Transmission lines

Electrical Department-3rd Stage

Lecture Three

By

Assistant Lecturer: Adnan Ali Abdullah

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❑ THE SMITH CHART

The Smith chart is the most commonly used of the graphical techniques. It is basically a graphical indication of the impedance of a transmission line and of the corresponding reflection coefficient as one moves along the line

Used for calculations of transmission line characteristics such as $\boldsymbol{\Gamma_L}$, s, and Zin.

By assuming that the transmission line to which the Smith chart will be applied is **lossless** $(\boldsymbol{Z_0 = R_0})$

The Smith chart is constructed within a circle of unit radius $(|\boldsymbol{\Gamma_L}| \leq \mathbf{1})$ as shown in Figure 1.4.

The construction of the chart is based on the relation in eq. (1. 13); that is

$$\boldsymbol{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}} \quad \dots \dots (\mathbf{1.13})$$

or

$$\boldsymbol{\Gamma = |\Gamma| \angle\theta_\Gamma = \Gamma_r + j\Gamma_i} \dots\dots\dots\dots\dots\dots\dots\dots\dots (1.23)$$

where $\boldsymbol{\Gamma_r}$ and $\boldsymbol{\Gamma_i}$ are the real and imaginary parts of the reflection coefficient $\boldsymbol{\Gamma}$.

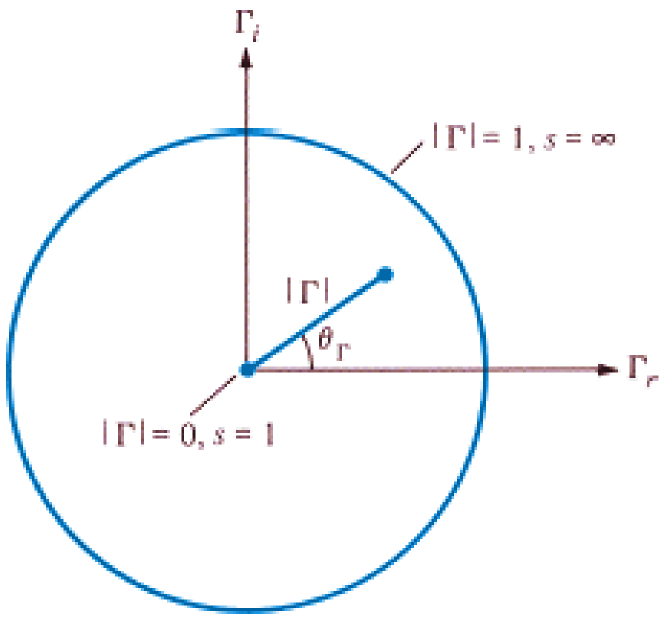


FIGURE 1.4 Unit circle on which the Smith chart is constructed.

Instead of having separate Smith charts for transmission lines with different characteristic impedances (e.g., Zo = 60, 100, 120 Ω), we prefer to have just one that can be used for any line. We achieve this by using a normalized chart in which all impedances are normalized with respect to the characteristic impedance Zo of the particular line under consideration. For the load impedance Z_L , for example, the normalized impedance Z_L is given by

$$z_L = \frac{Z_L}{Z_o} = r + jx \qquad \text{....(1.24)}$$

Substituting eq. (1. 24) into eqs. (1. 13) and (1. 23) gives

$$\Gamma = \Gamma_r + j\Gamma_i = \frac{z_L - 1}{z_L + 1} \qquad \text{....(1.25)}$$

or

$$z_L = r + jx = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} \qquad \text{....(1.26)}$$

Normalizing and equating real and imaginary components, we obtain

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \qquad \text{....(1.27)}$$

$$x = \frac{2\,\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2} \qquad \text{....(1.28)}$$

Rearranging terms in eqs. (1. 27 & 28) leads to

$$\boxed{\left[\Gamma_r - \frac{r}{1+r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2} \quad \text{....(1.29)}$$

and

$$\boxed{[\Gamma_r - 1]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2} \quad \text{....(1.30)}$$

Each of eqs. (11) and (1) is similar to

$$(x-h)^2 + (y-k)^2 = a^2$$

which is the general equation of a circle of radius a, centered at (h, k). Thus eq. (11.50) is an *r-circle (resistance circle)* with

