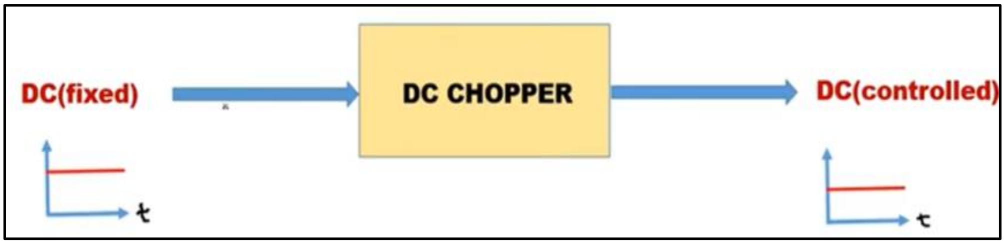
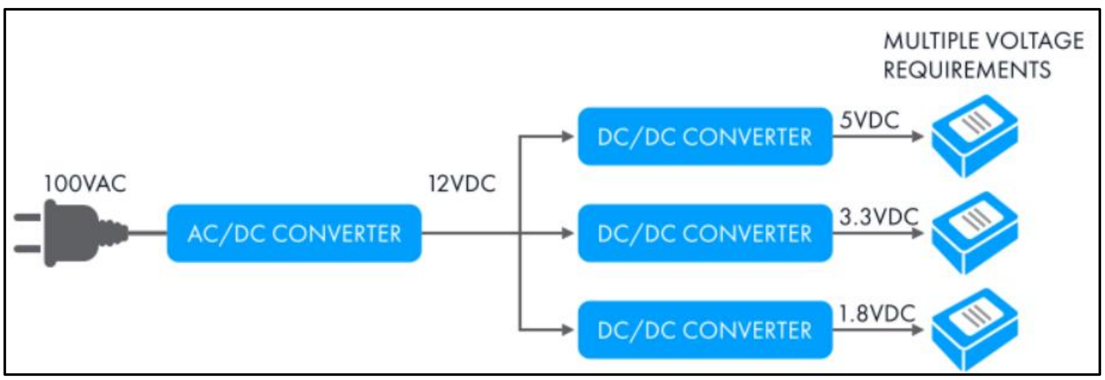
Lecture -7-

Dc - Dc Converter

Dc-dc converter are power electronic circuit that convert the input dc voltage (fixed) to a different dc voltage level at output (controlled voltage), often providing a regulated output.



For example, in a computer off-line power supply, the 120 V or 240 V ac voltage is rectified, producing a DC voltage of approximately 170 V or 340 V, respectively. A dc-dc converter then reduces the voltage to the regulated 5 V or 3.3 V required by the processor ICs.



DC-to-DC converters are placed between the power source (an AC-to-DC converter) and voltage consumers

These circuits help distribute and manage power properly to provide each power consumer with appropriate voltage or current level. It also protects highly-sensitive sub-circuits.

The functions of dc-dc converters are:(وظائف)

1. Convert a fixed dc input voltage Vs into controlled dc output voltage Vo.
2. Regulate the dc output voltage against load and line variations.
3. Reduce the voltage ripple on the output voltage.
4. Provide isolation between the input source and the load.
5. Protect the input supplied system from electromagnetic interference (EMI)
6. Satisfy various international safety standards.

Advantages of dc-dc converters

1. It has greater efficiency.
2. It has faster response.
3. It requires less maintenance
4. It has compact size.
5. It has smooth control.
6. It is cheap. Its cost is lower than motor-generator sets.

Applications of dc-dc converters are:

1. Battery chargers.
2. Spacecraft power systems,
3. laptop computers,
4. Telecommunications equipment,
5. Railway systems.
6. In DC motors as a speed controllers.

Methods of Converting Dc-Dc

- Linear voltage regulators.
- Switching converter (chopper).

- ***Linear voltage regulator*** is one method of converting a dc voltage to a lower dc voltage expressed as a simple circuit using a transistor controls the load current. The output voltage is

$$V_o = I_L R_L$$

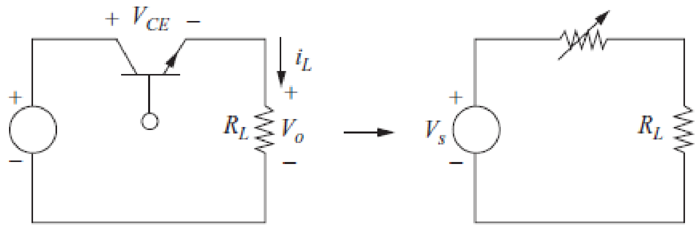


Figure A basic linear regulator.

By adjusting the transistor base current, the output voltage may be controlled over a range of 0 to about Vs. The base current can be adjusted to compensate for variations in the supply voltage or the load, thus regulating the output. This type of circuit is called a linear regulator because the transistor operates in the linear region, rather than in the saturation or cutoff regions. The transistor in effect operates as a variable resistance.

