Ministry of Higher Education and Scientific Research

Tikrit University

Engineering Collage –Al shirqat

FUNDAMENTALS OF ELECTRICAL ENGINEERING

LECTURE 8

NORTON’S THEOREM

PREPARED BY

TEACHING ASSISTANT

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General Objectives:

- **Simplifying complex electrical circuits**: The theory enables converting a complex circuit into an equivalent circuit consisting of a current source and a parallel resistor, making analysis easier.
- **Enhancing understanding of relationships between circuit components:** The theory helps in understanding the effect of various elements on the circuit's performance.
- **Improving circuit design efficiency:** The theory is used to simplify the design of electronic and electrical systems by identifying critical components.
- **Facilitating calculations and analysis:** It provides a direct method to calculate currents or voltages in a specific part of the circuit.

Specific objectives:

- **Determining the equivalent current source and equivalent resistance:** The goal is to represent any electrical network with an ideal current source and a parallel resistance.
- **Analyzing current and voltage at a specific point:** The theory helps in calculating the current or voltage across a specific component.
- **Applying the theory to solve practical problems:** The theory is used in applications such as power circuit design and analysis of complex electrical systems.
- **Understanding the concept of equivalent circuits:** Focuses on the practical understanding of circuit equations and how to simplify them into the Norton equivalent model.

Introduction

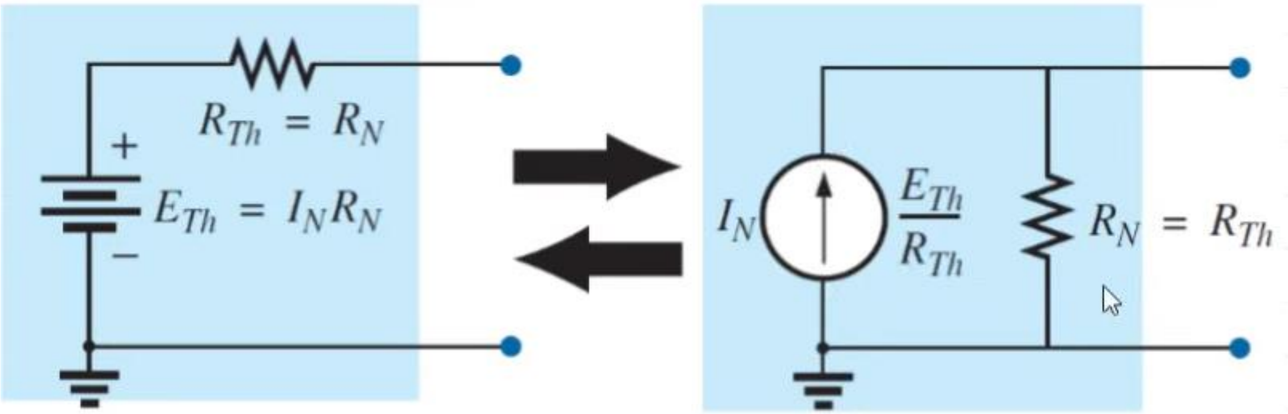
- **Norton's Theorem** is one of the fundamental theories in electrical circuit analysis. It allows the simplification of any linear electrical network containing power sources (such as voltage or current) and resistors into a simpler form. According to this theorem, any complex linear electrical network can be replaced by an equivalent current source connected in parallel with an equivalent resistance.
- **Key elements of Norton's Theorem:**
- **Equivalent Current Source (Norton Current):** This is the current that flows through an open circuit connected to the points where the circuit is being analyzed.
- **Equivalent Resistance (Norton Resistance):** This is the equivalent resistance determined by the network's resistance when all power sources are turned off.

Thevenin’s Theorem

Solution method:

- **1. Identify the part to be analyzed:**
  Determine the element (resistor) for which you want to calculate the current or voltage, and consider it the load R_L.
- **2. Remove the load R_L:**
  Disconnect the load R_L from the circuit to analyze the remaining network.
- **3. Calculate Norton current (I_N):**
  Calculate the current that flows at the location of the load R_L when it is replaced by a short circuit.
  The current resulting from the short circuit is I_N.
- **4. Calculate the equivalent Norton resistance (R_N):**
  Turn off the power sources (replace voltage sources with short circuits and current sources with open circuits).
  Calculate the total resistance of the circuit as seen from the terminals of the load.
  This resistance is R_N.

