The house and the interior surfaces of the wall are maintained at a constant temperature of 20 ^{0}C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfers, **determine** the rate of heat transfer through the roof, and the inner surface temperature of the roof.



If the house is heated by a furnace burning natural gas with an efficiency of 80 percent, and the price of natural gas is \$1.20/therm (1therm=105,500 kJ of energy content), **determine** the money lost through the roof that night during a 14 hours period.

Assumptions: *1. Steady operating conditions exist, 2 The emissivity and thermal conductivity of the roof are constant.*

Properties: *The thermal conductivity of the concrete is k = 2 W/m·°C. The emissivity of both surfaces of the roof is 0.9.*



In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), that must be equal to the heat transfer through the roof by conduction.

$$\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surrounding, conv+rad}}$$

- Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ *and* $T_{s,out}$*, respectively*

$$\dot{Q}_{\text{room to roof, conv+rad}} = h_i A\left(T_{room} - T_{s,in}\right) + \varepsilon\sigma A(T_{\text{room}}^4 - T_{\text{s,in}}^4)$$

$$= 5 \times 300\left(20 - T_{s,in}\right) + 0.9 \times 567 \times 10^{-8} \times 300\left[\left(20 + 273\right)^4 - \left(T_{s,in} + 273\right)^4\right]$$

$$\dot{Q}_{\text{roof, cond}} = kA\frac{T_{\text{s, in}} - T_{\text{s, out}}}{\text{L}} = 2 \times 300\left(\frac{T_{\text{s, in}} - T_{\text{s, out}}}{0.15}\right)$$

$$\dot{Q}_{\text{roof to surr., conv+rad}} = h_o A\left(T_{s,out} - T_{surr}\right) + \varepsilon\sigma A(T_{\text{s,out}}^4 - T_{\propto}^4)$$

$$= 12 \times 300\left(T_{s,out} - 10\right) + 0.9 \times 567 \times 10^{-8} \times 300\left[\left(T_{s,out} + 273\right)^4 - 100^4\right]$$

Solving the equations above simultaneously gives

$$\dot{Q} = 37440\ \mathrm{W},\ \mathrm{T}_{s,in} = 7.3\ {}^{0}\mathrm{C},\ \mathrm{T}_{s,out} = -2.1\ {}^{0}\mathrm{C}$$

- The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{\dot{Q}\Delta t}{0.80} = \frac{(37.440\mathrm{kJ})\times(14\times 60\times 60\mathrm{s})}{0.80}\left(\frac{1\ \mathrm{therms}}{105500\ \mathrm{kJ}}\right) = 22.36\ \mathrm{therms}$$

- The money lost through the roof

$$\mathrm{Money\ lost} = 22.36\ \mathrm{therms} \times \$1.20/\mathrm{therms} = \$26.8$$