Course Outcome 1 (CO1)

Students should be able to understand and analyse the concept of conduction, convection and radiation heat transfer.

Lesson Outcomes from CO1 (Part 2)

- To **explain** the mechanism of various modes and equations for the estimation of heat transfer
- To **derive** the generalize 3-dimensional heat conduction equation
- To **develop** the one-dimensional heat conduction form from the generalize 3-dimensional equation for various geometries

INTRODUCTION

- Heat transfer has direction as well as magnitude. It is a *vector* quantity.
- The driving force for any form of heat transfer is the *temperature difference.*

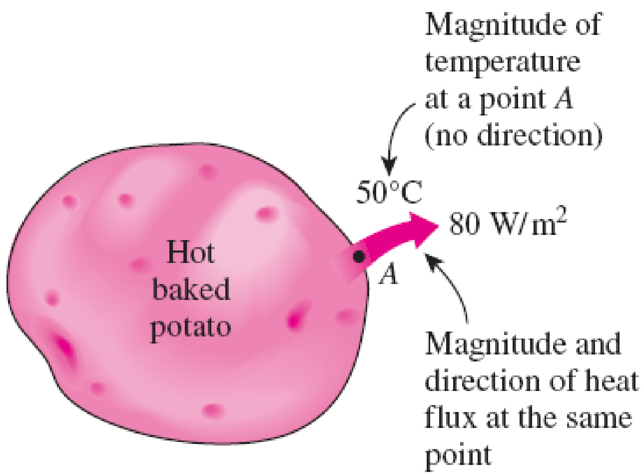


FIGURE 2–1
Heat transfer has direction as well as magnitude, and thus it is a *vector* quantity.

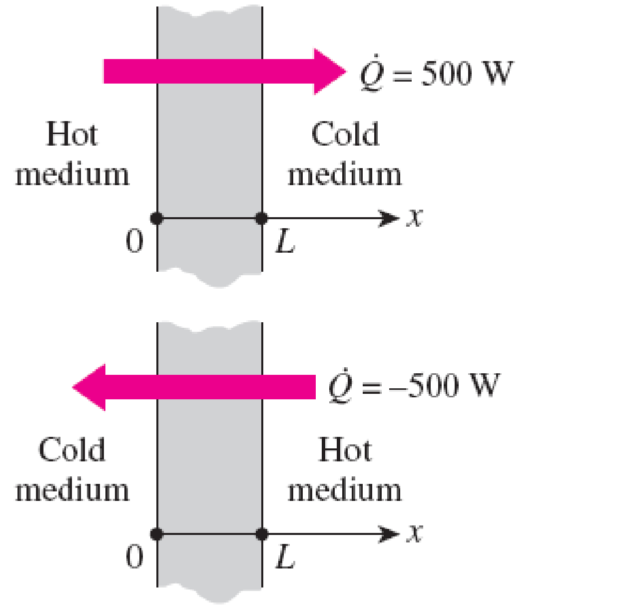
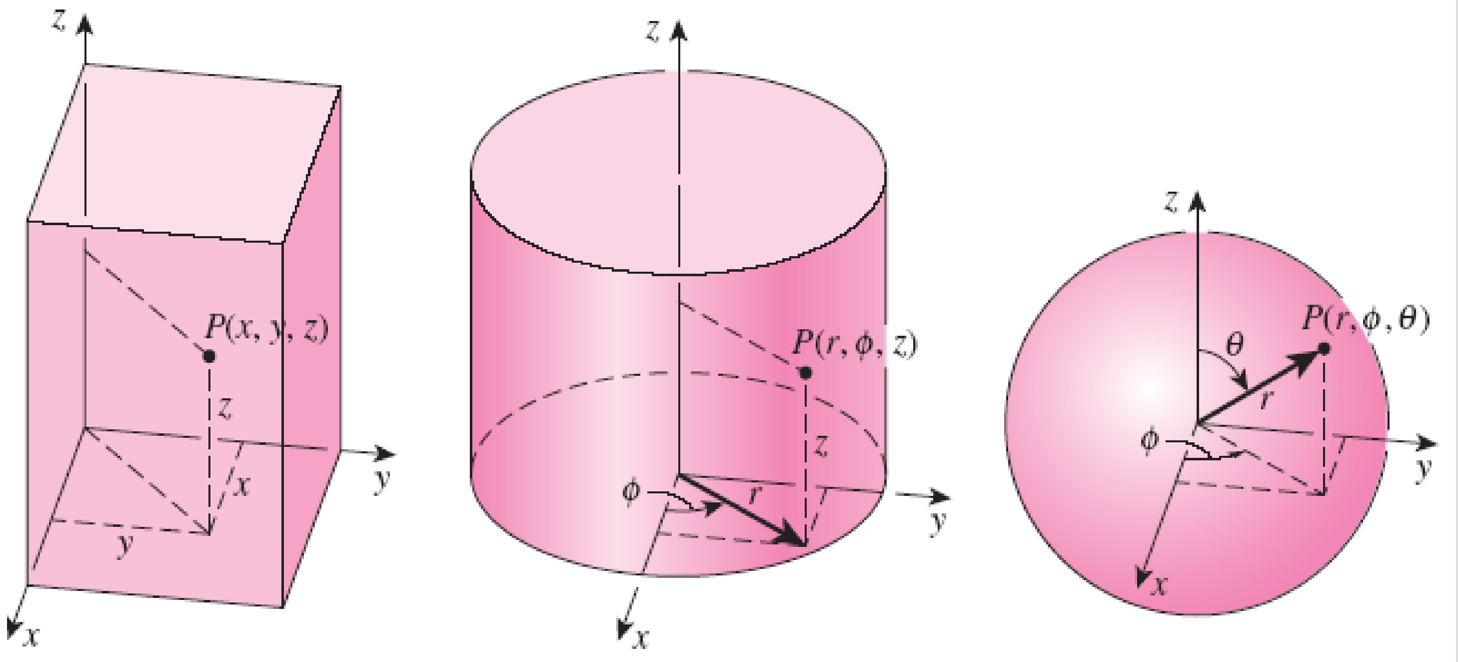