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(\frac{r}{1+r}, 0\right)$$

$$\text{radius} = \frac{1}{1+r}$$

TABLE 1 Radii and Centers of r-Circles for Typical Values of r

Normalized Resistance (r)	Radius $\left(\frac{1}{1+r}\right)$	Center $\left(\frac{r}{1+r}, 0\right)$
0	1	(0, 0)
1/2	2/3	(1/3, 0)
1	1/2	(1/2, 0)
2	1/3	(2/3, 0)
5	1/6	(5/6, 0)
∞	0	(1, 0)

TABLE 11.4 Radii and Centers of x-Circles for Typical Values of x

Normalized Reactance (x)	Radius $\left(\frac{1}{x}\right)$	Center $\left(1, \frac{1}{x}\right)$
0	∞	$(1, \infty)$
$\pm 1/2$	2	$(1, \pm 2)$
± 1	1	$(1, \pm 1)$
± 2	1/2	$(1, \pm 1/2)$
± 5	1/5	$(1, \pm 1/5)$
$\pm\infty$	0	$(1, 0)$

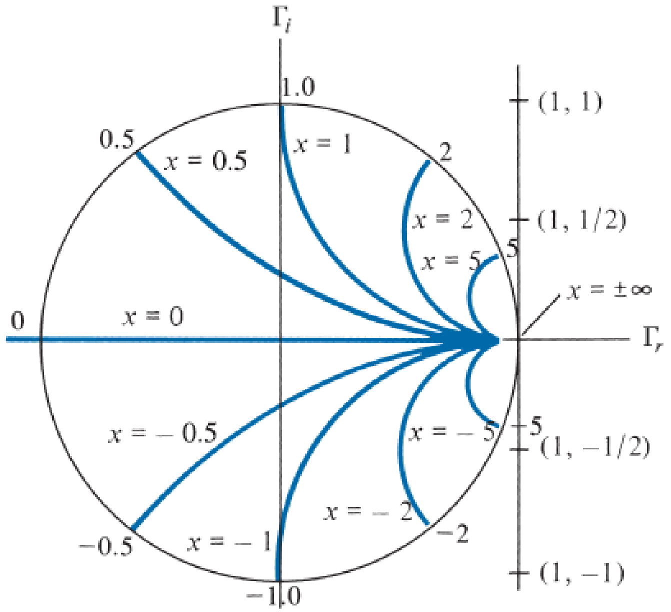


FIGURE 11.12 Typical x-circles for $x = 0, \pm 0.5, \pm 1, \pm 2, \pm 5, \pm\infty$.

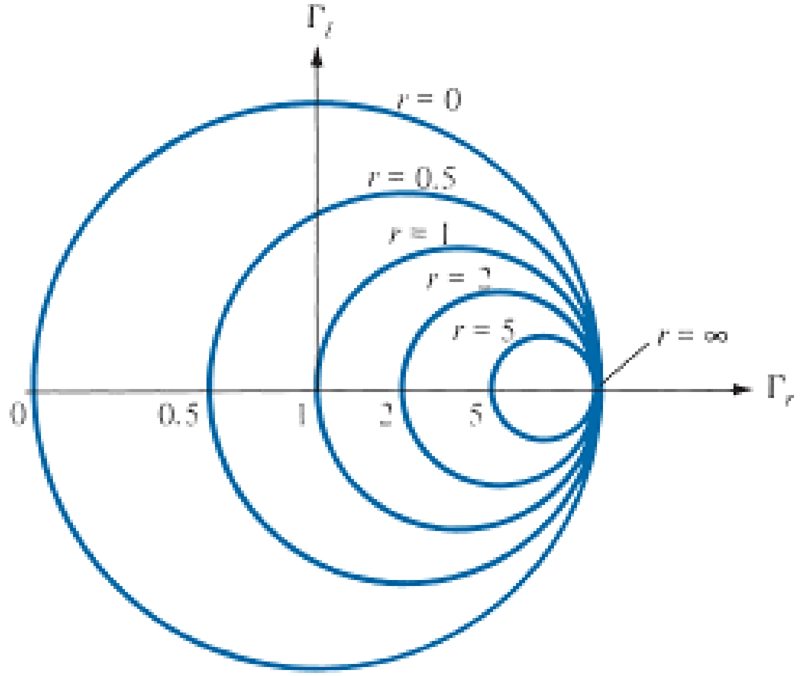


FIGURE 5 Typical r-circles for $r = 0, 0.5, 1, 2, 5, \infty$.

