Transmission lines

Electrical Department-3rd Stage

Lecture one

By
Assistant Lecturer: Adnan Ali Abdullah
2025-2024

❑ 1.Transmission lines

A Transmission line is a mechanism of guiding electrical energy from one place to another. (or) Transmission line is an electrical line which is used to transmit electrical waves from one point to another. Eg: i. Transfer of RF energy from transmitter to antenna. ii. Transmission lines can also be used as impedance transformers.

❑ 2.Transmission Line Equivalent circuit

A transmission line is a two-port network, with each port consisting of two terminals, as illustrated in Fig.1. One of the ports, the line's sending end, is connected to a source (also called the generator). The other port, the line's receiving end, is connected to a load.

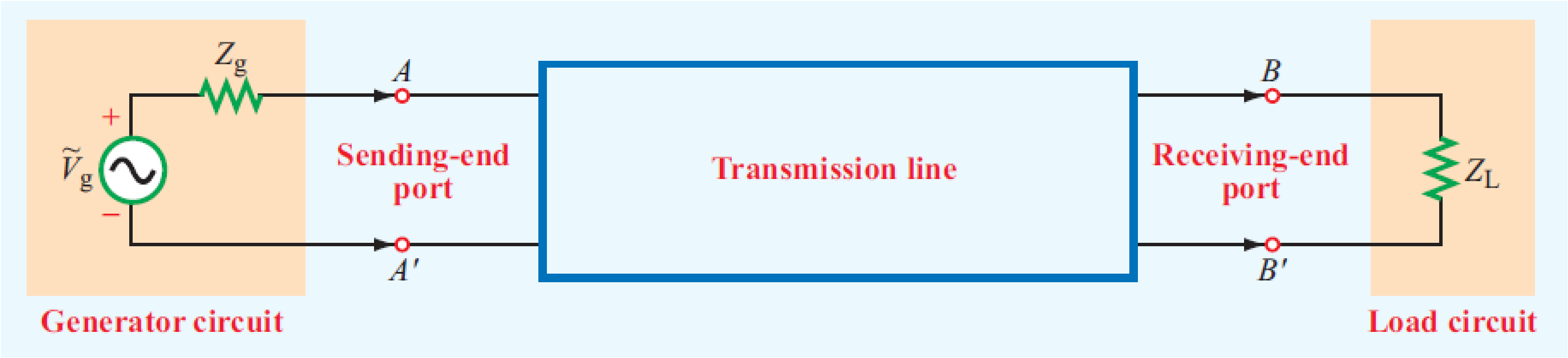


Fig.1

2.1 Lumped-element circuit of coaxial line

the lumped-element circuit model, consists of four basic elements with values that henceforth will be called the transmission line parameters. These are:

- R′: The combined resistance of both conductors per unit length, in Ω/m,
- L′: The combined inductance of both conductors per unit length, in H/m,
- G′: The conductance of the insulation medium between the two conductors per unit length, in S/m, and
- C′: The capacitance of the two conductors per unit length, in F/m.