Norton’s Theorem
The Laws Related to Norton's Theorem:



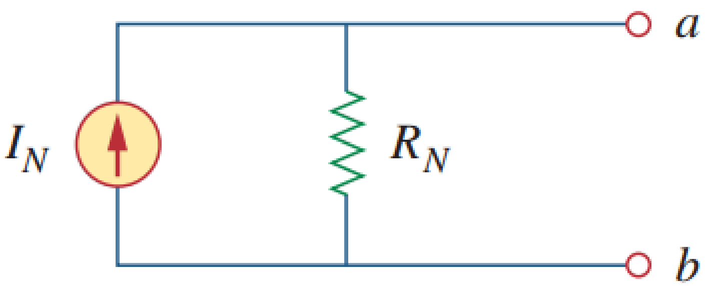


Figure 8-1 Norton equivalent circuit.

$$\boxed{R_N = R_{\text{Th}}}$$

$$\boxed{I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}}$$

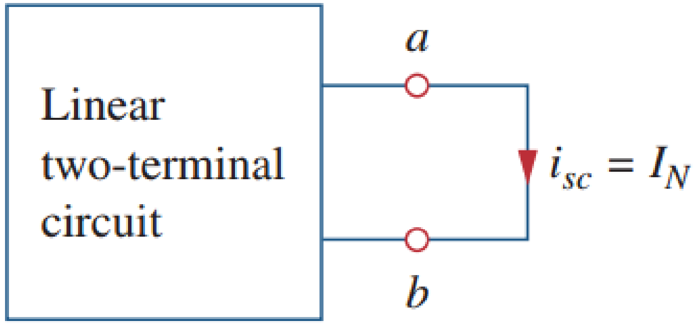


Figure 8-2 Finding Norton current I_N

Norton's Theorem

Example 1: Develop the Norton Equivalent for the circuit given below.

Solution:

Using Ohm's law, first replace the series combination of the 24 V source and the 4 Ω resistor with a parallel combination of a 6 A current source in shunt with the original 4 Ω resistor as shown below.

Combination of the two parallel current sources, 6 A and 3 A, results in a net, total, Norton current source of 9 A.

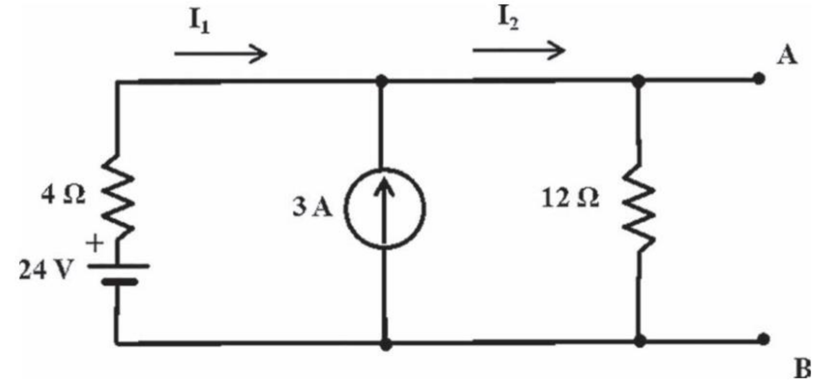
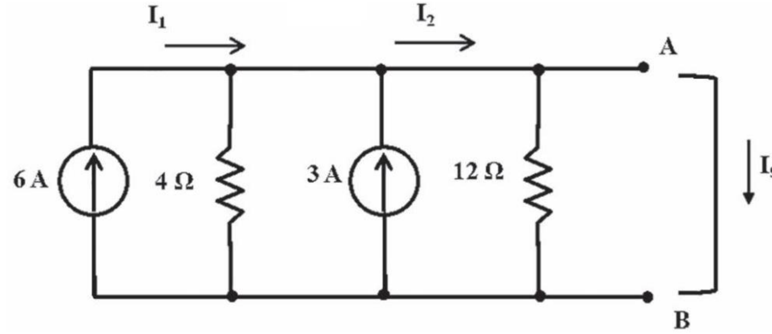


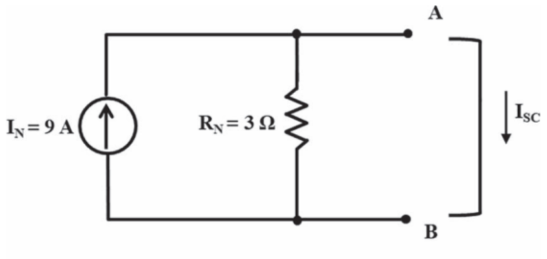
Figure 3-3 The Thevenin equivalent circuit for Example 1

$$I_1 = \frac{V}{R} = \frac{24}{4} = 6\text{A}$$



$$I_N = I_{sc} = (I_1 + I_1) = 6 + 3 = 9\text{A}$$

$$R_N = 12 \| 4 = \frac{12 \times 4}{12 + 4} = 3\ \Omega$$



Norton Equivalent

Example 2: Find the Norton equivalent circuit of the circuit in Fig.8.4 at terminals a-b and the voltage.

