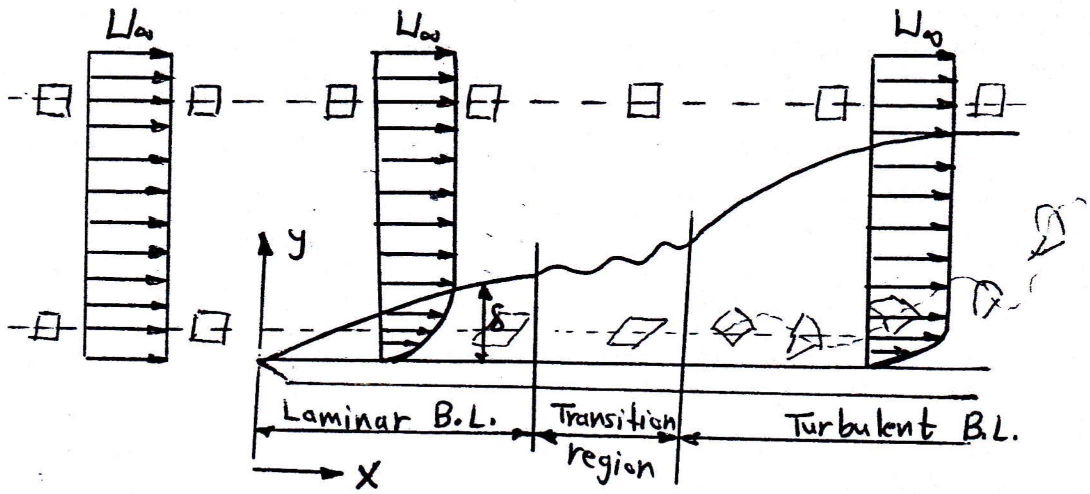
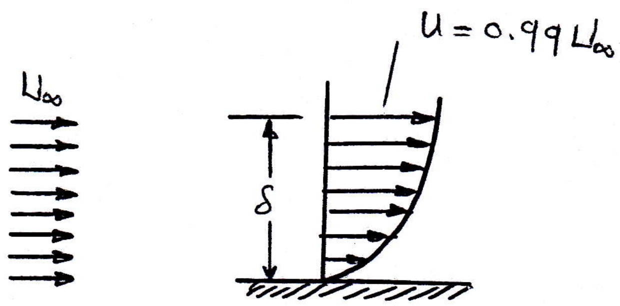
Chapter - 6 -

External Flow

6.1. Parallel Flow over Flat Plate



The development of the boundary layer over a flat plate, and the different flow regimes.



δ: B.L. thickness.

The transition from laminar to turbulent depends on
- surface geometry
- Surface roughness.
- upstream turbulence intensity.
- type of fluid.
- Value of surface temp.

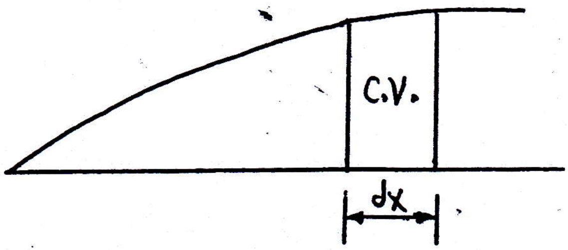
$Re_x = \dfrac{U_\infty x}{\nu}$ $\quad 2*10^5 — 3*10^6$

We will use $Re_{x,cr} = 5*10^5$

6.2. Momentum – Integral Boundary Layer Equation for a Flat plate

Assumptions

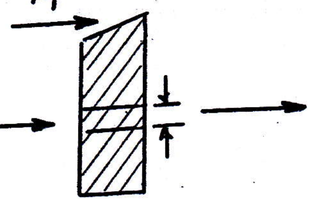
① Laminar boundary layer flow.
② Pressure constant throughout the flow field. (Zero Pressure gradient).



Continuity Equation

$$\dot{m}_{top} = \dot{m}_{out} - \dot{m}_{in} = \frac{\partial \dot{m}_{in}}{\partial x} dx$$

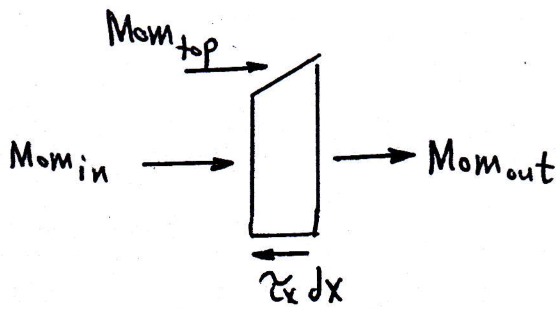
$$\dot{m}_{in} = \int_0^\delta \rho u \, dy$$