The power loss in the transistor makes this circuit inefficient, thus, the low efficiency is a serious drawback for power applications. Assuming a small base current, the power absorbed by the load is ($P=V_o I_L$), and power absorbed by the transistor is ($P_t = V_{CE}.I_L$). For example, if the output voltage is one-quarter of the input voltage, the load resistor absorbs one-quarter of the source power, which is an efficiency of 25 %, the transistor absorbs the other 75% of the power supplied by the source. Low output voltage result in lower efficiency. Therefore, the linear voltage regulator is suitable only for low-power applications.

- ***Switching converter*** is an efficient alternative to the linear regulator. In a switching converter circuit, the transistor operates as an electronic switch by being completely on or completely off (saturation or cutoff for a BJT). This circuit is also known as a dc **chopper.** Assuming the switch is ideal in Fig. below, the output is the same as the input when the switch is closed, and the output is zero when the switch is open.

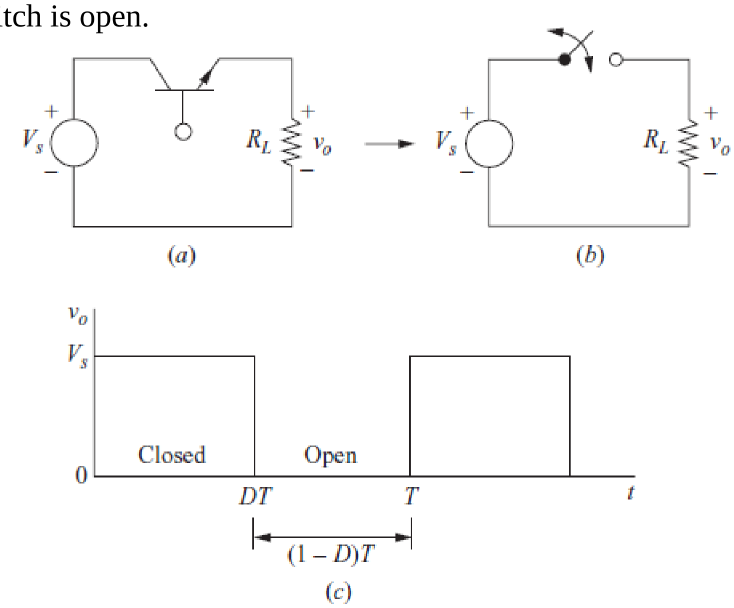


Figure (*a*) A basic dc-dc switching converter; (*b*) Switching equivalent; (*c*) Output voltage.

Periodic opening and closing of the switch results in the pulse output shown in Fig.*c*. The average or dc component of the output voltage is

$$\boxed{V_o = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{T}\int_0^{DT} V_s\,dt = V_s D}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{DT} V_s^2 .\, dt} = \sqrt{D}V_s$$

The dc component of the output voltage is controlled by adjusting the duty ratio D, defined as a ratio of the switch ON time to the sum of the ON and OFF times.

$$\boxed{D \equiv \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} = \frac{t_{\text{on}}}{T} = t_{\text{on}} f}$$

$$\mathrm{T = t_{on} + t_{off}} \qquad at \quad t_{on} = DT$$

$$\mathrm{T = DT + t_{off}}$$

$$\mathrm{t_{off} = T - DT = (1 - D)T}$$

where ***f*** is the switching frequency. The dc component of the output voltage will be less than or equal to the input voltage for this circuit.

The power absorbed by the ideal switch is zero. When the switch is open, there is no current in it. while the switch is closed, there is no voltage across it. Therefore, the load absorbs all power, and the energy efficiency is 100%. This method is sufficient for some applications, such as controlling the speed of a dc motor.

<u>Example</u>

The DC chopper has a resistive load of R=10 and input voltage is Vs=220V. When the chopper switch remains on its voltage drop =2V, duty cycle =50% and the chopping frequency is f=1kHz.determine a) The average output voltage, b) the rms output voltage ,c) chopper efficiency.

<u>Solution</u>

a) $\boldsymbol{V_o = DVs}$

$\mathbf{= 0.5 \times (220 - 2) = 109V}$

b) $V_{rms} = \sqrt{D}V_s = \sqrt{0.5} * (220 - 2) = 154.15\, V$

c) The output power is:

$$P_o = \frac{(V_s - V_{drop})^2}{R} D = \frac{(220-2)^2}{10} * 0.5 = 2376.2\ w$$

The input power is:

$$P_i = \frac{(V_s)^2}{R} D = \frac{(220)^2}{10} * 0.5 = 2398\ w$$

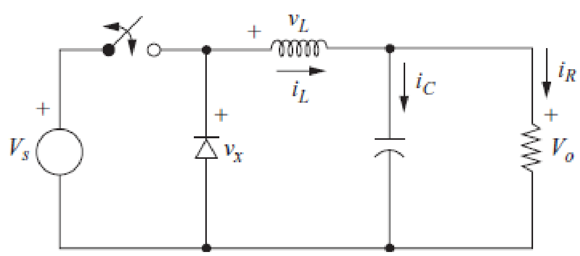
$$\text{efficiency} = \frac{P_o}{P_i} = \frac{2376.2}{2398} = 99.09\ \%$$

There are many type of dc-dc converter circuits.