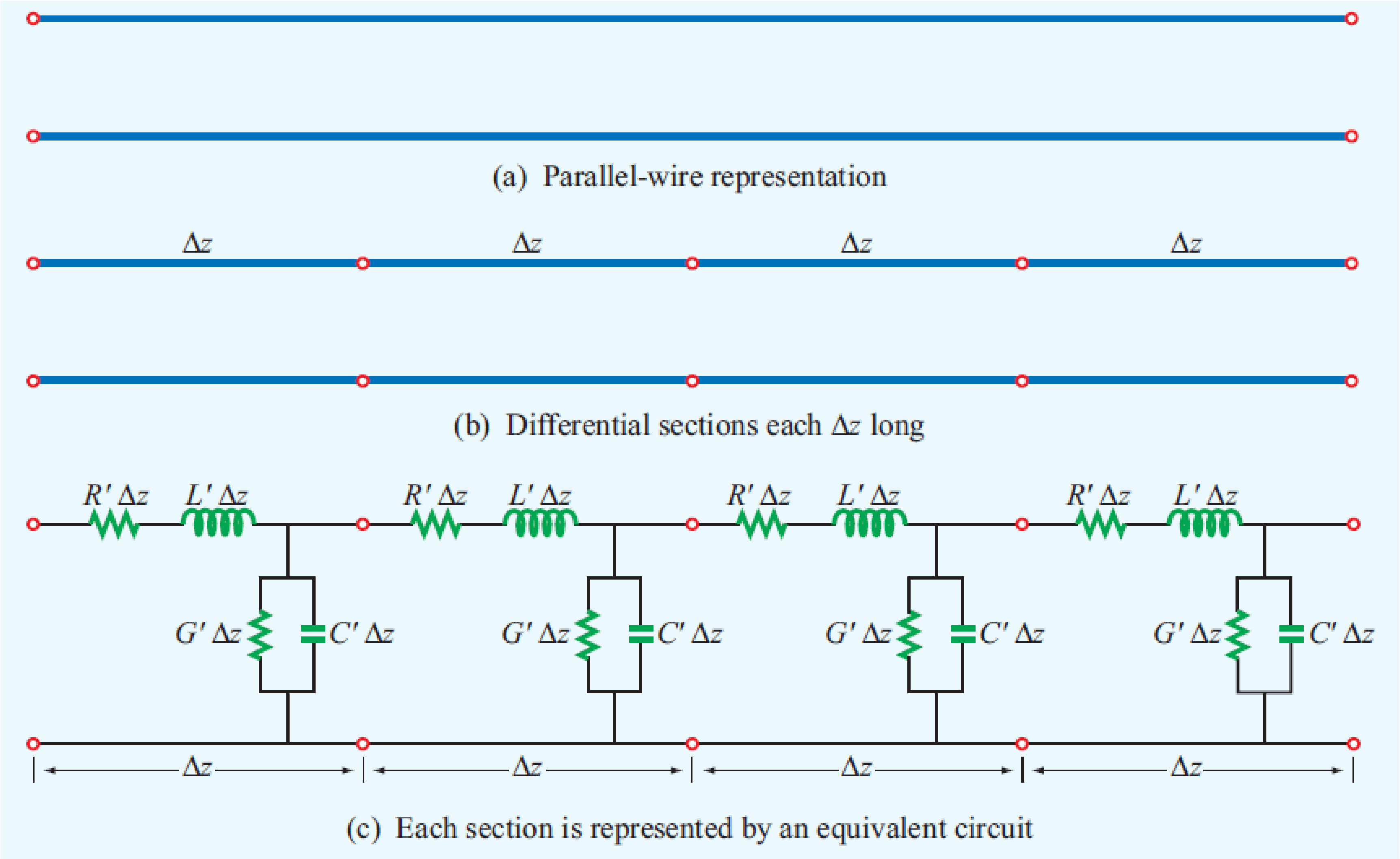


Fig.2

2.2 TRANSMISSION LINE PARAMETERS

To describe a transmission line in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C. Each of the lines shown in Figure 3 has specific formulas for finding R, L, G, and C.

For coaxial, two-wire, and planar lines, the formulas for calculating the values of R, L, G, and C are provided in Table 1.

TABLE 1 Distributed Line Parameters at High Frequencies*

Parameters	Coaxial Line	Two-Wire Line	Planar Line
$R\ (\Omega/\text{m})$	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$	$\frac{1}{\pi a\delta\sigma_c}$	$\frac{2}{w\delta\sigma_c}$
	$(\delta \ll a, c-b)$	$(\delta \ll a)$	$(\delta \ll t)$
$L\ (\text{H/m})$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
$G\ (\text{S/m})$	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
$C\ (\text{F/m})$	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ $(w \gg d)$

*$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\frac{d}{2a} \simeq \ln\frac{d}{a}$ if $\left[\frac{d}{2a}\right]^2 \gg 1$.

