Ministry of Higher Education and Scientific Research
Tikrit University
Engineering Collage –Al shirqat

FUNDAMENTALS OF ELECTRICAL ENGINEERING
LECTURE 10

MILLMAN’S THEOREM

PREPARED BY
TEACHING ASSISTANT

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General Objectives:

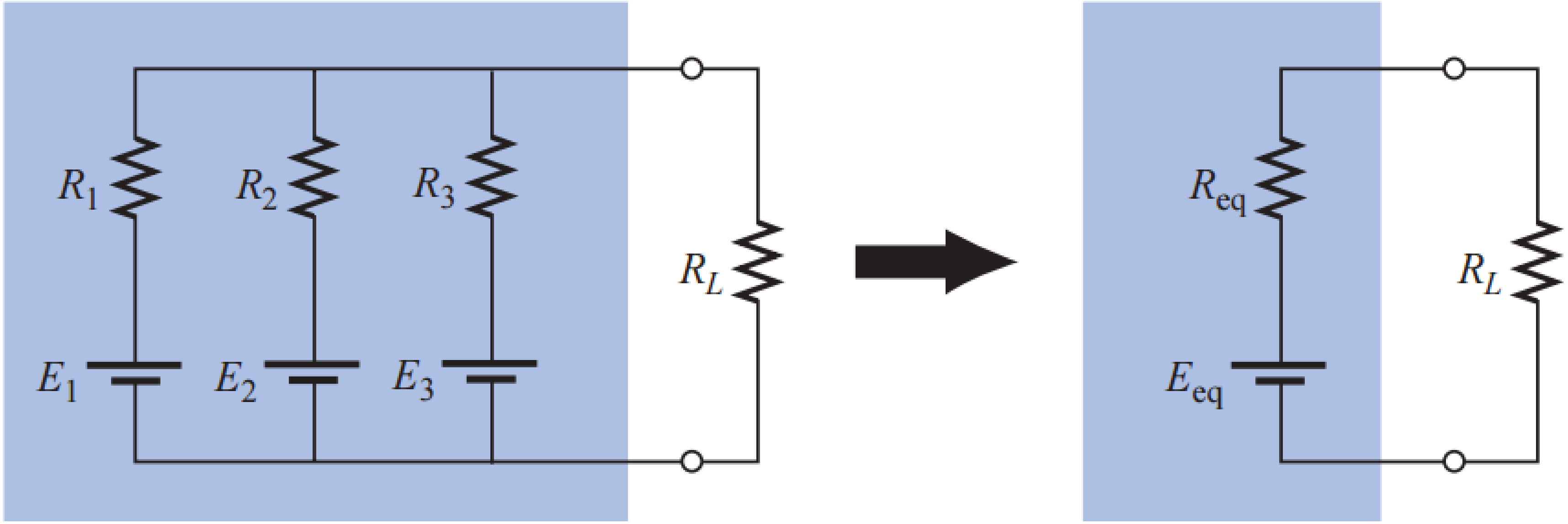
- **Simplifying the analysis of complex circuits:** Millman’s Theorem allows for finding the voltage at a common node of multiple parallel branches containing voltage sources and resistors, without using Kirchhoff’s Laws or nodal analysis.
- **Saving time and effort:** It reduces the number of calculation steps compared to traditional analysis methods.
- **Easily finding the equivalent voltage:** It helps directly calculate the equivalent voltage across the junction point of parallel elements.
- **Applicable to circuits with multiple sources:** The theorem is especially useful in circuits that have multiple voltage sources connected in parallel, where traditional methods may be cumbersome.

Specific objectives:

- To enable students to analyze electrical circuits with multiple parallel branches easily.
- To familiarize students with how to combine voltage sources and resistors connected in parallel into a single equivalent source.
- To develop students' skills in applying electrical laws to simplify networks.
- To enhance the ability to verify calculated results using other analytical methods (such as Kirchhoff's laws or nodal analysis).
- To build the capability to apply the theorem in solving real-life problems in power systems and electronics.
- To help students distinguish between cases where the theorem can be applied and where it cannot.

Millman’s theorem

Millman’s Theorem is one of the electrical circuit analysis methods used to simplify circuits that have multiple parallel branches, where each branch contains a voltage source and a resistor. Instead of using Kirchhoff’s Laws or nodal analysis, this theorem allows us to calculate the voltage at the junction point (the common node voltage) directly using a single formula. The theorem is especially useful in analyzing complex circuits with multiple sources, as it greatly simplifies the process of finding voltage and current.