Solution: We find R_N in the same way we find R_{TH} in the Thevenin equivalent circuit.

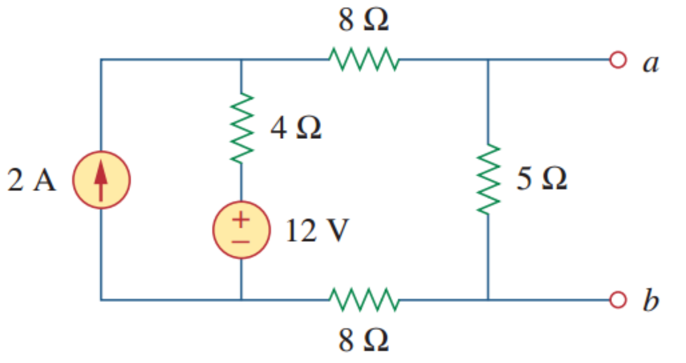


Figure 8-4 Equivalent Circuit for Example 2

$$R_N = 5\|(8+8+4) = 5\|20 = \frac{5\times 20}{5+20} = 4\ \Omega$$

To find we short-circuit terminals a and b, as shown in Fig. 8.5(b).

➢For loop 1 : $I_1 = 2\text{A}$,

We ignore the resistor 5Ω because it has been short-circuited.

Applying mesh analysis, we obtain

➢For Loop 2 : $-4I_1 + (4+8+8)I_2 - 12 = 0$

$-4\times 2 + 12I_2 - 12 = 0 \Rightarrow -8-12+20I_2=0$

$-20+20I_2=0$

From these equations, we obtain

$$I_2 = \frac{20}{20} = 1\ \text{A} = I_N = I_{sc}$$

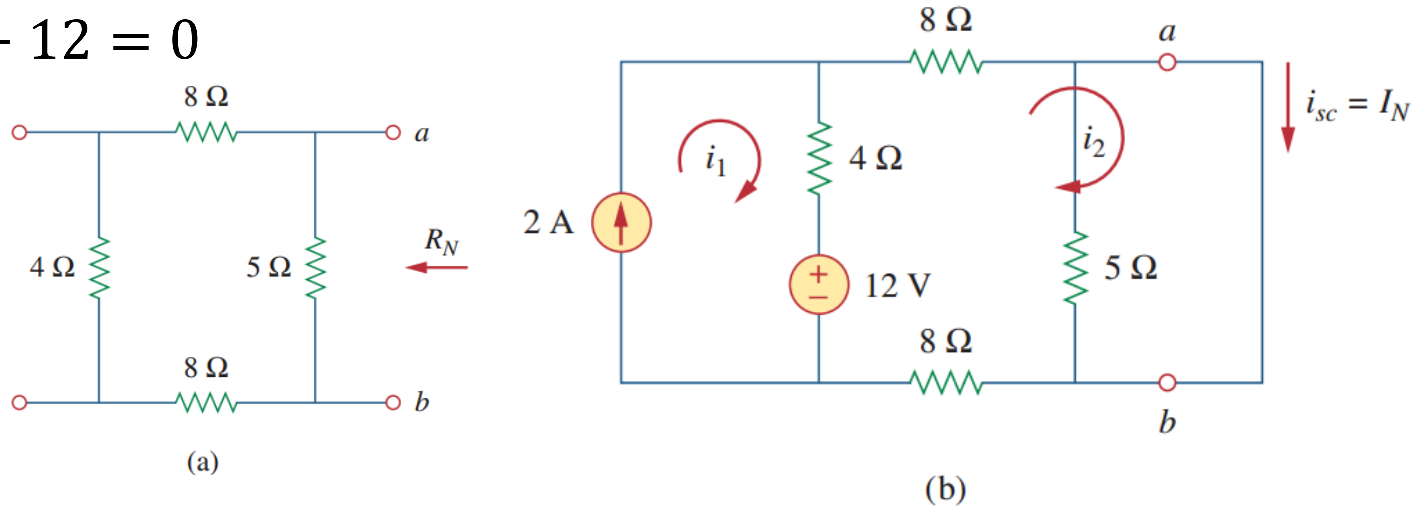


Figure 8-5(a) finding R_N.(b) finding I_N

Norton's Theorem

Alternatively, we may determine from We obtain

as the open-circuit voltage across terminals a and b in Fig. 8.5 (c).