1. Buck Converter (Step Down Chopper) .
2. Boost Converter (Step up Chopper).
3. Buck-Boost DC-DC converter (Step up/ Down Chopper).
4. Ćuk DC-DC converter.
5. SEPIC DC-DC converter(single-ended primary inductance converter).
6. Zeta DC-DC converter.

1. Buck Converter

Buck Converter is one way to produce a purely dc output by insert an LC low-pass filter to the basic converter after the switch, as shown in Fig. below . This circuit is called a ***step-down converter*** because the output voltage is less than the input *(Vo < Vi)* .

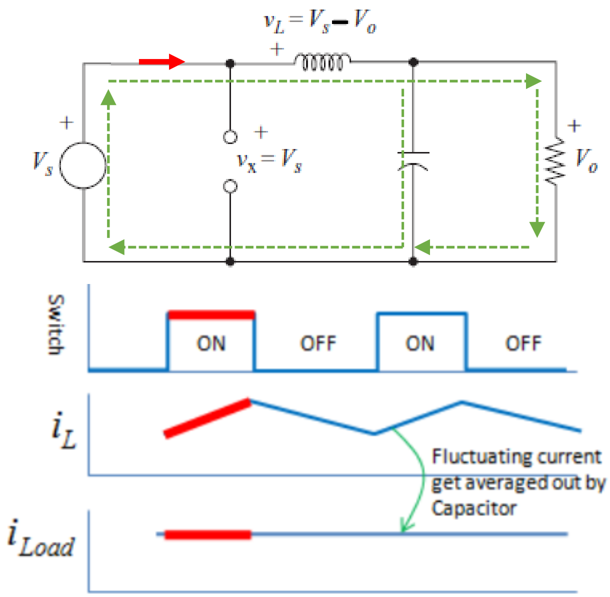


The Equivalent circuit consists of ideal components:

- Dc input voltage source *Vs*.
- Controllable switch S (as a thyristor, MOSFEs, IGBTs, BJTs, or GTOs,…)
- A diode (D) provides a path for the inductor current when the switch is opened and is reverse-biased when the switch is closed.
- An inductor (L) as energy storage element.
- A filter capacitor (C).
- Load resistance R.

→**Analysis for the switch closed (ON state):**

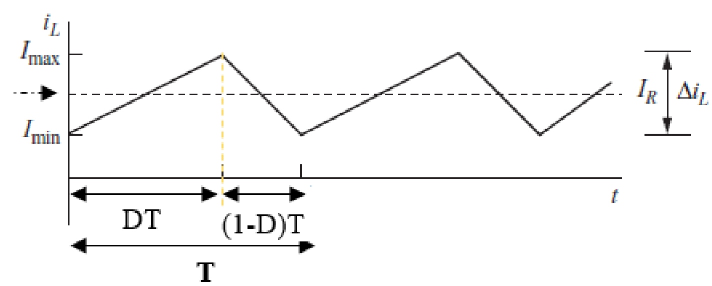
If the switch is *closed*, the current will flow through the switch to the circuit. As the current increases, inductor will store energy.thus, reduces the net voltage across the load (V_L=V_s-V_o). The diode is reverse-biased (v_x= V_s) at switch is closed, provided that the inductor current remains positive.



The switching period is *T*; the switch is closed for time *DT* . The voltage across the inductor is

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L} \qquad \text{switch closed}$$

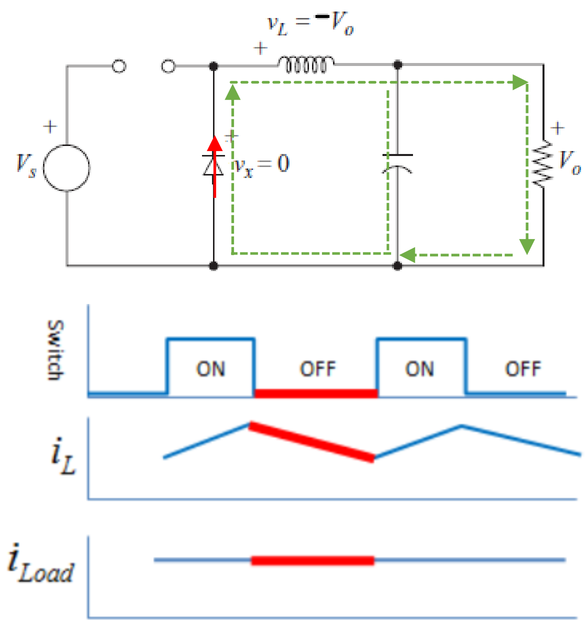


$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L}$$

$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L}\right)DT$$

→ Analysis For The Switch Open (OFF State)

When the switch is *opened*, the magnetic field of the inductor will collapse, and the current will flow from the inductor through the diode (forward-biased). The positive inductor current starts to decrease during the period $(1-D)T$, keeping the diode on. An inductor current that remains positive throughout the switching period is known as continuous current.