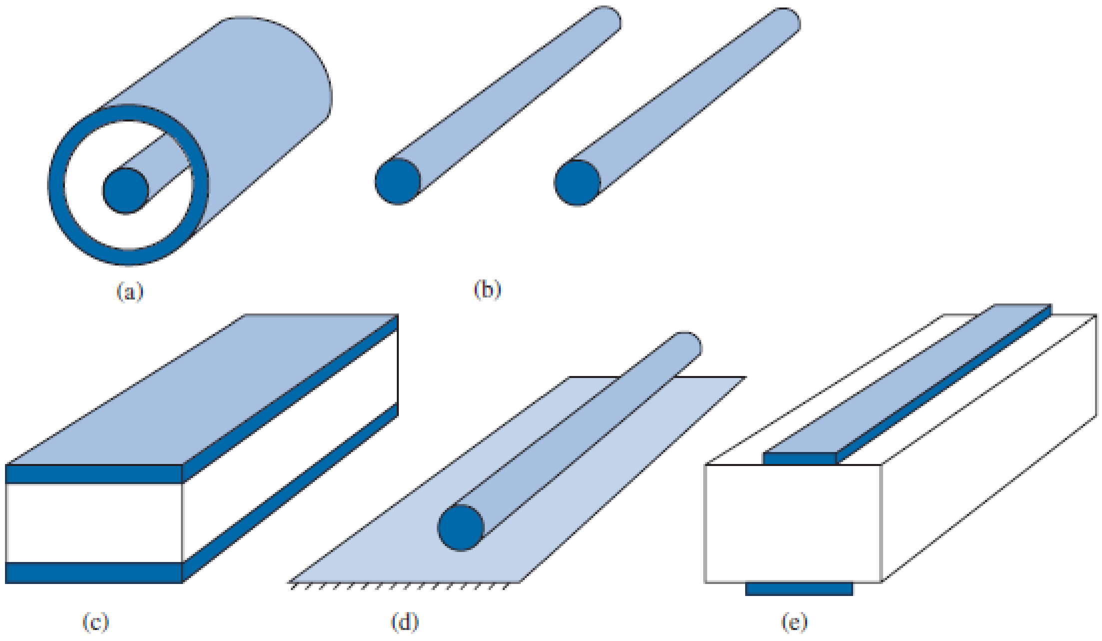


Fig.3

Typical transmission lines in cross-sectional view: **(a)** coaxial line, **(b)** two-wire line, **(c)** planar line, **(d)** wire above conducting plane, **(e)** microstrip line.

1. The line parameters R, L, G, and C are not discrete or lumped. Rather, they are distributed as shown in Figure 4. By this we mean that the parameters are uniformly distributed along the entire length of the line.

2. For each line, the conductors are characterized by $\sigma_c, \mu_c, \varepsilon_c = \varepsilon_o$, and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.

3. $G \neq 1/R$; R is the ac resistance per unit length of the conductors comprising the line, and G is the conductance per unit length due to the dielectric medium separating the conductors.
4. The value of L shown in Table 11.1 is the external inductance per unit length, that is, $L = L_{ext}$. The effects of internal inductance L_{in} $(= R/\omega)$ are negligible at the high frequencies at which most communication systems operate.
5. For each line,

$$LC = \mu\varepsilon \quad \text{and} \quad \frac{G}{C} = \frac{\sigma}{\varepsilon} \qquad (1\ \ 1)$$

