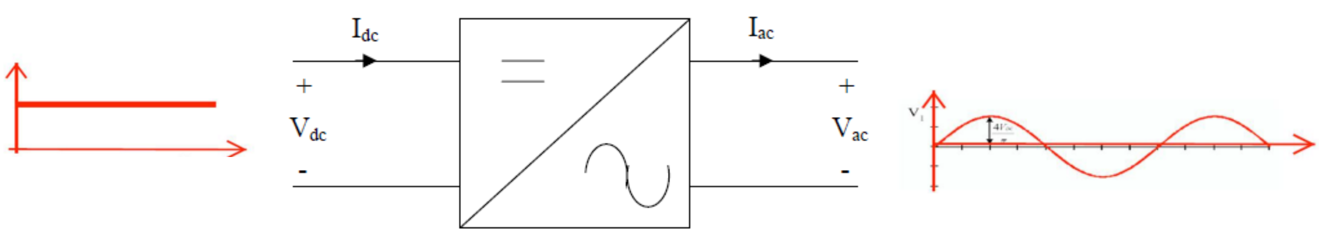
Lecture -6-

# Dc - Ac Inverters

## Inverters

Inverters are circuits that convert DC power into AC power at a specified output voltage and frequency. In the other word, Inverter performs the inverse process of a rectifier. The input source of the inverters can be battery, fuel cell, solar cell or Rectifier. The output voltage may be non-sinusoidal but can be made close to sinusoidal waveform. The general block diagram of an inverter is shown in Figure.




General block diagram of inverter

### Applications of Inverters

1. **Industrial Inverters:** adjustable-speed ac drives, induction heating, stand by aircraft power supplies and UPS (Uninterruptible Power Supplies) for computers.

2. **Renewable Energy Inverters:** Solar power systems and wind power.

3. **Automotive Inverters:** Used in electric vehicles and hybrid cars**.**

### Inverters can be classified According on several factors

- **According to the input phase numbers:**
  1. Single-phase half- and full bridge inverter.
  2. Three-phase inverter.
  3. Multi-inverter (high voltage)

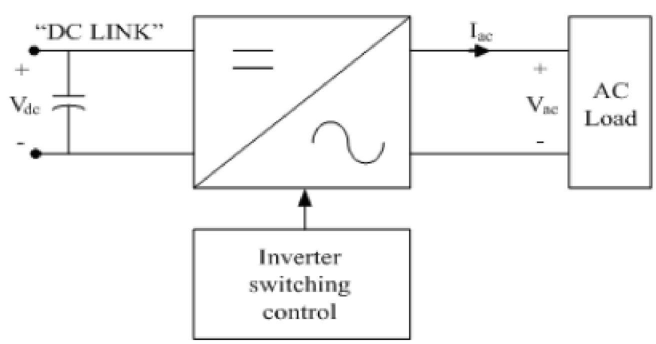
- **According to the output waveform:**
  1. **Square Wave Inverter**: Simple design but not suitable for sensitive electronics due to harmonic distortion.
  2. **Modified Sine Wave Inverter:** is not pure sine wave, used for household applications.
  3. **Pure Sine Wave Inverter:** Produces a smooth sine wave output, ideal for sensitive and precision devices

- **According to switching technique:**
  1. Pulse-width-modulated output (PWM)
  2. Quasi-square.

- **According to the input source:**
  1. voltage source inverters (VSI).
  2. current source inverters (CSI).
  3. current regulated inverters (hysteresis-type).

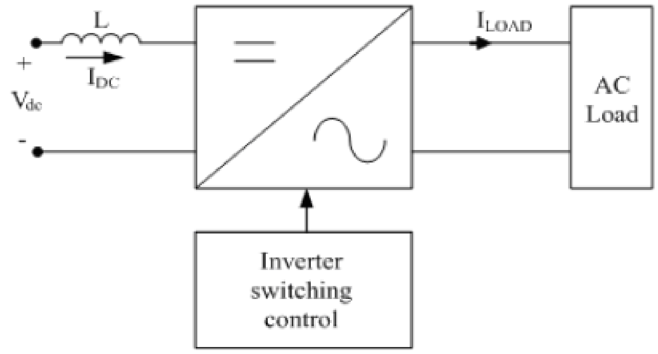
The *voltage source inverter* is the most commonly used type of inverter. The AC that's provided on the output side functions as a voltage source. The input DC voltage may be from the rectified output of an AC power supply or an independent source such as battery, which is called a 'DC link' inverter. A capacitor is connected to a voltage source inverter (VSI) to fix the i/p voltage, as called "pure voltage".as shown in fig a.

VSI Used for low or medium power about 5 kw. The traditional topologies for single-phase VSIs are half bridge and full bridge.