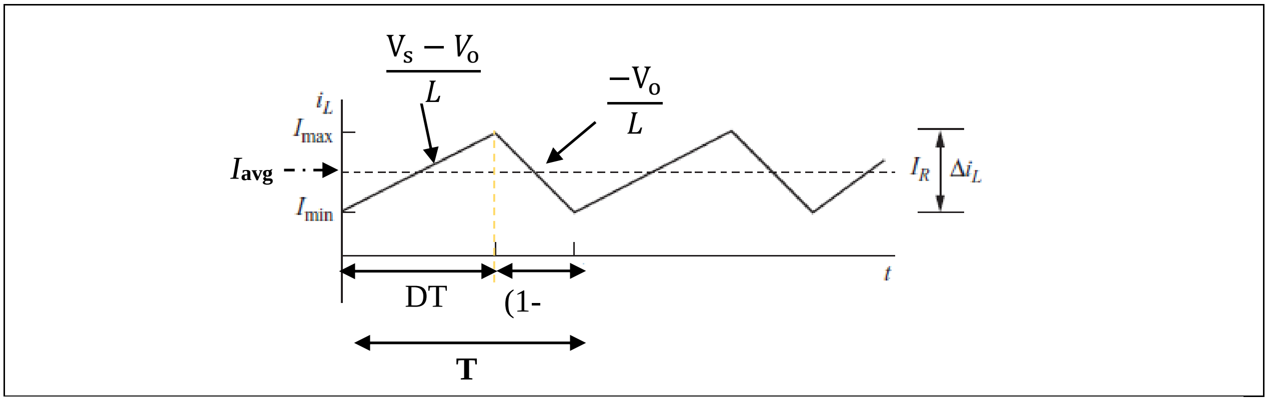
The switching period is *T*; the switch open for time *(1-D)T*.

$$v_L = -V_o = L\frac{di_L}{dt} \quad ,,, \quad \frac{di_L}{dt} = \frac{-V_o}{L} \quad \text{switch open}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T$$

A Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero as shown in fig. below.



This requires:

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\left(\frac{V_s - V_o}{L}\right)(DT) - \left(\frac{V_o}{L}\right)(1 - D)T = 0$$

Solving for V_o,

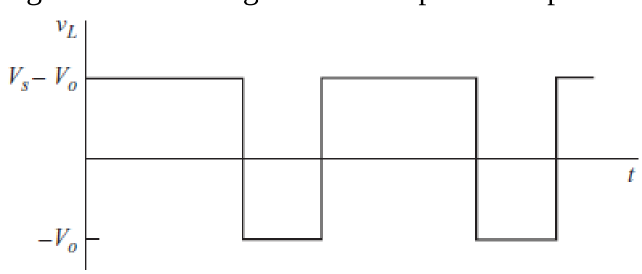
$$\frac{V_s}{L}DT - \cancel{\frac{V_o}{L}DT} = \frac{V_o}{L}T - \cancel{\frac{V_o}{L}DT}$$

$$\frac{V_s}{\cancel{L}}D\cancel{T} = \frac{V_o}{\cancel{L}}\cancel{T}$$

$$\boxed{V_o = V_s D}$$

Note that the output voltage depends on only the input and the duty ratio D. **The buck converter produces an output voltage that is less than or equal to the input**.

An alternative derivation of the output voltage is based on the inductor voltage, as shown in Fig.below. Since the average inductor voltage is zero for periodic operation,



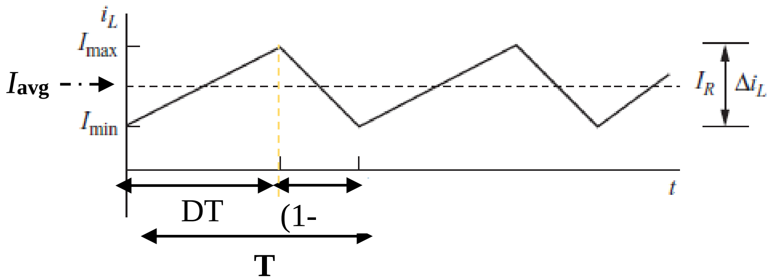
$$V_L = (V_s - V_o)DT + (-V_o)(1 - D)T = 0$$

$$(V_s - V_o)\, t_{on} = -\, V_o\, t_{off}$$

The average inductor current must be the same as the average current in the load resistor since the average capacitor current must be zero for steady-state operation:

$$I_L = I_R = \frac{V_o}{R}$$

Since the change in inductor current for ON and OFF states illustrated in Fig.below,



The maximum and minimum values of the inductor current are computed as:

$$I_{max} = I_L + \frac{\Delta i_L}{2}$$

$$= \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{min} = I_L - \frac{\Delta i_L}{2}$$

$$= \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

where $f = 1/T$ is the switching frequency.