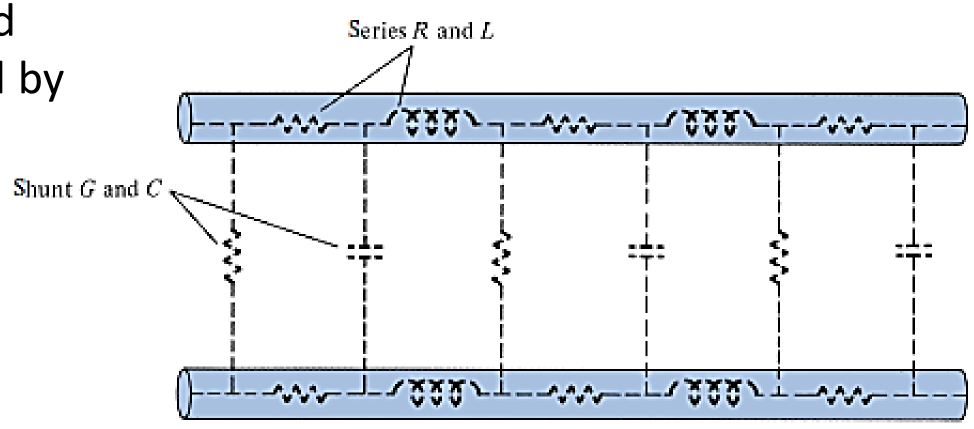


Fig.4

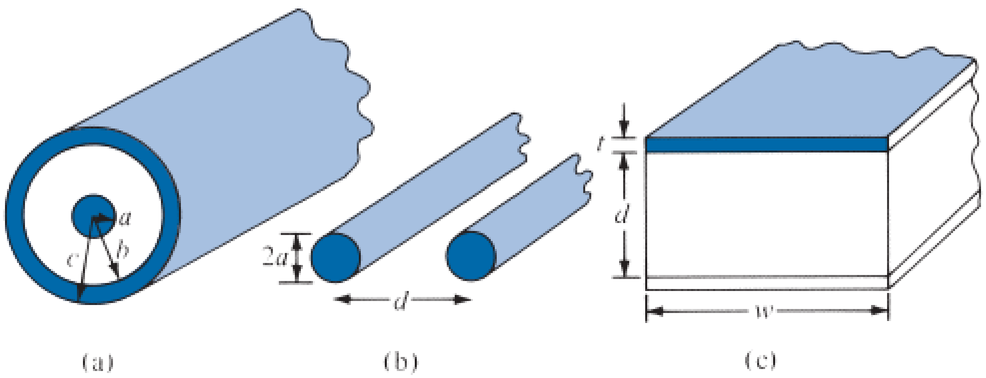


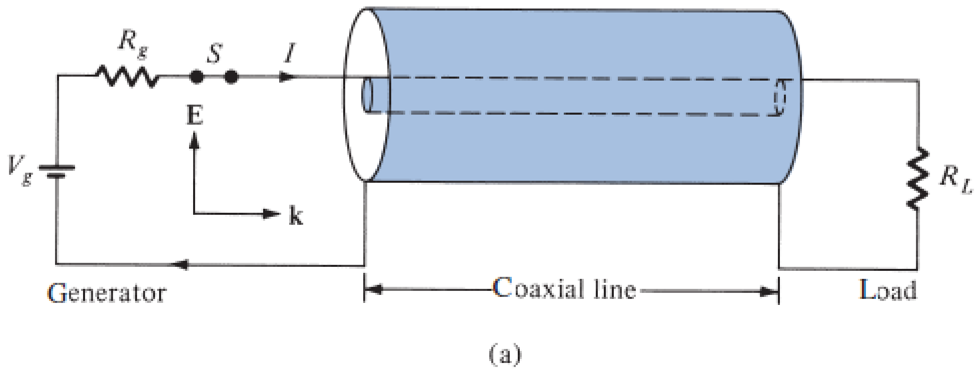
Fig.5

FIGURE 5 Common transmission lines: **(a)** coaxial line, **(b)** two-wire line, **(c)** planar line.

As a way of preparing for the next section, let us consider how an EM wave propagates through a two-conductor transmission line. For example, consider the coaxial line connecting the generator or source to the load as in Figure 6 (a). When switch S is closed, the inner conductor is made positive with respect to the outer one so that

the E field is radially outward as in Figure 6 (b). According to Ampere’s law, the H field encircles the current-carrying conductor as in Figure 6 (b).

The Poynting vector (E × H) points along the transmission line. Thus, closing the switch simply establishes a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line. This wave is a nonuniform plane wave, and by means of it, power is transmitted through the line.



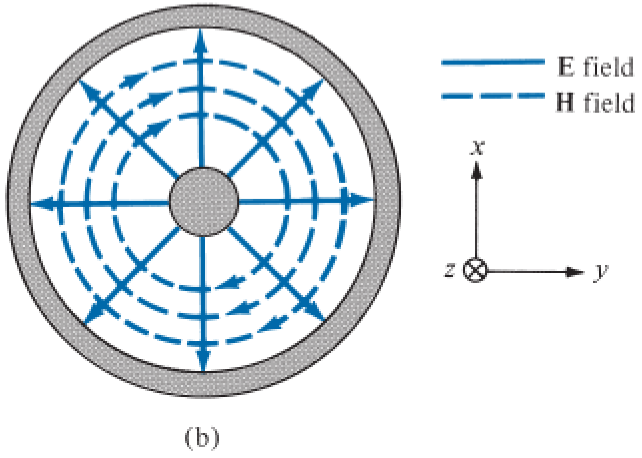


FIGURE 6 (a) Coaxial line connecting the generator to the load; **(b)** E and **H** fields on the coaxial line.

2.3 TRANSMISSION LINE EQUATIONS

A two-conductor transmission line supports a TEM wave; that is, the electric and magnetic fields on the line are perpendicular to each other and transverse to the direction of wave propagation. An important property of TEM waves is that the fields E and H are uniquely related to voltage V and current I, respectively:

$$V = -\int_L \mathbf{E} \cdot d\mathbf{l}, \quad I = \oint_L \mathbf{H} \cdot d\mathbf{l} \qquad (1\ 2)$$

we will use circuit quantities V and I in solving the transmission line problem instead of solving field quantities E and H

Let us examine an incremental portion of length ∇z of a two-conductor transmission line.