➢ Using mesh analysis, we obtain (For loop 1)

$I_3 = 2\text{ A}$

For Loop 2

$-4I_3 + (4+8+5+8)I_4 - 12 = 0$

$-4 \times 2 + 25\, I_4 - 12 = 0 \quad \Rightarrow \quad 25\, I_4 - 20 = 0 \quad \Rightarrow 25\, I_4 = 20$

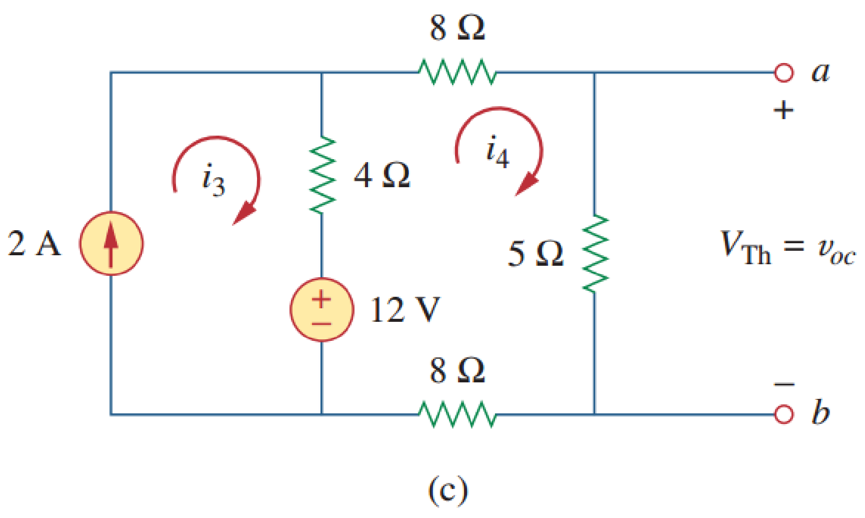
$I_4 = \frac{20}{25} = 0.8\text{ A}$

➢ To Find $v_{Th} = v_{oc}$

$v_{Th} = v_{oc} = 5I_4 = 5 \times 0.8 = 4\text{ V}$

$I_N = \frac{v_{Th}}{R_{Th}}$

$I_N = \frac{4}{4} = 1\text{ A}$



(c) calculating v_{TH}

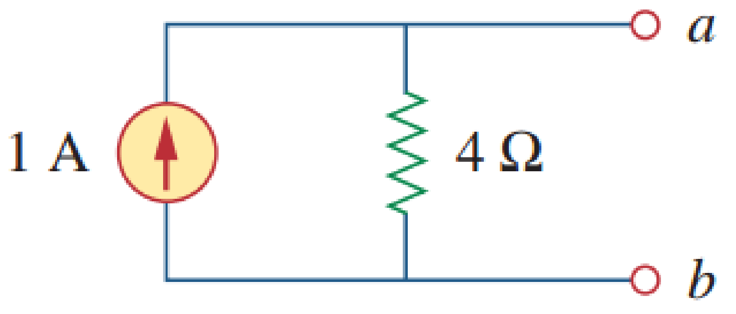
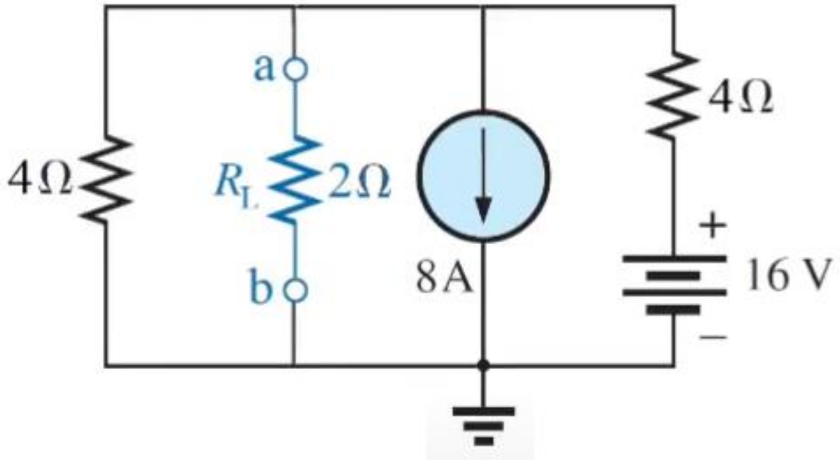


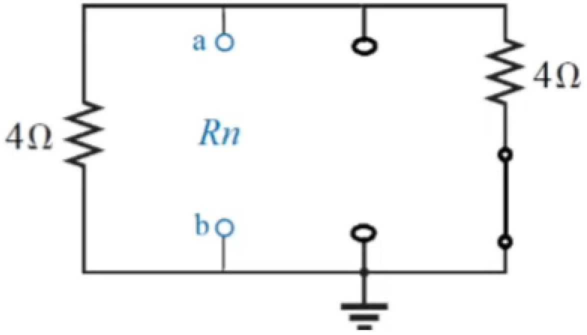
Figure 8-6 Norton equivalent of the circuit in Fig. 8.4.