The equation of I_{min} can be used to determine the combination of L and f that will result in continuous current. Since $I_{min}=0$ is the boundary between continuous and discontinuous current,

$$I_{min} = 0 = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$(Lf)_{min} = \frac{(1-D)R}{2}$$

$$\boxed{L_{min} = \frac{(1-D)R}{2f} \quad \text{for continuous current}}$$

where L_{min} is the minimum inductance required for continuous current.

to determine the value of inductance for a specified peak-to-peak inductor current for continuous-current operation:

$$\Delta i_L = \left(\frac{V_s - V_o}{L}\right)DT = \left(\frac{V_s - V_o}{Lf}\right)D$$

$$\Delta i_L = \frac{V_s D - V_o D}{Lf} \qquad at \ V_o = V_s D$$

$$\Delta i_L = \frac{V_s D - V_o D}{Lf} = \frac{V_o - V_o D}{Lf} = \frac{V_o(1-D)}{Lf}$$

$$L = \frac{V_o(1-D)}{\Delta i_L f} = (\frac{V_s - V_o}{\Delta i_L f})D$$

Since the converter components are assumed to be ideal, the power supplied by the source must be the same as the power absorbed by the load resistor.

$$P_s = P_o$$
$$V_s I_s = V_o I_o$$
$$\frac{V_o}{V_s} = \frac{I_s}{I_o}$$

The inductor value should be larger than L_{min} to ensure continuous current operation. Some designers select a value **25 percent larger than L_{min} .** Other designers use different criteria, such as setting the inductor current variation,

Output Voltage Ripple

the capacitor is assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$

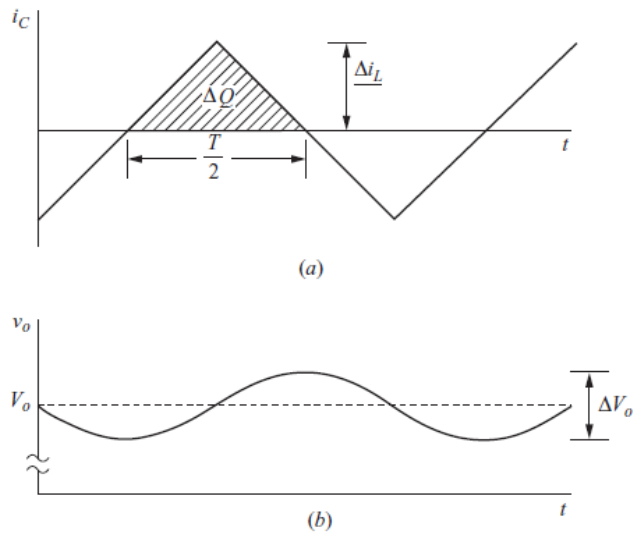


Figure Buck converter waveforms. (*a*) Capacitor current; (*b*) Capacitor ripple voltage.

While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,

$$Q = CV_o$$

$$\Delta Q = C\Delta V_o$$

$$\Delta V_o = \frac{\Delta Q}{C}$$

The change in charge ΔQ is the area of the triangle above the time axis

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$$

resulting in

$$\Delta V_o = \frac{T\Delta i_L}{8C}$$

Using Eq. for Δi_L

$$\Delta V_o = \frac{TV_o}{8CL}(1-D)T = \frac{V_o(1-D)}{8LCf^2}$$

It is also useful to express the ripple as a fraction of the output voltage,

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{1 - D}{8LCf^2}}$$

$$\boxed{C = \frac{1 - D}{8L(\Delta V_o/V_o)f^2}}$$

Example:

A buck dc-dc converter has the following parameters:

$V_s = 50$ V
$D = 0.4$
$L = 400\ \mu$H
$C = 100\ \mu$F
$f = 20$ kHz
$R = 20\ \Omega$

Assuming ideal components, calculate (*a*) the output voltage V_o, (*b*) the maximum and minimum inductor current, and (*c*) the output voltage ripple.

■ **Solution**

(a) The inductor current is assumed to be continuous, and the output voltage is computed from Eq.

$$V_o = V_s D = (50)(0.4) = 20 \text{ V}$$

(b) Maximum and minimum inductor currents are computed from Eqs.

$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1 - D}{2Lf}\right)$$

$$= 20\left[\frac{1}{20} + \frac{1 - 0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75 \text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1 - D}{2Lf}\right)$$

$$= 1 - \frac{1.5}{2} = 0.25 \text{ A}$$

$$I_{avg} = \frac{I_{max} + I_{min}}{2} = \frac{1.75 + 0.25}{2} = 1 \text{ A}$$

$$I_{max} - I_{min} = 1.75 - 0.25 = 1.5 \text{ A}$$

The average inductor current is 1 A, and $\Delta i_L = 1.5$ A. Note that the minimum inductor current is positive, verifying that the assumption of continuous current was valid.