The model in Figure 7 is in terms of the line parameters R, L, G, and C, and may represent any of the two-conductor lines

The model is called the L-type equivalent circuit; there are other possible types

In the model of Figure 7, we assume that the wave propagates along the +z-direction, from the generator to the load.

By applying Kirchhoff ’s voltage law to the outer loop of the circuit in Figure 7, we obtain

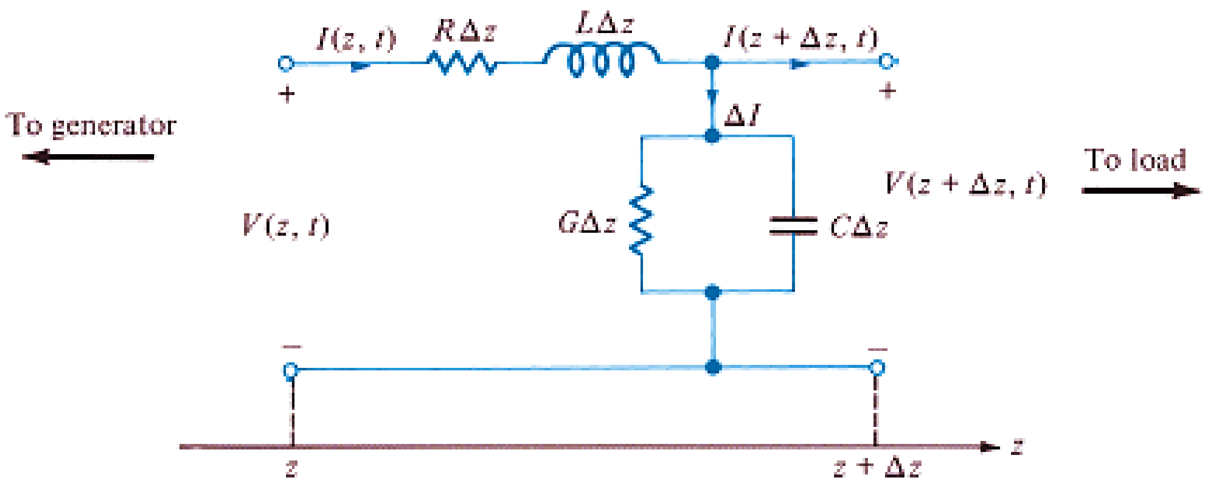


FIGURE 7 An L-type equivalent circuit model of a two-conductor transmission line of differential length ∇z.

γ = *propagation constant*

β = the phase constant (in radians per meter).

u = *wave velocity*

λ= wave length

α = *attenuation constant*

$$\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$\lambda = \frac{2\pi}{\beta}$$

$$\boxed{u = \frac{\omega}{\beta} = f\lambda}$$

$$V_s(z) = \underset{\longrightarrow +z}{V_o^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{V_o^- e^{\gamma z}}$$

and

$$I_s(z) = \underset{\longrightarrow +z}{I_o^+ e^{-\gamma z}} + \underset{-z \longleftarrow}{I_o^- e^{\gamma z}}$$

where V_o^+, V_o^-, I_o^+, and I_o^- are wave amplitudes; the + and − signs, respectively, denote waves traveling along $+z$- and $-z$-directions, as is also indicated by the arrows. We obtain the instantaneous expression for voltage as

$$\begin{aligned} V(z, t) &= \mathrm{Re}[V_s(z)\, e^{j\omega t}] \\ &= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z) \end{aligned}$$

The **characteristic impedance** $\boldsymbol{Z_o}$ of the line is the ratio of the positively traveling voltage wave to the current wave at any point on the line.

The characteristic impedance Zo is analogous to η, the intrinsic impedance of the medium of wave propagation.

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

$$\boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o}$$

where R_o and X_o are the real and imaginary parts of Z_o. Do not mistake R_o for R—while R is in ohms per meter, R_o is in ohms. The propagation constant γ and the characteristic impedance Z_o are important properties of the line because both depend on the line parameters R, L, G, and C and the frequency of operation. The reciprocal of Z_o is the characteristic admittance Y_o, that is, $Y_o = 1/Z_o$.

A. Lossless Line ($R = 0 = G$)

A transmission line is said to be **lossless** if the conductors of the line are perfect ($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma \simeq 0$).