If we superpose the r-circles and x-circles, what we have is the Smith chart shown in Figure 7. On the chart, we locate a normalized impedance z =2+j, for example, as the point of intersection of the r = 2 circle and the x = 1 circle. This is point **P1** in Figure 7. Similarly, z = 1-j0.5 is located at P2, where the r =1 circle and the x = -0.5 circle intersect. centered at the origin with s varying from 1 to ∞.

The value of the standing wave ratio s is determined by locating where an s-circle crosses the ***Γ***r axis

Typical examples of s-circles for s 5 1, 2, 3, and ∞

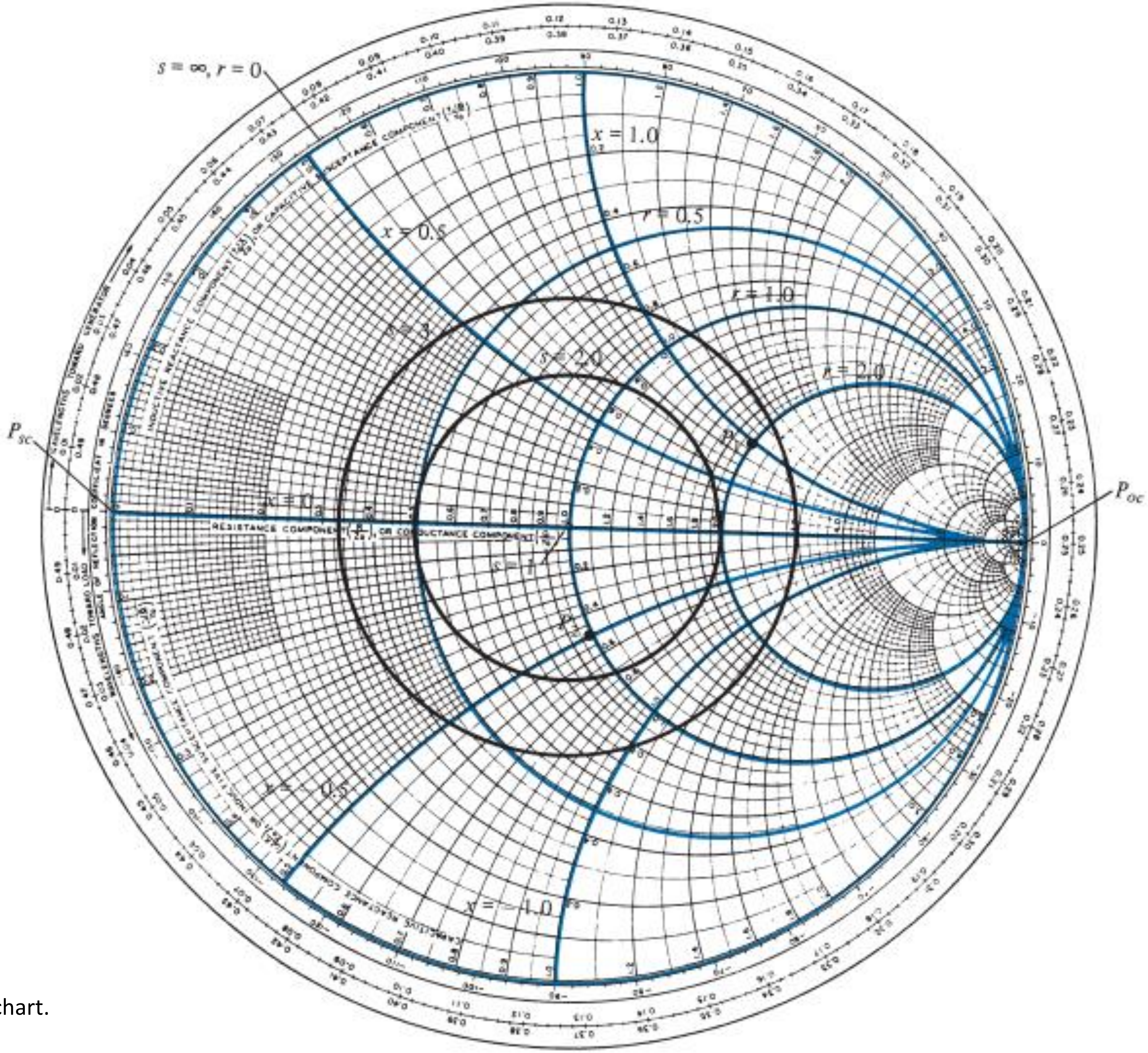
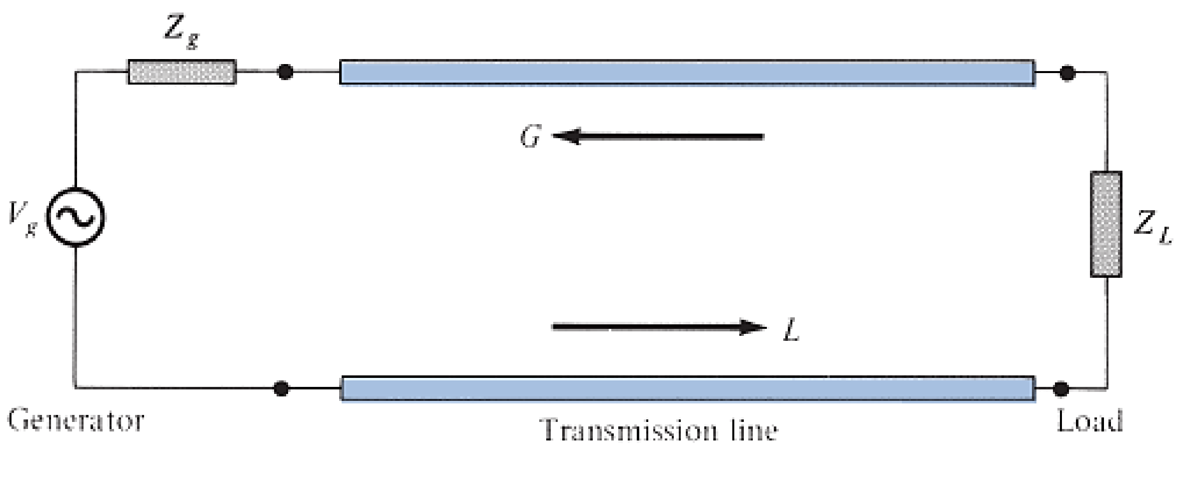