(c) The output voltage ripple is computed from Eq

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$
$$= 0.00469 = 0.469\%$$

Example: Design a buck converter to produce an output voltage of 18 V across a 10-Ω_ load resistor. The output voltage ripple must not exceed 0.5 percent. The dc supply is 48 V with switching frequency of 40kHz. Design for continuous inductor current. Specify a) the duty ratio, b) the values of the inductor and capacitor, c) The maximum and minimum inductor.

Solution:

a) the duty ratio

$$D = \frac{V_o}{V_s} = \frac{18}{48} = 0.375$$

b) the values of the inductor and capacitor

$$L_{\min} = \frac{(1-D)(R)}{2f} = \frac{(1-0.375)(10)}{2(40{,}000)} = 78\,\mu\text{H}$$

Let the inductor be 25 percent larger than the minimum to ensure that inductor current is continuous.

$$L = 1.25L_{\min} = (1.25)(78\ \mu\text{H}) = 97.5\,\mu\text{H}$$

$$C = \frac{1-D}{8L(\Delta V_o/V_o)f^2} = \frac{1-0.375}{8(97.5)(10)^{-6}(0.005)(40{,}000)^2} = 100\ \mu\text{F}$$

c) The maximum and minimum inductor.

Average inductor current and the change in current are determined from.

$$I_{\max} = I_L + \frac{\Delta i_L}{2}$$

$$I_L = \frac{V_o}{R} = \frac{18}{10} = 1.8\text{ A}$$

$$\Delta i_L = \left(\frac{V_s - V_o}{L}\right)DT = \frac{48-18}{97.5(10)^{-6}}(0.375)\left(\frac{1}{40{,}000}\right) = 2.88\text{ A}$$

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = 1.8 + 1.44 = 3.24\ \text{A}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = 1.8 - 1.44 = 0.36\ \text{A}$$

Q/A buck converter has an input of 6 V and an output of 1.5 V. The load resistor is 3 Ω, the switching frequency is 400 kHz, L = 5 μH, and C =10 μF. Determine

(a) the duty ratio

(b) the average and peak inductor currents

(c) the average source current,

Solution:

(a) the duty ratio

$$D = \frac{V_o}{V_{in}} = \frac{1.5}{6} = 0.25$$

(b) the average and peak inductor currents

$$\mathrm{I_{avg}} = \frac{\mathrm{I_{max}} + \mathrm{I_{min}}}{2} = \frac{V_o}{\mathrm{R}} = \frac{1.5}{3} = 0.5\ \mathrm{A}$$

$$I_{max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right) = 1.5\left(\frac{1}{3} + \frac{1-0.25}{2*5*10^{-6}*4*10^5}\right) = 0.78\ A$$

(c) the average source current,

$$\mathrm{P_i} = \mathrm{P_o}$$

$$\mathrm{V_s I_s} = \frac{(\mathrm{V_o})^2}{R}$$

$$\mathrm{V_s I_s} = \frac{(D\ \mathrm{V_s})^2}{R}$$

$$\mathrm{I_s} = \frac{D^2 \mathrm{V_s}}{R} = \frac{(0.25)^2 * 6}{3} = 0.125\ \ A$$

Q/A buck converter has an input of 50 V and an output of 25 V. The switching frequency is 100 kHz, and the output power to a load resistor is 125 W. Determine

(a) the duty ratio

(b) the value of inductance to limit the peak inductor current to 6.25 A.

(c) the minimum inductor current.

Solution:

(a) the duty ratio

$$D = \frac{V_o}{V_{in}} = \frac{25}{50} = 0.5$$

(b) the value of inductance to limit the peak inductor current to 6.25 A.

$$R = \frac{V_o^2}{P_{out}} = \frac{(25)^2}{125} = 5\ \Omega$$

$$I_{max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right) = 25\left(\frac{1}{5} + \frac{1-0.5}{2*L*10^5}\right)$$

$$I_{max} = 6.25\ A \quad \longrightarrow \quad L = 50\ \mu H$$

(c) the minimum inductor current.

$$I_{min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right) = 25\left(\frac{1}{5} - \frac{1-0.5}{2*50*10^{-6}*10^5}\right) = 3.75\ A$$

Q/A buck converter has the following parameters: *Vs* =24 V, *D*= 0.65, *L* = 25 =μH, *C* = 15 μF, and *R* =10 Ω. The switching frequency is 100 kHz. Determine (*a*) the output voltage, (*b*) the maximum and minimum inductor currents and (*c*) the output voltage ripple.

Solution:

$$a)\ \ V_o = V_s D = (24)(0.65) = 15.6\ V.$$

$$b)\ \ I_L = I_R = \frac{V_o}{R} = \frac{15.6}{10} = 1.56\ A$$

$$\Delta i_L = \frac{V_o}{L}(1-D)T = \frac{15.6}{25(10)^{-6}}(1-0.65)\frac{1}{100,000} = 2.18\ A$$

$$I_{L,\max} = I_L + \frac{\Delta i_L}{2} = 1.56 + \frac{2.18}{2} = 2.65\ A.$$

$$I_{L,\min} = I_L - \frac{\Delta i_L}{2} = 1.56 - \frac{2.18}{2} = 0.47\ A.$$

Q/

The buck converter : has $V_s = 30$ V, $V_o = 20$ V, and a switching frequency of 40 kHz. The output power is 25 W. Determine the size of the inductor such that the minimum inductor current is 25 percent of the average inductor current.