FIGURE 2–2
Indicating direction for heat transfer (positive in the positive direction; negative in the negative direction).

- Three prime coordinate systems:
  - ✓ rectangular $T(x, y, z, t)$
  - ✓ cylindrical $T(r, \phi, z, t)$
  - ✓ spherical $T(r, \phi, \theta, t)$.

FIGURE 2–3
The various distances and angles involved when describing the location of a point in different coordinate systems.



(a) Rectangular coordinates (b) Cylindrical coordinates (c) Spherical coordinates

- The rate of heat conduction through a medium in a specified direction is expressed by **Fourier's law of heat conduction** for one-dimensional heat conduction as:

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Heat is conducted in the direction of decreasing temperature, and thus the temperature gradient is negative when heat is conducted in the positive *x*-direction.

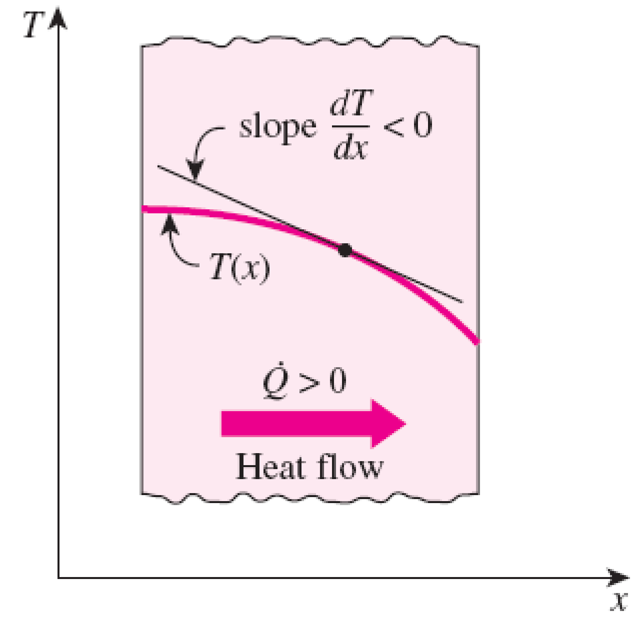


FIGURE 2–7

The temperature gradient dT/dx is simply the slope of the temperature curve on a T-x diagram.