FIGURE 7 Illustration of the *r*-, *x*-, and *s*-circles on the Smith chart.

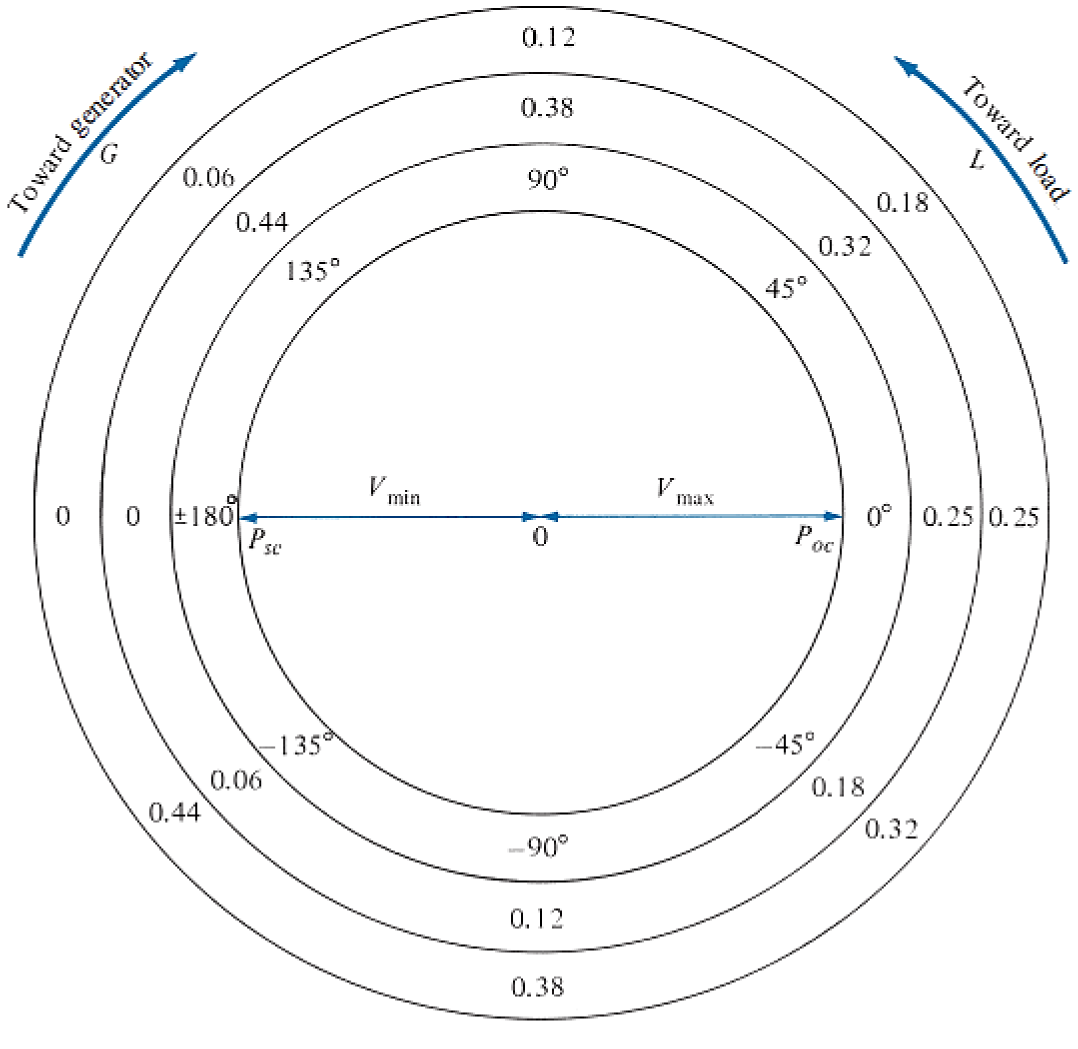
The following points should be noted about the Smith chart.