Solution:

$$I_o = I_L = \frac{P_o}{V_o} = \frac{25}{30} = 1.25\ A.$$

$$D = \frac{V_o}{V_s} = \frac{20}{30} = 0.667$$

$$I_{L,\min} = (0.25)(1.25) = 0.31\ A. = I_L - \frac{\Delta i_L}{2}$$

$$\Delta i_L = (I_L - I_{L,\min})2 = (1.25 - 0.31)2 = 1.88\ A.$$

$$\Delta i_L = \frac{V_o}{L}(1-D)T$$

$$L = \frac{V_o}{\Delta i_L}(1-D)\frac{1}{f} = \frac{20}{1.88}(1-.667)\frac{1}{40000} = 89\ \mu H$$

Q/

A buck converter has an input voltage that varies between 50 and 60 V and a load that varies between 75 and 125 W. The output voltage is 20 V. For a switching frequency of 100 kHz, determine the minimum inductance to provide for continuous current for every operating possibility.

Solution:

$$L_{\min} = \frac{(1-D)R}{2f}$$

$$D = \frac{V_o}{V_s};\ D_{\max} = \frac{20}{50} = 0.4;\ D_{\min} = \frac{20}{60} = 0.33$$

$$I_L = I_R = \frac{P_o}{V_o};\ I_{R,\min} = \frac{75}{20} = 3.75\ A;\ I_{R,\max} = \frac{125}{20} 6.25\ A$$

$$R = \frac{V_o}{P};\ R_{\max} = \frac{20^2}{75} = 5.33\ \Omega;\ R_{\min} = \frac{20^2}{125} = 3.20\ \Omega$$

$$L_{\min} = \frac{(1-D_{\min})R_{\max}}{2f} = \frac{(1-.33)(5.33)}{2(100,000)} = 17.76\ \mu H$$

Q/

Design a buck converter that has an output of 12 V from an input of 18 V. The output power is 10 W. The output voltage ripple must be no more than 100 mV p-p. Specify the duty ratio, switching frequency, and inductor and capacitor values. Design for continuous inductor current. Assume ideal components.

$$D = \frac{V_o}{V_s} = \frac{12}{18} = 0.667$$

$$Let\ f = 200 kHz\ \ (for\ example)$$

$$I_L = I_o = \frac{P_o}{V_o} = \frac{10W}{12V} = 0.833\,A$$

$$Let\ \Delta i_L = 40\%\ of\ I_L = 0.40(0.833) = 0.333\,A$$

$$L = \frac{(V_s - V_o)\,D}{(\Delta i_L)\,f} = \frac{(18-12)\,0.667}{(0.333)\,200,000} = 60\,\mu H$$

$$\Delta V_o = \frac{V_o(1-D)}{8LCf^2}$$

$$C = \frac{1-D}{8L(\Delta V_o/V_o)f^2}$$

$\dfrac{\Delta V_o}{V_o}$=(100/1000)/12=0.0083

C=3.5μ

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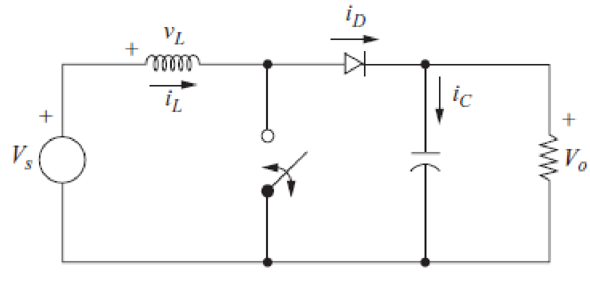
2. Boost Converter

The boost converter is shown in Fig. below. This is another switching converter that operates by periodically opening and closing an electronic switch. It is called a boost converter because the output voltage is larger than the input.

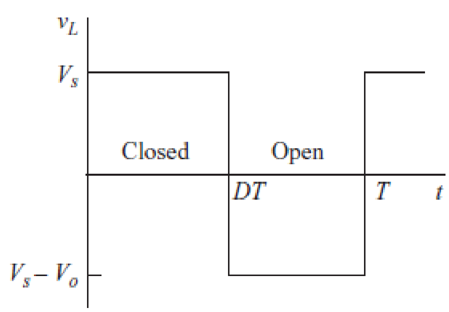


Voltage and Current Relationships

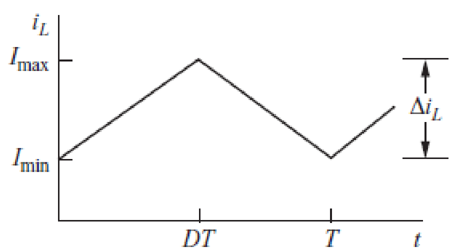
The analysis assumes the following:

1. Steady-state conditions exist.
2. The switching period is *T*, and the switch is closed for time *DT* and open for (1-*D*)*T*.
3. The inductor current is continuous (always positive).
4. The capacitor is very large, and the output voltage is held constant at voltage *Vo*.
5. The components are ideal.