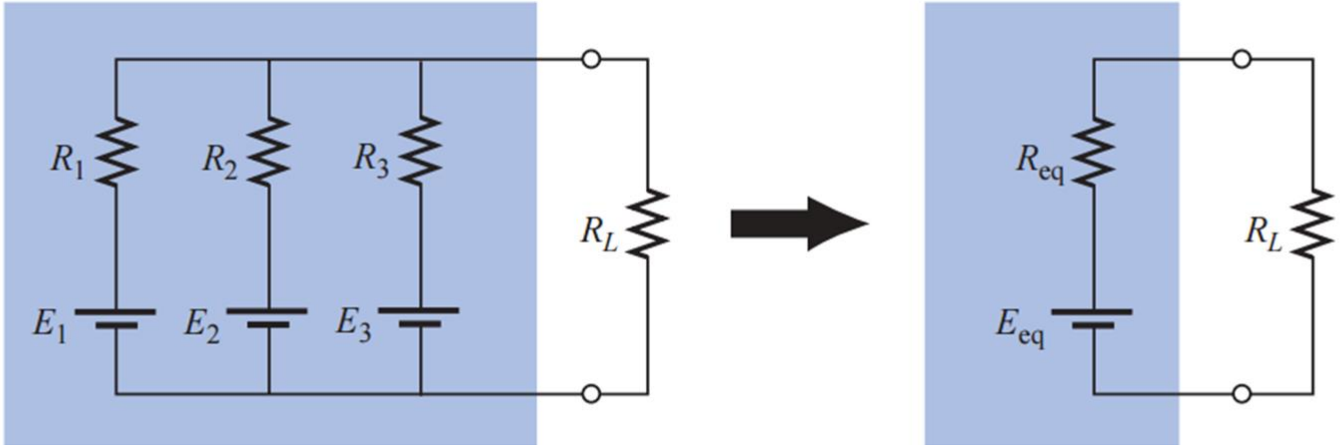


Demonstrating the effect of applying Millman’s theorem

Millman's theorem

$$E_{eq} = \frac{\pm \dfrac{E_1}{R_1} \pm \dfrac{E_2}{R_2} \pm \dfrac{E_3}{R_3} \pm \cdots \pm \dfrac{E_N}{R_N}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}$$

$$E_{eq} = \frac{I_T}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$



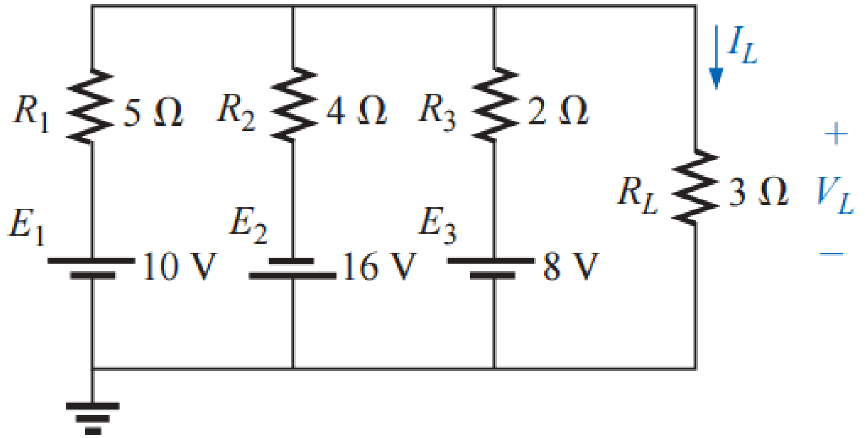
$$E_{eq} = \frac{\pm E_1 G_1 \pm E_2 G_2 \pm E_3 G_3 \pm \cdots \pm E_N G_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$

$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}$$

$$R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + \cdots + G_N}$$

Millman's theorem

EX1: Using Millman's theorem, find the current through and voltage across the resistor R_L of the figure below.



Solution:

$$E_{eq} = \frac{+\dfrac{E_1}{R_1} - \dfrac{E_2}{R_2} + \dfrac{E_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$E_{eq} = \frac{+\dfrac{10\text{ V}}{5\ \Omega} - \dfrac{16\text{ V}}{4\ \Omega} + \dfrac{8\text{ V}}{2\ \Omega}}{\dfrac{1}{5\ \Omega} + \dfrac{1}{4\ \Omega} + \dfrac{1}{2\ \Omega}} = \frac{2\text{ A} - 4\text{ A} + 4\text{ A}}{0.2\text{ S} + 0.25\text{ S} + 0.5\text{ S}}$$