Example 3: Find the Norton equivalent circuit of the circuit in the figure below at terminals a-b.

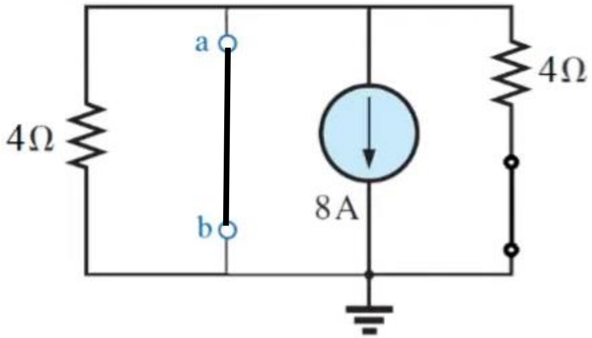


Solution:

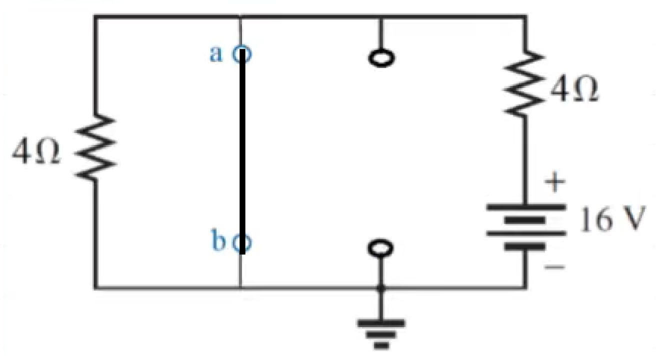
$$R_N = 4\|4 = \frac{4\times4}{4+4} = 2\ \Omega$$



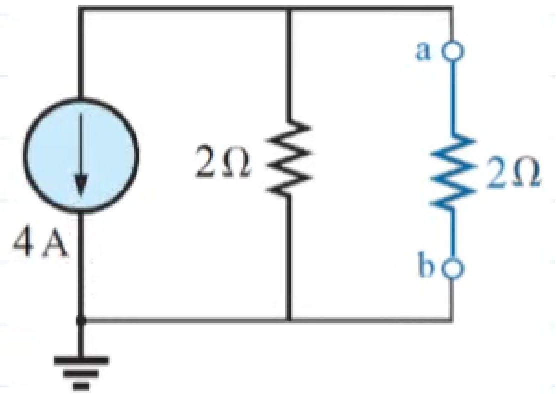
$$I'_N = I_s = 8\ A$$



$$I''_N = \frac{V_{s1}}{R_{4\Omega}} = \frac{16}{4} = 4\ A$$



$$I_N = I'_N - I''_N = 8A - 4A = 4A$$

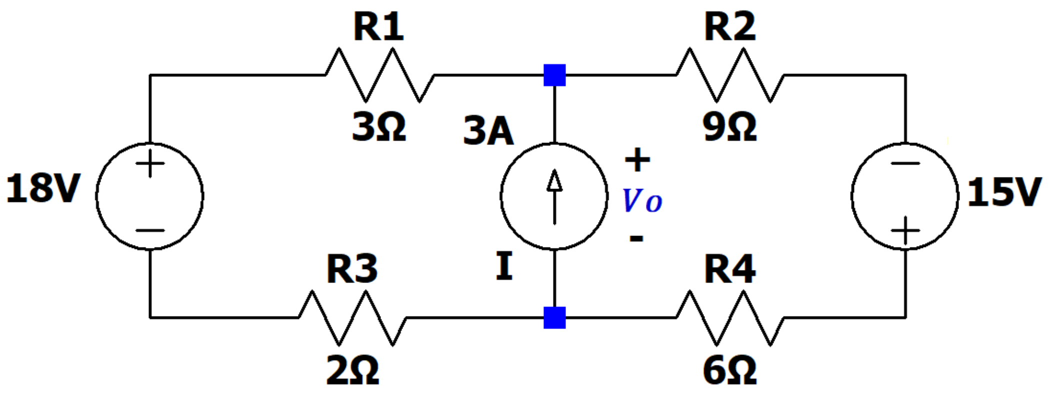


$$I_{2\Omega} = \frac{4A}{2\ resistors} = 2A$$

Or using a current divider

$$I_{2\Omega} = \frac{4 \times 2}{2 + 2} = 2A$$

Example 4: Following the circuit, find the value of V_o using Norton's theorem.