$$\dot{m}_{out} = \dot{m}_{in} + \frac{\partial \dot{m}_{in}}{\partial x} dx$$

Momentum

$$\Sigma F_x = Mom_{out} - Mom_{in}$$



$$-\tau_w dx = Mom_{out} - Mom_{in} - Mom_{top}$$

$$-\tau_x dx = \cancel{Mom_{in}} + \frac{\partial Mom_{in}}{\partial x} dx - \cancel{Mom_{in}} - \underbrace{Mom_{top}}_{U_\infty \dot{m}_{in}}$$

$$\tau_x dx = \left[\frac{\partial}{\partial x} \int_0^\delta \rho u^2 dy\right] dx - U_\infty \left[\frac{\partial}{\partial x} \int_0^\delta \rho u \, dy \; dx\right] \quad \div dx$$

$$\tau_w = U_\infty \frac{\partial}{\partial x}\int_0^\delta \rho u\, dy - \frac{\partial}{\partial x}\int_0^\delta \rho u^2 dy$$

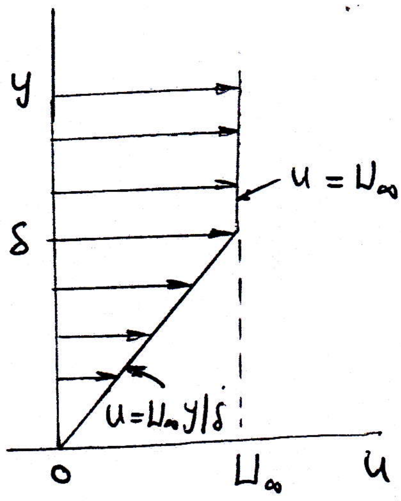
$$\tau_w = \frac{\partial}{\partial x}\int_0^\delta \rho u\,(U_\infty - u)\, dy$$

or

$$\boxed{\tau_w = \rho U_\infty^2 \frac{\partial}{\partial x}\int_0^\delta \frac{u}{U_\infty}\left(1 \frac{u}{U_\infty}\right) dy} \quad \text{------} \quad (6.1)$$

Example Consider the laminar flow of an incompressible fluid past a flat plate at $y=0$. The boundary layer profile is approximated as $u = U_\infty\, y/\delta$ for $0 \leq y \leq \delta$ and $u = U_\infty$ for $y > \delta$. Determine :-

1. The local boundary layer thickness
2. The Shear stress.
3. The local friction coefficient.
4. The drag coefficient.



Here

$\frac{u}{U_\infty} = \frac{y}{\delta}$ Sub into equation (6.1).

$$\tau_w = \rho U_\infty^2 \frac{\partial}{\partial x}\int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

or

$$\tau_w = \rho U_\infty^2 \frac{\partial}{\partial x}\,\delta \int_0^1 \eta\,(1-\eta)\, d\eta \qquad \text{where } \eta = \frac{y}{\delta}$$

$$\tau_w = \rho U_\infty^2 \frac{\partial \delta}{\partial x}\,\frac{1}{6}$$

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \frac{1}{6}\,\rho U_\infty^2 \frac{d\delta}{dx}$$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{U_\infty}{\delta} \longleftarrow$$ from velocity profile

$$\therefore \mu \frac{\cancel{U_\infty}}{\delta} = \rho U_\infty^{\cancel{2}} \frac{1}{6} \frac{d\delta}{dX}$$

$$\frac{6\nu}{U_\infty} dx = \delta \, d\delta$$

Integrating

$$\frac{6\nu}{U_\infty} x = \frac{\delta^2}{2} + C \qquad \text{at } (x=0 \quad \delta=0) \text{ (leading edge)}$$
$$(C=0)$$

$$\delta^2 = \frac{12\nu X}{U_\infty} \longrightarrow \boxed{\delta = 3.46\sqrt{\frac{\nu X}{U_\infty}}} \quad \text{or} \quad \boxed{\frac{\delta}{X} = \frac{3.46}{\sqrt{Re_x}}}$$

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu \frac{U_\infty}{\delta}$$

$$\therefore \boxed{\tau_w = \mu U_\infty \frac{1}{3.46}\sqrt{\frac{U_\infty}{\nu X}} = 0.289\, U_\infty^{3/2}\sqrt{\frac{\rho\mu}{x}}}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{0.289\, U_\infty^{3/2}}{\frac{1}{2}\rho U_\infty^2}\sqrt{\frac{\rho\mu}{x}}$$

$$\boxed{\therefore C_f = \frac{0.578}{\sqrt{Re_x}}}$$

$$C_D = \frac{1}{L}\int_0^L C_f \, dx = \frac{1}{L}\int_0^L \frac{0.578}{\sqrt{Re_x}} dx$$

$$\boxed{C_D = \frac{1.156}{\sqrt{Re_L}}} \quad , \quad Re_L = \frac{U_\infty L}{\nu}$$