$$= \frac{2\text{ A}}{0.95\text{ S}} = \mathbf{2.105\ V}$$

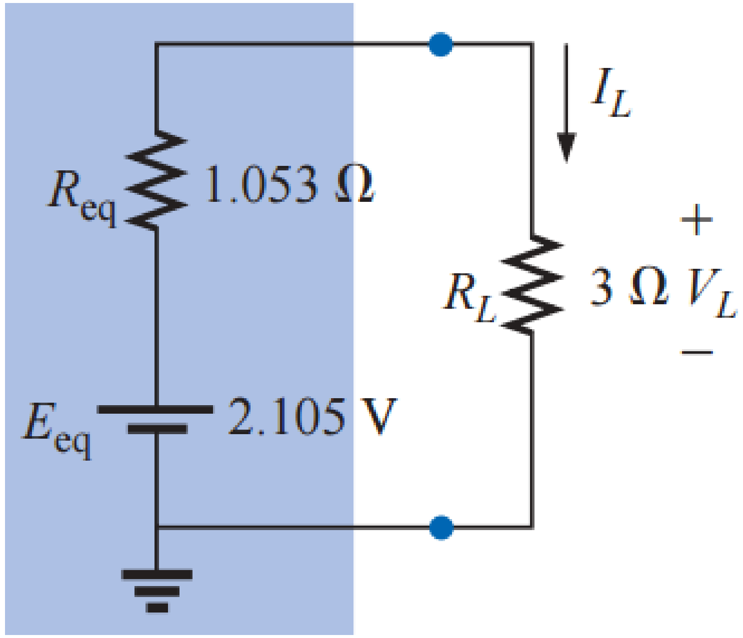
Millman's theorem

with $$R_{eq} = \frac{1}{\frac{1}{5\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}} = \frac{1}{0.95\ \text{S}} = \mathbf{1.053\ \Omega}$$

The resultant source is shown in Fig. and

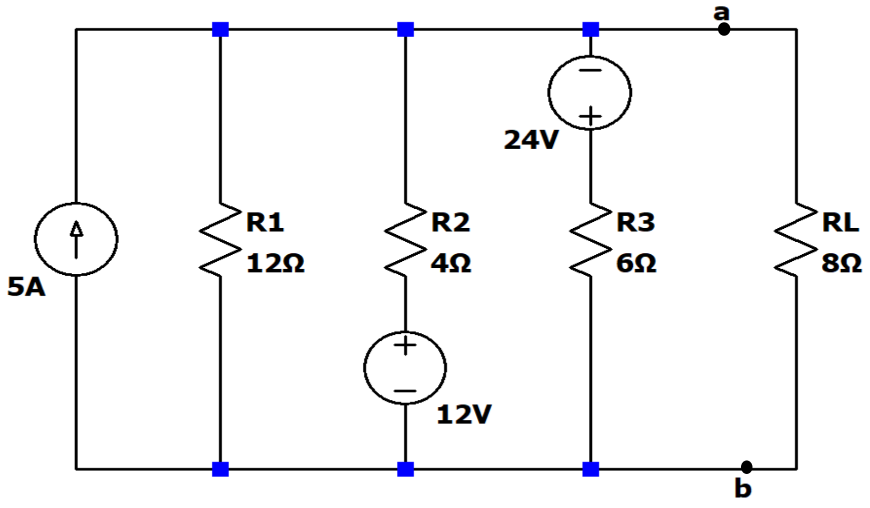
$$I_L = \frac{2.105\ \text{V}}{1.053\ \Omega + 3\ \Omega} = \frac{2.105\ \text{V}}{4.053\ \Omega} = \mathbf{0.519\ A}$$

with $$V_L = I_L R_L = (0.519\ \text{A})(3\ \Omega) = \mathbf{1.557\ V}$$



Millman's theorem

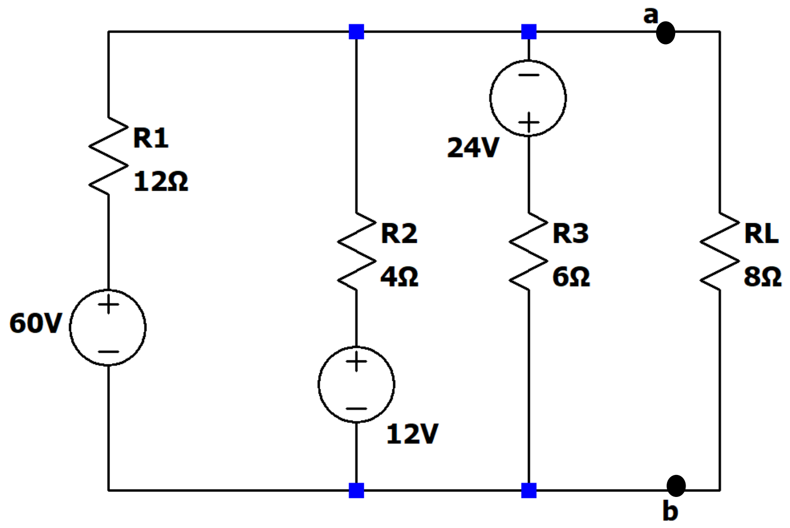
EX2: Using Millman's theorem, to find the voltage across and the through the load (R_L) in the circuit below.