1. At point P_{sc} on the chart $r = 0, x = 0$; that is, $Z_L = 0 + j0$, showing that P_{sc} represents a short circuit on the transmission line. At point P_{oc}, $r = \infty$ and $x = \infty$, or $Z_L = \infty + j\infty$, which implies that P_{oc} corresponds to an open circuit on the line. Also at P_{oc}, $r = 0$ and $x = 0$, showing that P_{oc} is another location of a short circuit on the line.
2. A complete revolution (360°) around the Smith chart represents a distance of $\lambda/2$ on the line. Clockwise movement on the chart is regarded as moving toward the generator (or away from the load) as shown by the arrow G in Figure 11.14(a) and (b). Similarly, counterclockwise movement on the chart corresponds to moving toward the load (or away from the generator) as indicated by the arrow L in Figure 11.14. Notice from Figure 11.14(b) that at the load, moving toward the load does not make sense (because we are already at the load). The same can be said of the case when we are at the generator end.
3. There are three scales around the periphery of the Smith chart as illustrated in Figure 11.14(a). The three scales are included for the sake of convenience but they are actually meant to serve the same purpose; one scale should be sufficient. The scales are used in determining the distance from the load or generator in degrees or wavelengths. The outermost scale is used to determine the distance on the line from the generator end in terms of wavelengths, and the next scale determines the distance from the load end in terms of wavelengths. The innermost scale is a protractor (in degrees) and is primarily used in determining θ_Γ; it can also be used to determine the distance from the load or generator. Since a $\lambda/2$ distance on the line corresponds to a movement of 360° on the chart, λ *distance on the line corresponds*

4. The voltage V_{max} occurs where $Z_{in,\,max}$ is located on the chart [see eq. (11.39a)], and that is on the positive Γ_r-axis or on OP_{oc} in Figure 11.14(a). The voltage V_{min} is located at the same point where we have $Z_{in,\,min}$ on the chart, that is, on the negative Γ_r-axis or on OP_{sc} in Figure 11.14(a). Notice that V_{max} and V_{min} (or $Z_{in,\,max}$ and $Z_{in,\,min}$) are $\lambda/4$ (or 180°) apart.
5. The Smith chart is used both as impedance chart and admittance chart ($Y = 1/Z$). As admittance chart (normalized admittance $y = Y/Y_o = g + jb$), the g- and b-circles correspond to r- and x-circles, respectively.



(b)



(a)

FIGURE 11.14 **(a)** Smith chart illustrating scales around the periphery and movements around the chart. **(b)** Corresponding movements along the transmission line.

