Chapter -4-

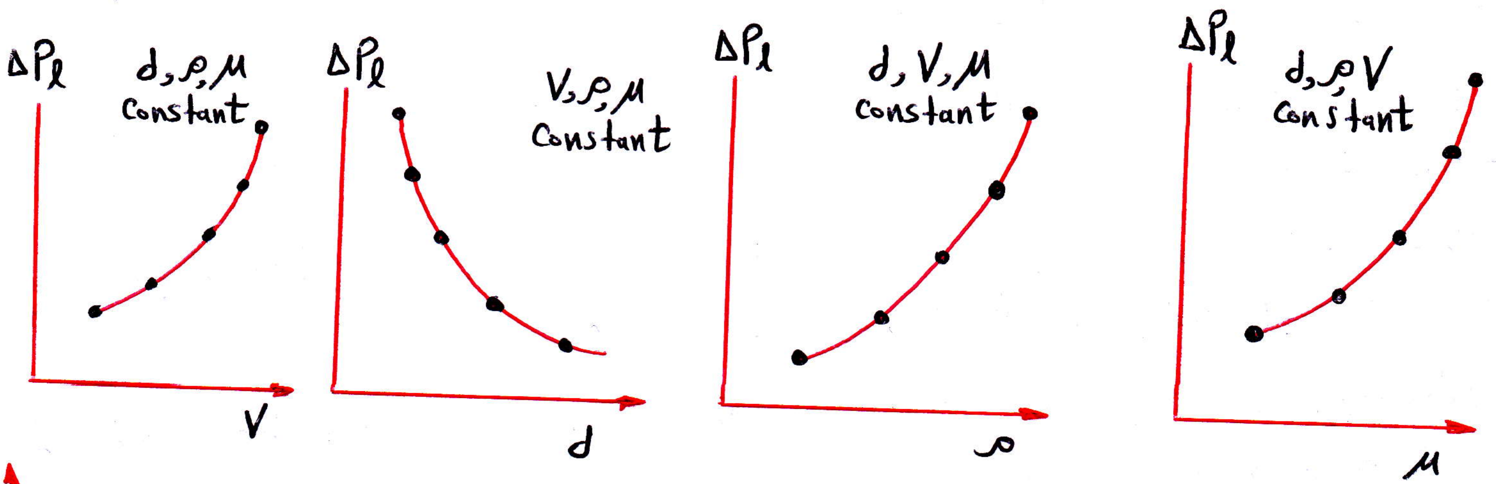
Dimensional Analysis and Similarity

4.1 Introduction

If the pressure drop per unit length (ΔP_l) in pipe depend on the diameter (d), the fluid density (ρ) & the fluid viscosity (μ) & and the mean velocity (V), thus we can write

$$\Delta P_l = f(d, \rho, \mu, V)$$

The objective of experiment is to determine the nature of this function.

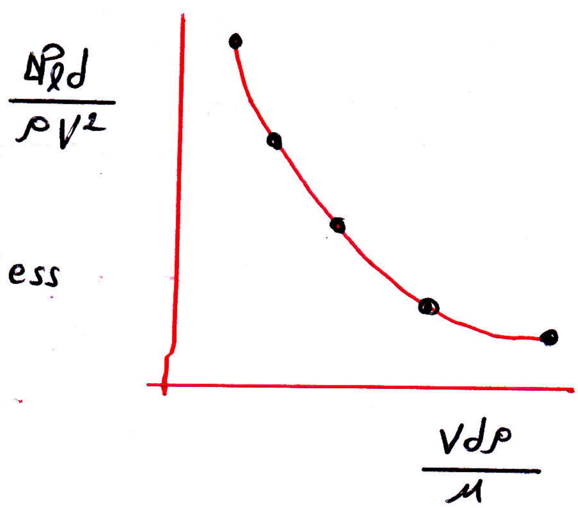


Valid only for specific pipe and for specific fluid.

By dimensional analysis

$$\frac{d\,\Delta P_l}{\rho V^2} = f\left(\frac{V d \rho}{\mu}\right) \quad \text{------------} \quad (4.1)$$

This curve is Valid for any pipe and incompressible Newtonian fluid. Thus the experiment is much simpler, easier, and less expensive.