Solution:

First thing to do is convert the current source

into a voltage source. Then circuit become as follows:



$$E_{\mathrm{eq}} = \frac{\pm\dfrac{E_1}{R_1} \pm \dfrac{E_2}{R_2} \pm \dfrac{E_3}{R_3} \pm \cdots \pm \dfrac{E_N}{R_N}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}$$

$$V_{ab} = \frac{\left(\frac{60}{12}\right) + \left(\frac{12}{4}\right) - \left(\frac{24}{6}\right)}{\left(\frac{1}{12}\right) + \left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)} = \frac{4 + 3 - 4}{0.5} = 8\ \mathrm{V}$$

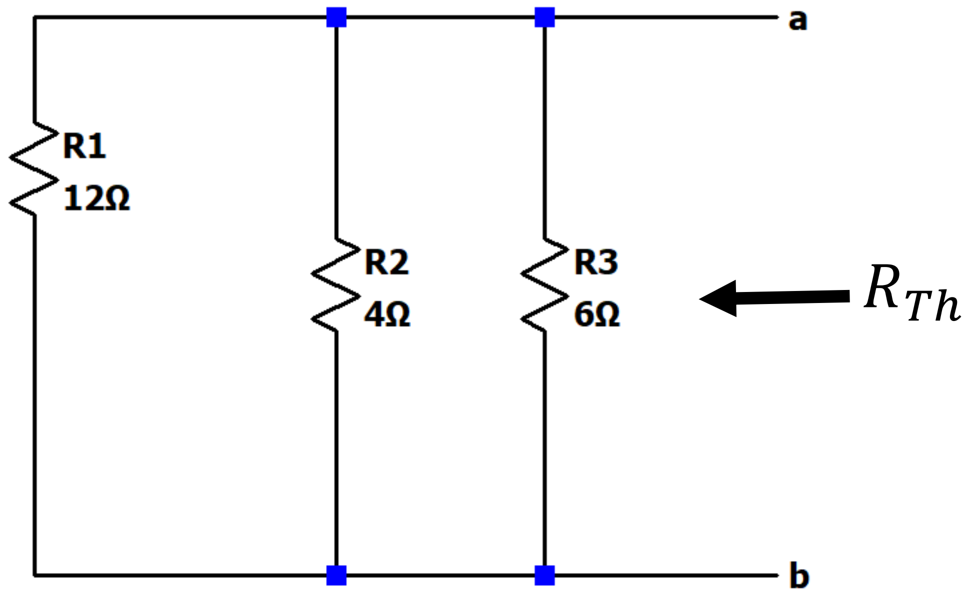
Millman's theorem

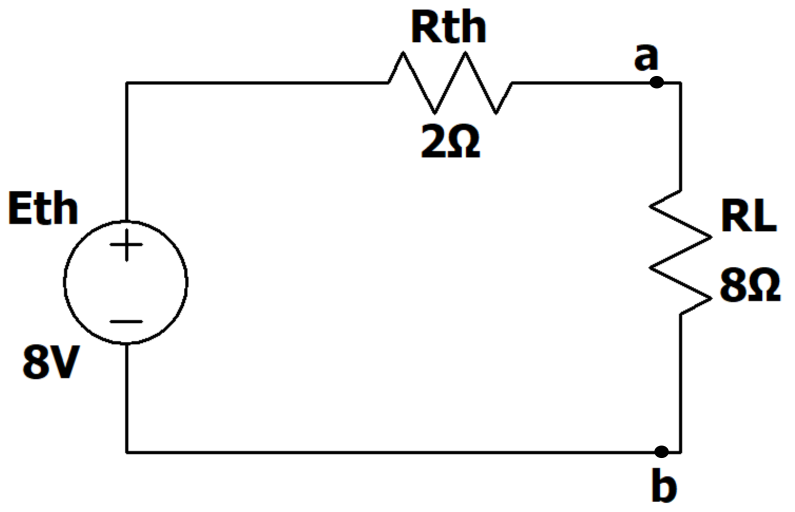
$\therefore \boldsymbol{E_{Th} = 8\ V}$

$R_{Th} = 12 \| 4 \| 6 = 2\ \Omega$

$$I_L = \frac{8}{2+8} = 0.8\ \text{A}$$

$$V_L = 0.8 \times 8 = 6.4\ \text{V}$$





Thank you for listening