For such a line, it is evident from Table 11.1 that when $\sigma_c \simeq \infty$ and $\sigma \simeq 0$,

$$\boxed{R = 0 = G}$$

This is a necessary condition for a line to be lossless. Thus for such a line, eq. (11.20) forces eqs. (11.11), (11.14), and (11.19) to become

$$\alpha = 0, \quad \gamma = j\beta = j\omega\sqrt{LC} \tag{11.21a}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{11.21b}$$

$$X_o = 0, \quad Z_o = R_o = \sqrt{\frac{L}{C}} \tag{11.21c}$$

B. Distortionless Line ($R/L = G/C$)

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line because α is frequency dependent. Since, in general, the phase velocity of each frequency component is also frequency dependent, this will result in distortion.

> A **distortionless line** is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.

From the general expression for α and β [in eq. (11.11)], a distortionless line results if the line parameters are such that

$$\boxed{\frac{R}{L} = \frac{G}{C}} \tag{11.22}$$

Thus, for a distortionless line,

$$\begin{aligned}\gamma &= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)} \\ &= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta\end{aligned}$$

or

$$\alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC} \tag{11.23a}$$

showing that α does not depend on frequency, whereas β is a linear function of frequency. Also

$$Z_o = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o + jX_o$$

or

$$R_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \quad X_o = 0 \tag{11.23b}$$

and

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{11.23c}$$

Note the following important properties.

1. The phase velocity is independent of frequency because the phase constant β linearly depends on frequency. We have shape distortion of signals unless α and u are independent of frequency.

TABLE 1.2 Transmission Line Characteristics

Case	Propagation Constant $\gamma = \alpha + j\beta$	Characteristic Impedance $Z_o = R_o + jX_o$
General	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\dfrac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\dfrac{L}{C}} + j0$

2. Both u and Z_o remain the same as for lossless lines.
3. A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

A summary of our discussion in this section is in Table 11.2. For the greater part of our analysis, we shall restrict our discussion to lossless transmission lines.

Example 1

An air line has a characteristic impedance of 70 Ω and a phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

Solution:

An air line can be regarded as a lossless line because $\sigma \simeq 0$ and $\sigma_c \rightarrow \infty$. Hence

$$R = 0 = G \quad \text{and} \quad \alpha = 0$$

$$Z_o = R_o = \sqrt{\frac{L}{C}} \tag{11.1.1}$$

$$\beta = \omega\sqrt{LC} \tag{11.1.2}$$

Dividing eq. (11.1.1) by eq. (11.1.2) yields

$$\frac{R_o}{\beta} = \frac{1}{\omega C}$$

or

$$C = \frac{\beta}{\omega R_o} = \frac{3}{2\pi \times 100 \times 10^6(70)} = 68.2 \text{ pF/m}$$

From eq. (11.1.1),

$$L = R_o^2 C = (70)^2(68.2 \times 10^{-12}) = 334.2 \text{ nH/m}$$

Example 2

A distortionless line has $Z_o = 60\ \Omega$, $\alpha = 20$ mNp/m, $u = 0.6c$, where c is the speed of light in a vacuum. Find R, L, G, C, and λ at 100 MHz.

Solution:

For a distortionless line,

$$RC = GL \quad \text{or} \quad G = \frac{RC}{L}$$

and hence

$$Z_o = \sqrt{\frac{L}{C}} \tag{11.2.1}$$

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_o} \tag{11.2.2a}$$

or

$$R = \alpha Z_o \tag{11.2.2b}$$

But

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \tag{11.2.3}$$

From eq. (11.2.2b),

$$R = \alpha Z_o = (20 \times 10^{-3})(60) = 1.2\ \Omega/\text{m}$$

Dividing eq. (11.2.1) by eq. (11.2.3) results in

$$L = \frac{Z_o}{u} = \frac{60}{0.6\,(3 \times 10^8)} = 333\ \text{nH/m}$$

From eq. (11.2.2a),

$$G = \frac{\alpha^2}{R} = \frac{400 \times 10^{-6}}{1.2} = 333\ \mu\text{S/m}$$

Multiplying eqs. (11.2.1) and (11.2.3) together gives

$$uZ_o = \frac{1}{C}$$

or

$$C = \frac{1}{uZ_o} = \frac{1}{0.6\,(3 \times 10^8)\,60} = 92.59\ \text{pF/m}$$

$$\lambda = \frac{u}{f} = \frac{0.6\,(3 \times 10^8)}{10^8} = 1.8\ \text{m}$$