4.2. Basic Dimensions

Time T
Mass M } MLT (System)
Length L

4.3. Buckingham π- theorem

$$f(\pi_1, \pi_2, \pi_3, -------- \pi_{n-m}) = 0 \quad -------- (4.2)$$

where

π's (dimensionless groups) or π-Parameters

equation (4.2) can be written as:-

$$\pi_1 = f(\pi_2, \pi_3, -----, \pi_{n-m})$$

n = No. of Variables in the Problem.
m = No. of the basic dimensions

1. Write the functional form of the dependent Variable depending on the $(n-1)$ independent Variables.
2. Identify m repeating variables Variable that will be combined with each remaining Variables to form the π-parameter. The repeating Variables selected from the independent variables must include all the basic dimensions but they must not form a π-Parameter themselves.
3. Form π-Parameters by combining the repeating variables with each of the remaining Variables.
4. Write $\pi_1 = f(\pi_2, \pi_3, ---- \pi_{n-m})$

4.4. Dimensionless Groups in Fluid Mechanics

Example The drag force (F) on a cylinder is depend on velocity (V), cylinder diameter (d), cylinder length (l), fluid viscosity (μ), and fluid density (ρ). write the functional form of the dimensionless variables.

① $F = f(V, \rho, \mu, d, l)$ $\quad n = 6$, by MLT system

$F = \frac{ML}{T^2}$, $V = \frac{L}{T}$, $\rho = \frac{M}{L^3}$, $\mu = \frac{M}{LT}$, $d = L$, $l = L$

② Repeating variables $m = 3$, ρ, V, d contain (MLT) not form a $-\pi$ group

$\therefore\ n - m = 3\ \pi$ groups

③ $\pi_1 = F \rho^a V^b d^c = M^0 L^0 T^0$

$$\frac{ML}{T^2}\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b (L)^c = M^0 L^0 T^0$$

M: $1 + a = 0$

L: $1 - 3a + b + c = 0$

T: $-2 - b = 0$

$a = -1$, $b = -2$, $c = -2$

$\pi_1 = F \rho^{-1} V^{-2} d^{-2} \longrightarrow \boxed{\pi_1 = \frac{F}{\rho V^2 d^2}}$

$\pi_2 = \mu \rho^e V^f d^g = M^0 L^0 T^0$

$$\frac{M}{LT}\left(\frac{M}{L^3}\right)^e \left(\frac{L}{T}\right)^f (L)^g = M^0 L^0 T^0$$

M: $1 + e = 0$

L: $-1 - 3e + f + g = 0$

T: $-1 - f = 0$

$e = -1$, $f = -1$, $g = -1$

$\pi_2 = \mu V^{-1} d^{-1} \rho^{-1} \longrightarrow \boxed{\pi_2 = \frac{\mu}{V d \rho}}$

$\pi_3 = l (\rho)^h (V)^i (d)^j$

$\pi_3 = L \left(\frac{M}{L^3}\right)^h \left(\frac{L}{T}\right)^i (L)^j = M^0 L^0 T^0$

M: $h = 0$

L: $1 + i + j - 3h = 0 \longrightarrow j = -1$

T: $-i = 0$

$\pi_3 = l d^{-1} \longrightarrow \boxed{\pi_3 = l/d}$

④ $\pi_1 = f(\pi_2, \pi_3)$

4.5. Similarity

Geometeric Similarity

Requires that the model and the Prototype having the same Shape.

Dynamic Similarity

Requires

$\pi_m = \pi_p$

$$\boxed{Eu_m = Eu_p \quad , \quad Re_m = Re_p \qquad Fr_m = Fr_p} \quad \text{-----------} \; (4.3)$$

Kinematic Similarity

Requires that the Velocity ratio is constant in all points of the flow field (between model and Prototype)

Example The aerodynamic drag of a new sport car is to be predicted at a speed of 50 mi/h at an air temperature of 25 °C. Automotive engineers build a one-fifth scale model of the car to test in the wind tunnel. It is winter and the wind tunnel is located in an unheated building, the temperature of the wind tunnel air is 5 °C. Determine how fast the engineers should run the wind tunnel in order to achieve similarity between the model and the prototype.

at $T = 25\,°C$ $\quad \rho = 1.184\ kg/m^3 \, , \; \mu = 1.849 \times 10^{-5}\ kg/m.s \longrightarrow$ Prototype

at $T = 5\,°C$ $\quad \rho = 1.269\ kg/m^3 \, , \; \mu = 1.754 \times 10^{-5}\ kg/m.s \longrightarrow$ Model

$Re_m = Re_p \qquad \boxed{\frac{L_p}{L_m} = 5}$

$$\frac{V_m L_m \rho_m}{\mu_m} = \frac{V_p L_p \rho_p}{\mu_p} \longrightarrow V_m = V_p \left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)$$

$$V_m = 50 \left(\frac{1.754 \times 10^{-5}}{1.849 \times 10^{-5}}\right)\left(\frac{1.184}{1.269}\right)(5) = \underline{221\ mi/h}$$