Figure below shows the voltage and current waveforms for the boost converter.



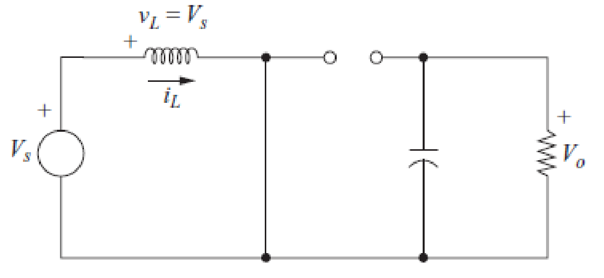
(*a*)



The analysis proceeds by examining the inductor voltage and current for the switch closed and again for the switch open.

→Analysis for the switch closed (ON state):

When the switch is closed, in Fig.



the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

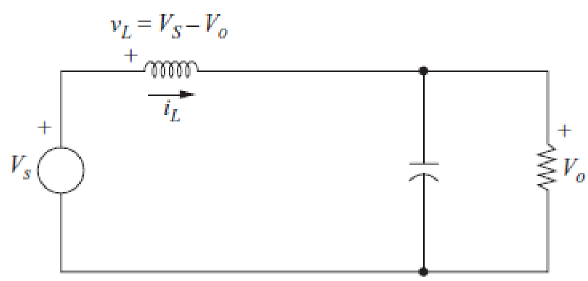
$$v_L = V_s = L\frac{di_L}{dt} \quad \text{or} \quad \frac{di_L}{dt} = \frac{V_s}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L}$$

$$(\Delta i_L)_{\text{closed}} = \frac{V_s DT}{L}$$

→Analysis for the switch opened (OFF state):

When the switch is opened,



the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current. Assuming that the output voltage *Vo* is a constant, the voltage across the inductor is

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L}$$

$$(\Delta i_L)_{\text{open}} = \frac{(V_s - V_o)(1-D)T}{L}$$

For steady-state operation, the net change in inductor current must be zero.

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s DT}{L} + \frac{(V_s - V_o)(1 - D)T}{L} = 0$$

Solving for V_o,

$$V_s(D + 1 - D) - V_o(1 - D) = 0$$

$$\boxed{V_o = \frac{V_s}{1 - D}}$$

This Equation shows that if the switch is always open and D is zero, the output voltage is the same as the input. As the duty ratio is increased, the denominator of the Eq. becomes smaller, resulting in a larger output voltage. *The boost converter produces an output voltage that is greater than or equal to the input voltage*. However, the output voltage cannot be less than the input, as was the case with the buck converter.

The average current in the inductor is determined by recognizing that the average power supplied by the source must be the same as the average power absorbed by the load resistor. Output power is

$$P_o = \frac{V_o^2}{R} = V_o I_o$$

$$V_s I_s = V_s I_L$$

$$V_s I_L = \frac{V_o^2}{R} = \frac{[V_s/(1 - D)]^2}{R} = \frac{V_s^2}{(1 - D)^2} R$$

$$\boxed{I_L = \frac{V_s}{(1 - D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s}}$$

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1 - D)^2 R} + \frac{V_s DT}{2L}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1 - D)^2 R} - \frac{V_s DT}{2L}$$

A condition necessary for continuous inductor current is for $I_{\min}$ to be positive. Therefore, the boundary between continuous and discontinuous inductor current is determined from

$$I_{\min} = 0 = \frac{V_s}{(1 - D)^2 R} - \frac{V_s DT}{2L}$$

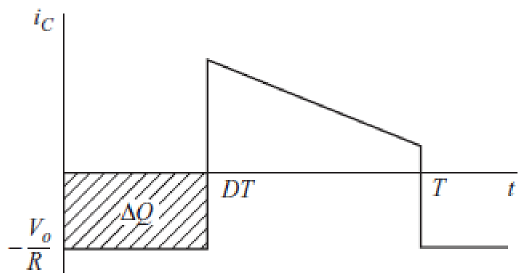
or

$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf}$$

$$(Lf)_{\min} = \frac{D(1-D)^2 R}{2}$$

$$\boxed{L_{\min} = \frac{D(1-D)^2 R}{2f}}$$

Output Voltage Ripple



$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o$$

An expression for ripple voltage is then

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{D}{RCf}}$$

$$C = \frac{D}{R(\Delta V_o / V_o) f}$$

Summary

1. A switched-mode dc-dc converter is much more efficient than a linear converter because of reduced losses in the electronic switch.
2. A buck converter has an output voltage less than the input.
3. A boost converter has an output voltage greater than the input.