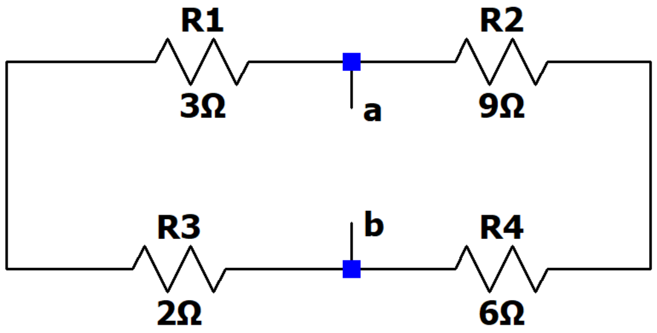


Solution:

$$R_N = (2+3)||(9+6) = \frac{5\times15}{5+15} = 3.75\ \Omega$$

Loop1 : $I_1(3+2) - 18 = 0 \Rightarrow 5I_1 = 18$

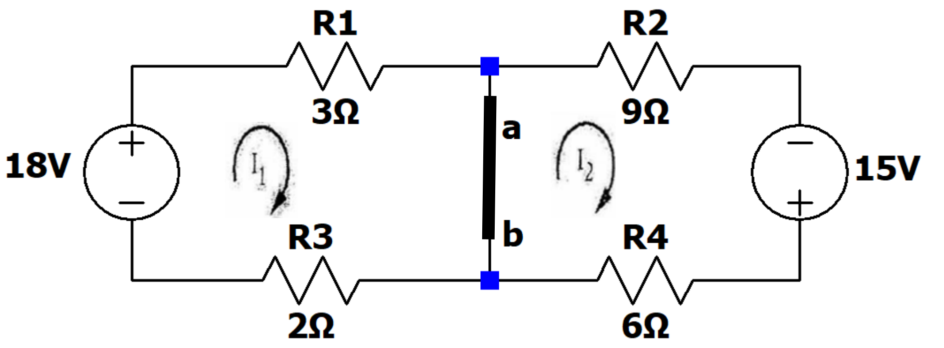
$I_1 = \frac{18}{5} = 3.6$ A



Loop 2: $I_2(6+9) - 15 = 0 \Rightarrow 15I_2 = 15$

$I_2 = \frac{15}{15} = 1$ A

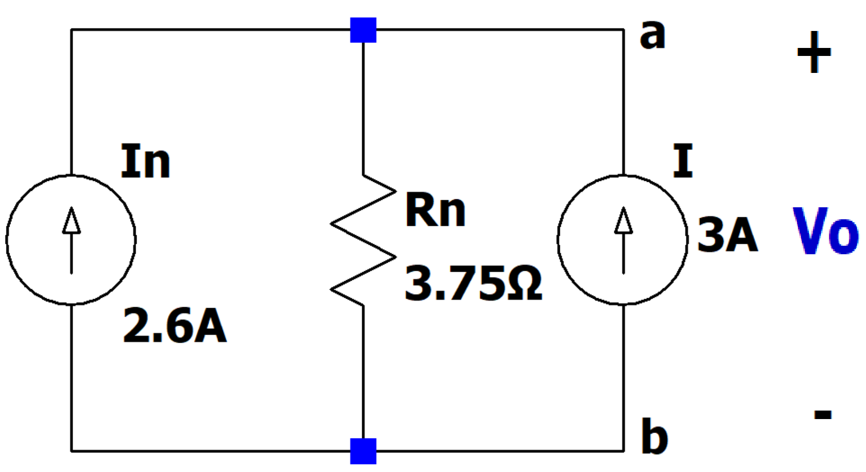
$I_N = I_1 - I_2 = 3.6 - 1 = 2.6$ A



$$V_o = (I_N + I_s) \times R_N$$

$$V_o = (2.6 + 3) \times 3.75$$

$$V_o = 5.6 \times 3.75 = 21\ V$$

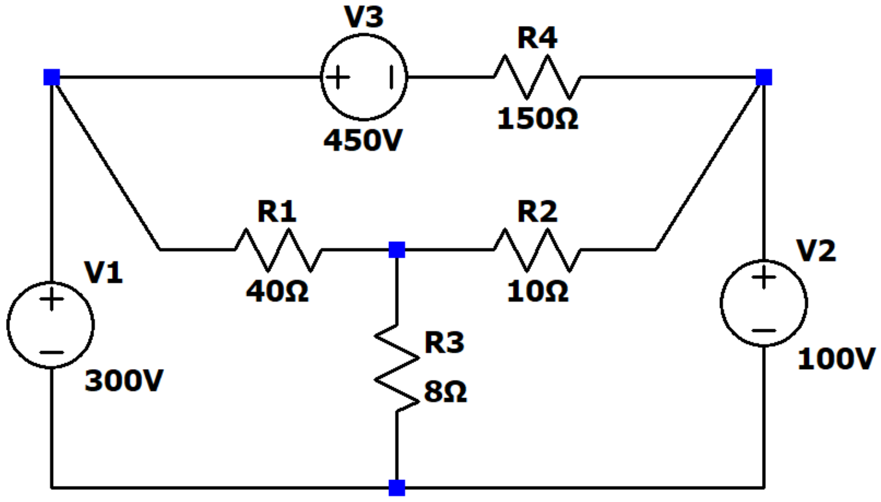