Example 1

A lossless transmission line with $Z_o = 50\ \Omega$ is 30 m long and operates at 2 MHz. The line is terminated with a load $Z_L = 60 + j40\ \Omega$. If $u = 0.6c$ on the line, find

(a) The reflection coefficient Γ

(b) The standing wave ratio s

(c) The input impedance

Solution:

(a) Calculate the normalized load impedance

$$z_L = \frac{Z_L}{Z_o} = \frac{60 + j40}{50}$$
$$= 1.2 + j0.8$$

Locate z_L on the Smith chart of Figure 11.15 at point P, where the $r = 1.2$ circle and the $x = 0.8$ circle meet. To get Γ at z_L, extend OP to meet the $r = 0$ circle at Q and measure OP and OQ. Since OQ corresponds to $|\Gamma| = 1$, then at P,

$$|\Gamma| = \frac{OP}{OQ} = \frac{3.2\text{ cm}}{9.1\text{ cm}} = 0.3516$$

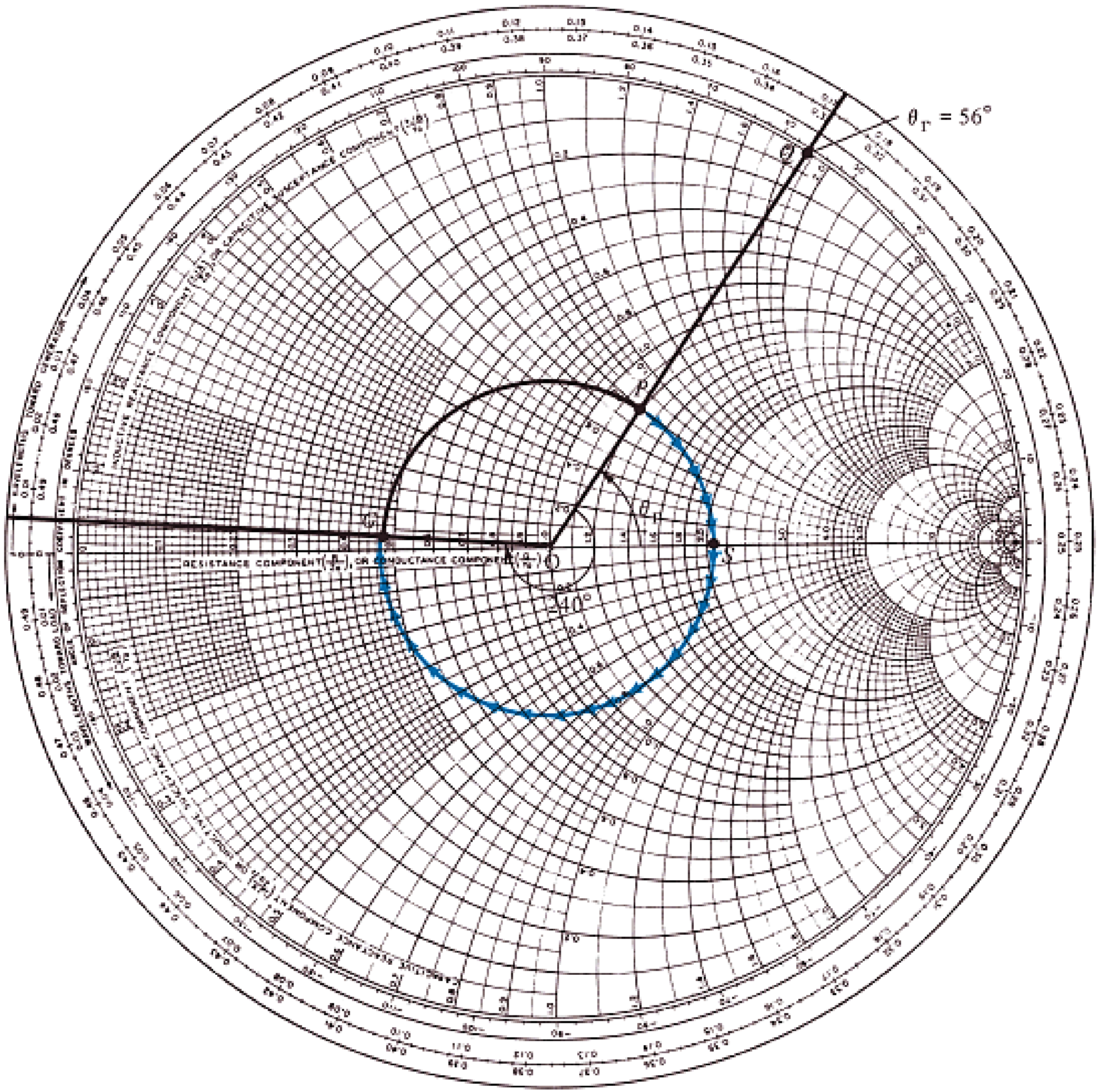


FIGURE 11.15 Smith chart for Example 11.4.