- If *n* is the normal of the isothermal surface at point *P (heat flux vector)*, the rate of heat conduction at that point can be expressed by Fourier's law as

$$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \qquad \text{(W)}$$

$$\vec{\dot{Q}}_n = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k}$$

$$\dot{Q}_x = -kA_x\frac{\partial T}{\partial x}, \qquad \dot{Q}_y = -kA_y\frac{\partial T}{\partial y},$$

$$\dot{Q}_z = -kA_z\frac{\partial T}{\partial z}$$

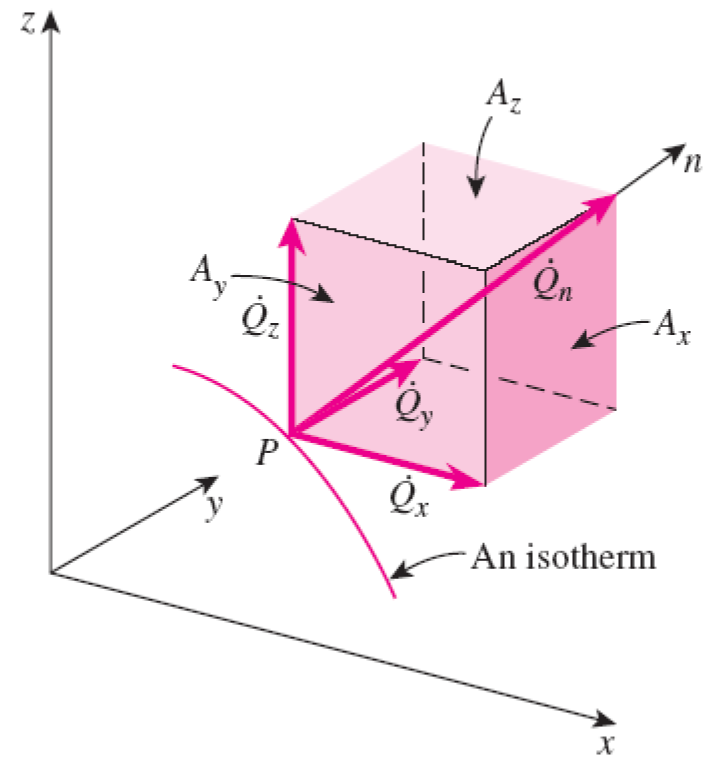


FIGURE 2–8
The heat transfer vector is always normal to an isothermal surface and can be resolved into its components like any other vector.

ONE-DIMENSIONAL HEAT CONDUCTION EQUATION

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{E}_{\text{gen, element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \qquad \textbf{(2-6)}$$

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta x (T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{gen, element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta x$$

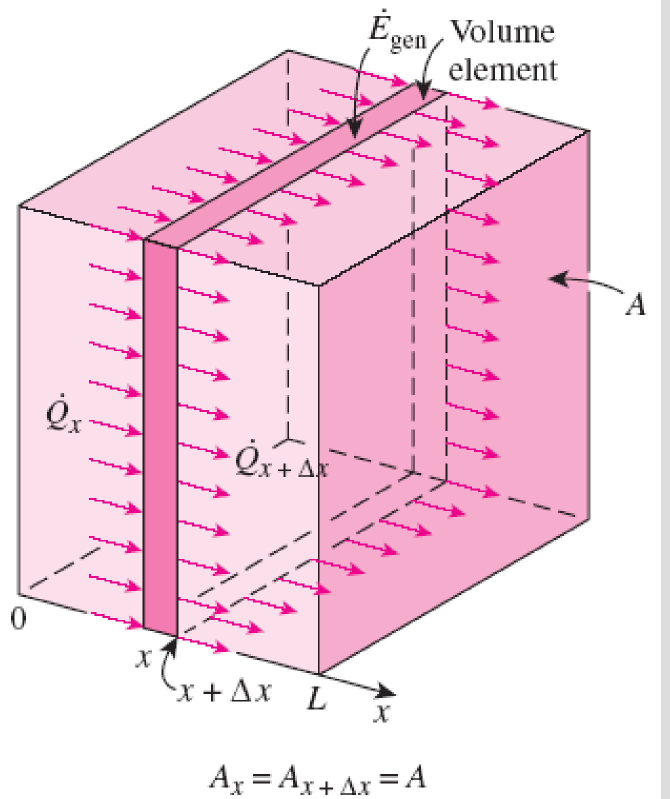


FIGURE 2–12

Heat Conduction Equation in a Large Plane Wall

Substituting into Eq. 2–6, we get

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{e}_{gen} A\Delta x = \rho c A \Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $A\Delta x$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{e}_{gen} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{e}_{gen} = \rho c \frac{\partial T}{\partial t}$$

$$\lim_{\Delta x \to 0} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{\partial \dot{Q}}{\partial x} = \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)$$