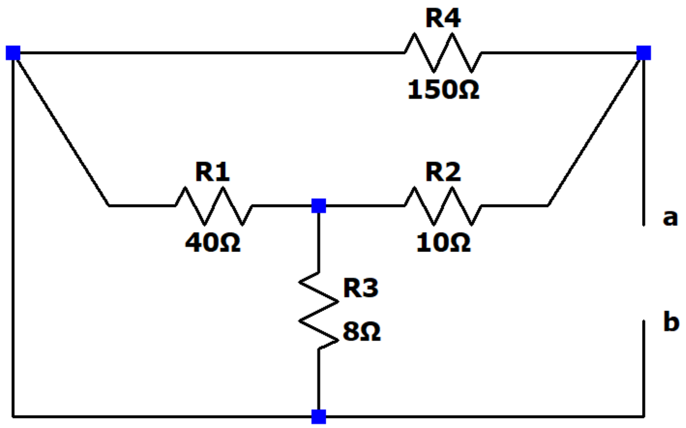
Example 5: Find the power 100 V source using Norton's theorem.

Solution:

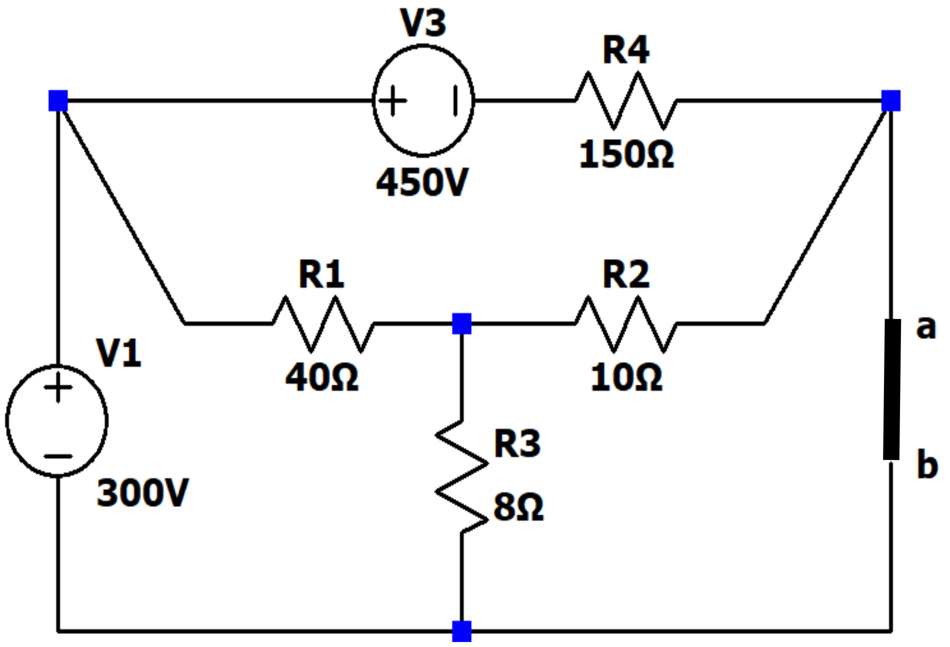
$$R_a = (40\|8) + 10 = \frac{40\times8}{40+8} + 10 = \frac{50}{3}\,\Omega$$

$$R_N = \frac{50}{3}\|150 = \frac{\frac{50}{3}\times150}{\frac{50}{3}+150} = 15\,\Omega$$





$$\text{Loop1}: \ I_1(40+8) - 8I_N - 40I_3 - 300 = 0$$
$$48I_1 - 8I_N - 40I_3 = 300 \ldots\ldots(1)$$
$$\text{Loop 2}: I_N(8+10) - 8I_1 - 10I_3 = 0$$
$$-8I_1 + 18I_N - 10I_3 = 0 \ldots.(2)$$