Note that $OP = 3.2$ cm and $OQ = 9.1$ cm were taken from the Smith chart used by the author; the Smith chart in Figure 11.15 is reduced, but the ratio of OP/OQ remains the same.

Angle θ_Γ is read directly on the chart as the angle between OS and OP; that is,

$$\theta_\Gamma = \text{angle } POS = 56^\circ$$

Thus

$$\Gamma = 0.3516 \,\underline{/56^\circ}$$

(b) To obtain the standing wave ratio s, draw a circle with radius OP and center at O. This is the constant s or $|\Gamma|$ circle. Locate point S where the s-circle meets the Γ_r-axis. [This is easily shown by setting $\Gamma_i = 0$ in eq. (11.49a).] The value of r at this point is s; that is,

$$\begin{aligned} s &= r(\text{for } r \geq 1) \\ &= 2.1 \end{aligned}$$

(c) To obtain Z_{in}, first express ℓ in terms of λ or in degrees:

$$\lambda = \frac{u}{f} = \frac{0.6(3 \times 10^8)}{2 \times 10^6} = 90 \text{ m}$$

$$\ell = 30 \text{ m} = \frac{30}{90}\lambda = \frac{\lambda}{3} \rightarrow \frac{720^\circ}{3} = 240^\circ$$

Since λ corresponds to an angular movement of 720° on the chart, the length of the line corresponds to an angular movement of 240°. That means we move toward the generator (or away from the load, in the clockwise direction) 240° on the s-circle from point P to point G. At G, we obtain

$$z_{in} = 0.47 + j0.03$$

Hence

$$Z_{in} = Z_o z_{in} = 50(0.47 + j0.03) = 23.5 + j1.5\ \Omega$$

Example 2

A load of $100 + j150\ \Omega$ is connected to a $75\ \Omega$ lossless line. Find:

(a) Γ

(b) s

(c) The load admittance Y_L

(d) Z_{in} at 0.4λ from the load

(e) The locations of V_{max} and V_{min} with respect to the load if the line is 0.6λ long

(f) Z_{in} at the generator.

Solution:

(a) We can use the Smith chart to solve this problem. The normalized load impedance is

$$z_L = \frac{Z_L}{Z_o} = \frac{100 + j150}{75} = 1.33 + j2$$

We locate this at point P on the Smith chart of Figure 11.16. At P, we obtain

$$|\Gamma| = \frac{OP}{OQ} = \frac{6 \text{ cm}}{9.1 \text{ cm}} = 0.659$$

$$\theta_\Gamma = \text{angle } POS = 40^\circ$$

Hence,

$$\Gamma = 0.659 \underline{/40^\circ}$$

Check:

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100 + j150 - 75}{100 + j150 + 75}$$

$$= 0.6598 \underline{/39.94^\circ}$$

(b) Draw the constant s-circle passing through P and obtain

$$s = 4.82$$

Check:

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.659}{1 - 0.659} = 4.865$$

(c) To obtain Y_L, extend PO to POP' and note point P' where the constant s-circle meets POP'. At P', obtain

$$y_L = 0.228 - j0.35$$

The load admittance is

$$Y_L = Y_o y_L = \frac{1}{75}(0.228 - j0.35) = 3.04 - j4.67 \text{ mS}$$

Check:

$$Y_L = \frac{1}{Z_L} = \frac{1}{100 + j150} = 3.07 - j4.62 \text{ mS}$$

or

$$Z_{in} = 21.9 + j47.6\ \Omega$$

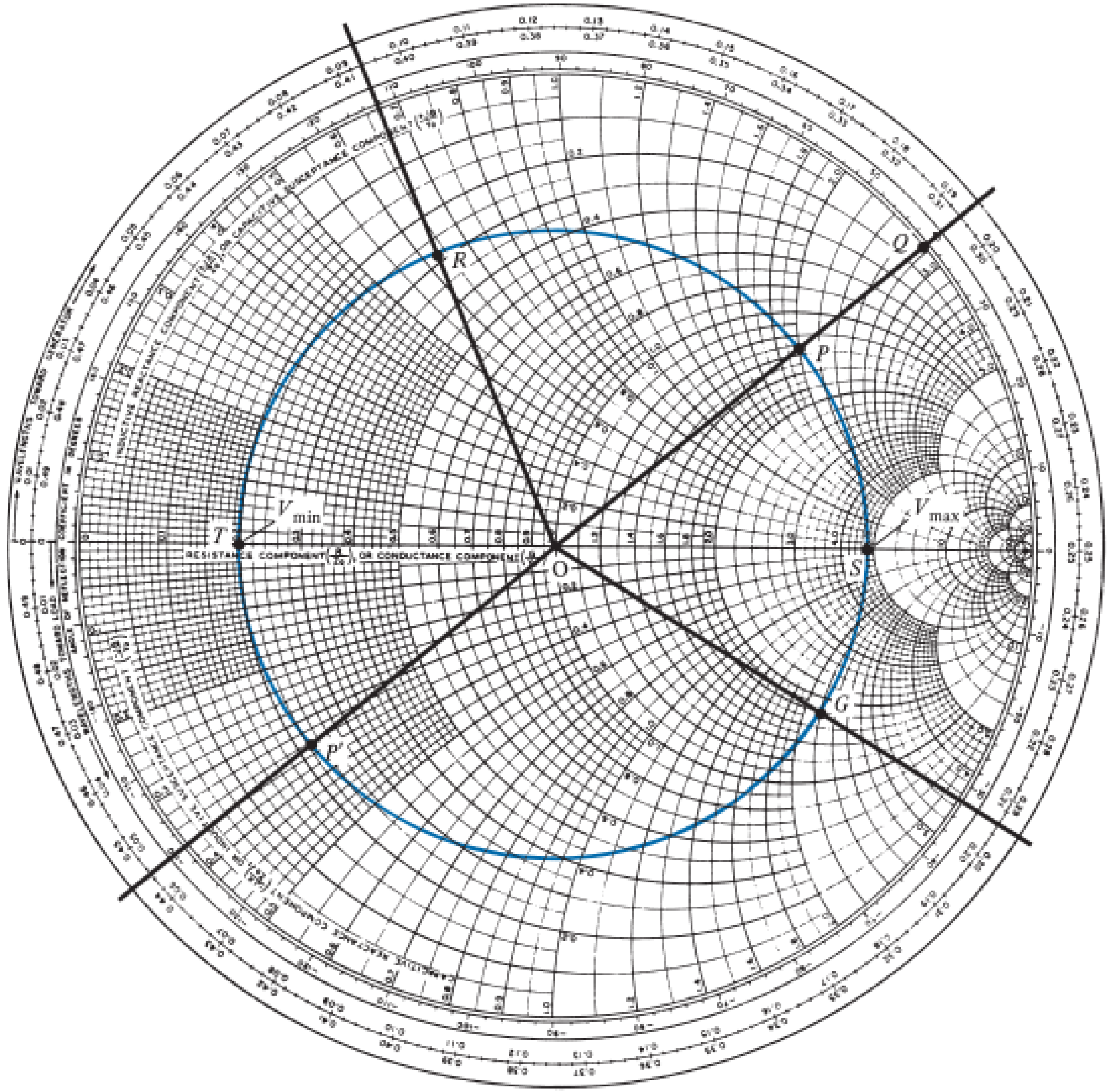


FIGURE 11.16 Smith chart for Example 11.5.

(e) The 0.6λ corresponds to an angular movement of

$$0.6 \times 720° = 432° = 1 \text{ revolution} + 72°$$

Thus, we start from P (load end), move clockwise along the s-circle 432°, or one revolution plus 72°, and reach the generator at point G. Note that to reach G from P, we have passed through point T (location of V_{min}) once and point S (location of V_{max}) twice. Thus, from the load,

$$1\text{st } V_{max} \text{ is located at } \frac{40°}{720°}\lambda = 0.055\lambda$$

$$2\text{nd } V_{max} \text{ is located at } 0.0555\lambda + \frac{\lambda}{2} = 0.555\lambda$$

and the only V_{min} is located at $0.055\lambda + \lambda/4 = 0.3055\lambda$

(f) At G (generator end),

$$z_{in} = 1.8 - j2.2$$

$$Z_{in} = 75(1.8 - j2.2) = 135 - j165\ \Omega$$

This can be checked by using eq. (11.34), where $\beta\ell = \frac{2\pi}{\lambda}(0.6\lambda) = 216°$.

We can see how much time and effort are saved by using the Smith chart.