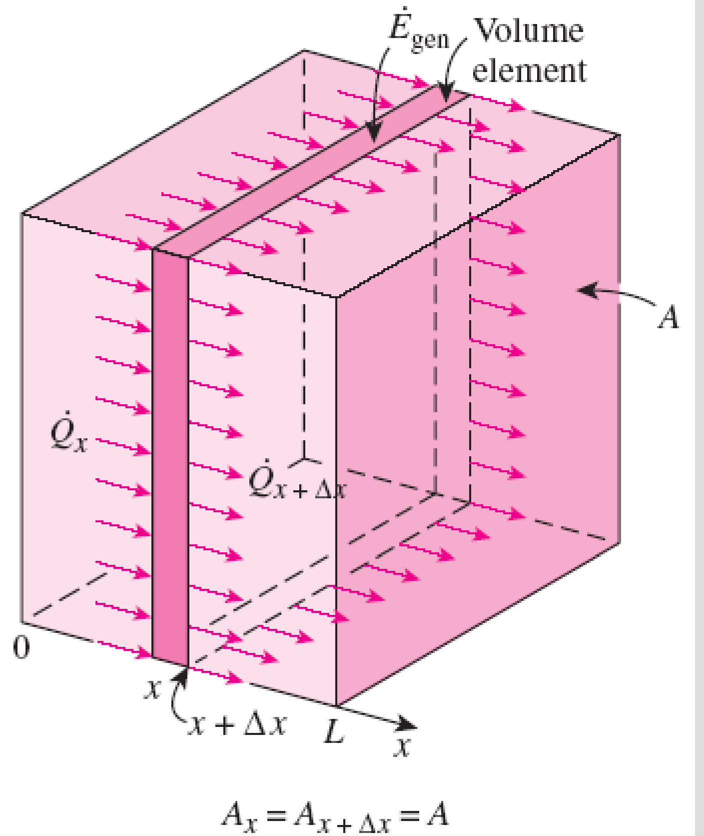


FIGURE 2–12

One-dimensional heat conduction through a volume element in a large plane wall.

Variable conductivity: $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t}$$

Constant conductivity: $$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\alpha = k/\rho c$ thermal diffusivity

(1) *Steady-state:*
$(\partial/\partial t = 0)$
$$\frac{d^2 T}{dx^2} + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

(2) *Transient, no heat generation:*
$(\dot{e}_{\text{gen}} = 0)$
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:*
$(\partial/\partial t = 0$ and $\dot{e}_{\text{gen}} = 0)$
$$\frac{d^2 T}{dx^2} = 0$$

Heat Conduction Equation in a Long Cylinder

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r+\Delta r\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{\text{gen, element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta r (T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{gen, element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta r$$

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{e}_{\text{gen}} A \Delta r = \rho c A \Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

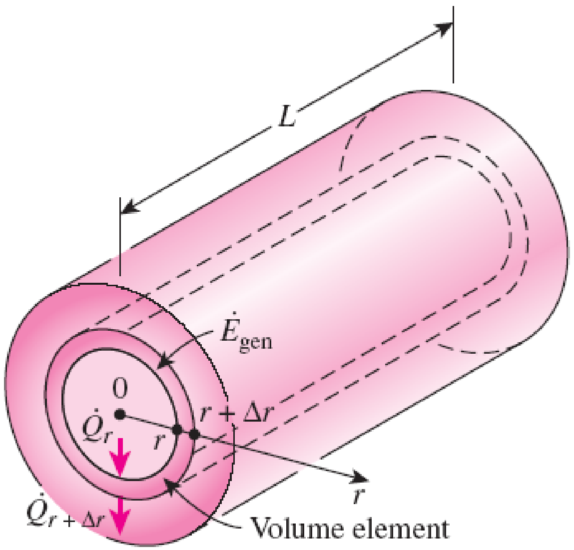


FIGURE 2–14

One-dimensional heat conduction through a volume element in a long cylinder.