$$\text{Loop3}: -40I_1 - 10I_N + (40+10+150)I_3 = -450$$
$$-40I_1 - 10I_N + 200I_3 = -450 \ldots\ldots(3)$$

$$48I_1 - 8I_N - 40I_3 = 300 \ \ldots\ldots(1)$$
$$-8I_1 + 18I_N - 10I_3 = 0 \ \ldots\ldots(2)$$
$$-40I_1 - 10I_N + 200I_3 = -450 \ \ldots\ldots(3)$$

$$48I_1 - 8I_N - 40I_3 = 300 \;\ldots\ldots(1)$$

$$-8I_1 + 18I_N - 10I_3 = 0 \;\ldots\ldots(2)$$

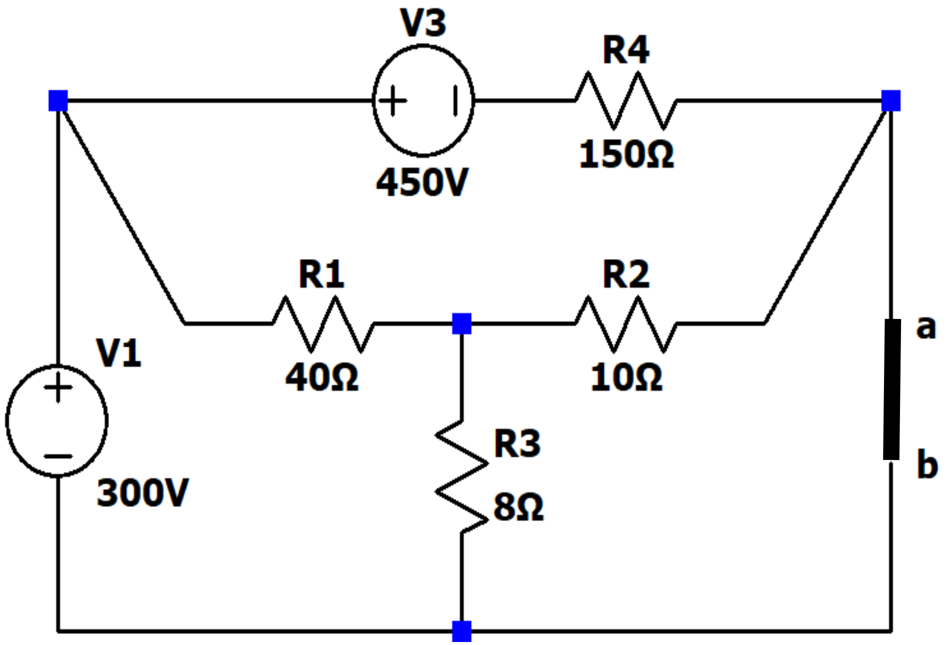
$$-40I_1 - 10I_N + 200I_3 = -450 \;\ldots\ldots(3)$$

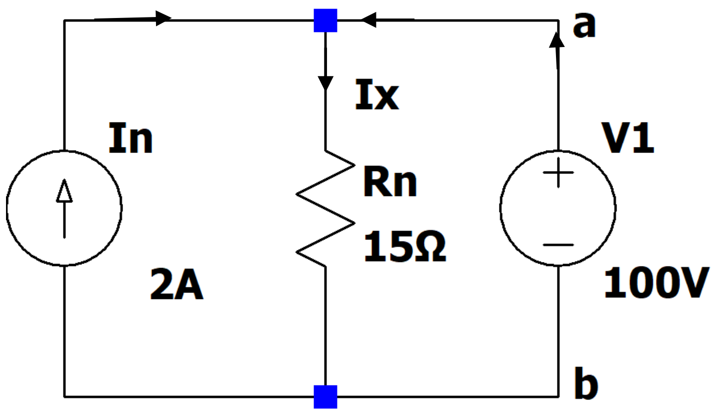
$$I_N = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 48 & 300 & -40 \\ -8 & 0 & -10 \\ -40 & -450 & 200 \end{vmatrix}}{\begin{vmatrix} 48 & -8 & -40 \\ -8 & 18 & -10 \\ -40 & -10 & 200 \end{vmatrix}} = 2A$$

$$I_x = \frac{100}{15} = \frac{20}{3}A$$

$$2 + I_{100V} = I_x \Rightarrow I_{100V} = \frac{20}{3} - 2 = \frac{14}{3}A$$

$$P = VI = 100 \times \frac{14}{3} = 466.67W$$





فيديو توضيحي لنظرية Norton’s Theorem

Let’s Start!

Thank you for listening