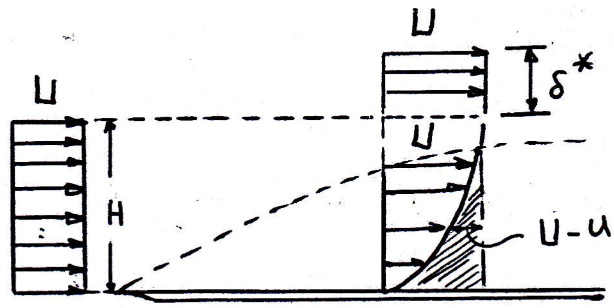
Parabolic

$$\frac{u}{U_\infty} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \qquad C_f = \frac{0.73}{\sqrt{Re_x}} \qquad C_D = \frac{1.46}{\sqrt{Re_L}}$$

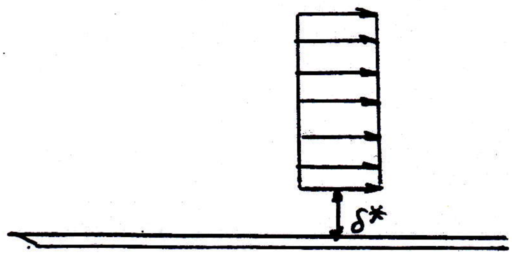
Cubic

$$\frac{u}{U_\infty} = 3(y/\delta)/2 - (y/\delta)^3/2 \qquad \frac{\delta}{x} = \frac{4.46}{\sqrt{Re_x}} \qquad C_f = \frac{0.646}{\sqrt{Re_x}} \qquad C_D = \frac{1.292}{\sqrt{Re_L}}$$

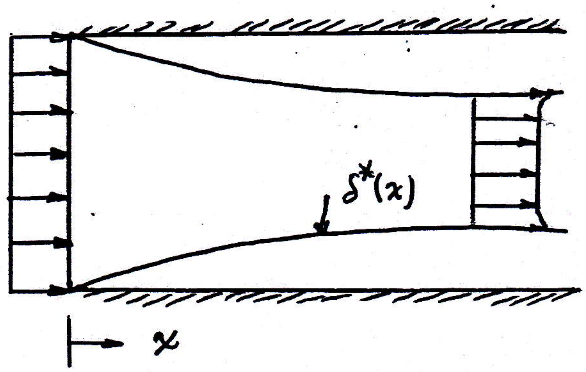
Displacement Thickness



$$\delta^* U * b = \int_0^\delta (U-u)\,dy * b$$

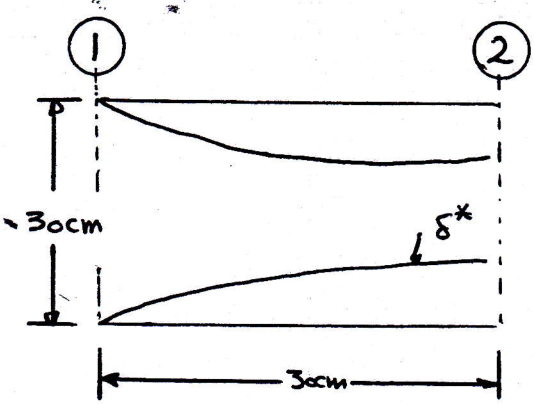


$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy} \quad \text{------ (6.2)}$$



Example A Laminar flow wind tunnel has a test section that is 30 cm in diameter and 30 cm in length. The air is at 19°C. At a uniform air speed of 4 m/s at the test section inlet, by how much will the centerline air speed accelerate by the end of the test section?

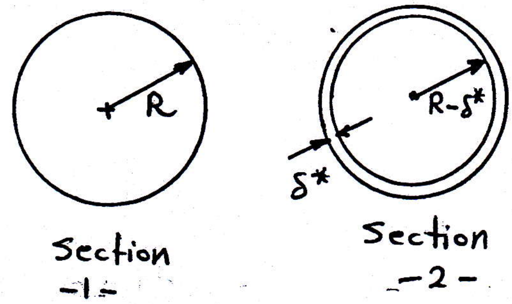
use $\delta^* = \dfrac{1.72x}{\sqrt{Re_x}}$



at 19°C $\nu = 1.507 \times 10^{-5}\ m^2/s$

$$Re_x = \frac{Vx}{\nu} = \frac{4 \times 0.3}{1.507 \times 10^{-5}} = 7.96 \times 10^4$$

$$\delta^* = \frac{1.72 * 0.3}{\sqrt{7.96 \times 10^4}} \doteq 1.83 \times 10^{-3} m = \underline{\underline{1.83\ mm}}$$



$$V_1 A_1 = V_2 A_2$$

$$4 * \cancel{\pi} R^2 = V_2\ \cancel{\pi}(R - \delta^*)^2$$

$$V_2 = 4R^2 \frac{1}{(R-\delta^*)^2} = 4\,(0.15)^2 \frac{1}{(0.15 - 1.83 \times 10^{-3})^2} = 4.1\ m/s\ .$$

∴ The air speed increases by $\underline{\underline{2.5\ \%}}$