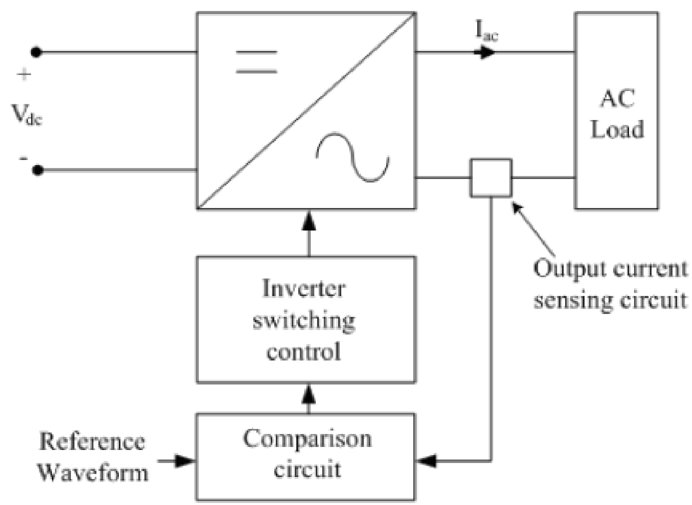

(a) Voltage source inverter

in the *current source inverter*, the output side is functions as AC current source. This type also has a DC link inverter but it functions like a DC current source. A conductor is connected to the source(CSI) to fix the i/p current. CSI Used for high power motors. as shown in fig b.




(b) Current source inverter

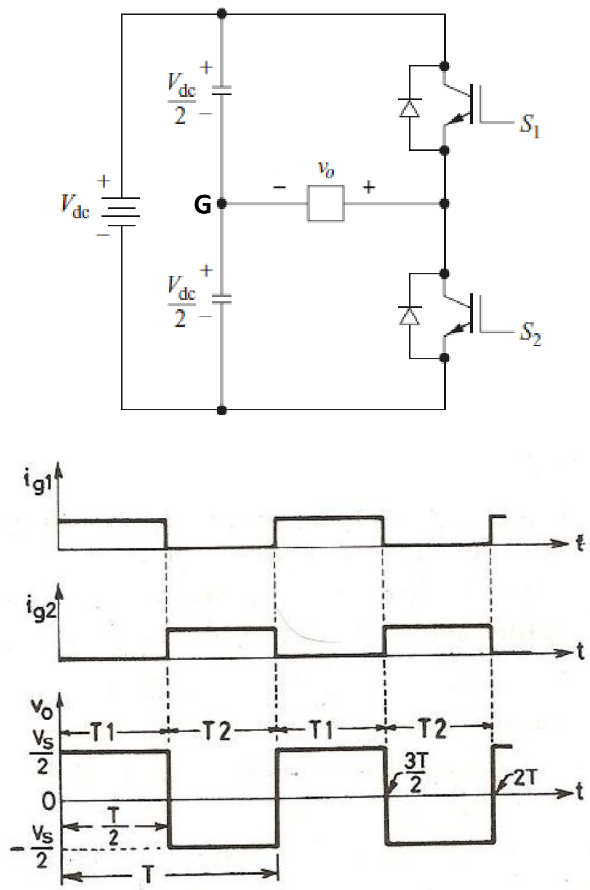
The *current regulated inverters* are becoming popular, especially for speed control of AC motors. The input DC is the same as in a conventional voltage source inverter. In this type, there is a current-sensing circuit that senses the actual value of the current at every instant. The sensed value is compared against the value found by the reference waveform, and the switching inside the inverter is regulated as necessary to correct any error. as shown in fig c.




(c) Current regulated inverter

## 1. Single-phase half-bridge inverter.

The half-bridge inverter in fig. below is constructed with two equal capacitors connected in series across the dc input source, and their junction is at a midpoint or center point G. The number of switches called as 'inverter leg' for half bridge inverter is reduced to one leg, which only consists of two switches. The dc source voltage is divided into two parts that have the same value of voltages (one half voltages) as $V_{dc}/2$ across it. When $S_1$ is closed, the load voltage is $+V_{dc}/2$ and at $S_2$ is closed, the load voltage is - $V_{dc}/2$.




(single phase half-bridge VSI)

**T: is the total time that is going to be taken care of the circuit.**

Diodes are usually connected to switches in inverter circuits with an inductive load to overcome the problems of inductance stored energy. While there is no need for these diodes in the resistive load. In other word, feedback diodes are required to provide current continuity for inductive loads.

Overlap of the switch "on" times will result in a short circuit, sometimes called a shoot-through fault across the dc source. The time allowed for switching is called blanking time. So, blanking time for the switches is required to prevent a short circuit across the source.

The load voltage is a square wave of amplitude $V_{dc}/2$ and the load current waveform is an exponentially rising and falling waveform determined by the load impedance. For a resistive load, two switches in half wave bridge and four switches in full wave bridge would suffice because the load current Io and load voltage Vo would always be in phase with each other. thus, the current waveform follows the voltage waveform. while for an inductive load, the current waveform lags the voltage waveform by an angle that is approximately equal to the load power factor angle.

The load voltage is

$$V_o = \frac{V_{dc}}{2}$$

$$V_{o,rms} = \frac{2V_{dc}}{\sqrt{2}\,\pi} = \frac{\sqrt{2}\,V_{dc}}{\pi} = 0.45\,V_{dc}$$

**Example**

The single-phase half-bridge inverter has a resistive load of R = 2.4Ω and the DC input voltage is 48V. Determine:

(a) the rms output voltage at the fundamental frequency
(b) the output power
(c) the average and peak current of each transistor.

**Solution**

$V_{DC}$ = 48V and R = 2.4Ω

(a) The fundamental rms output voltage, V = $0.45V_{DC}$ = 0.45x48 = 21.6V
(b) For single-phase half-bridge inverter, the output voltage $V_o = V_{DC}/2$
Thus, the output power,

$$P_o = V_o^2 / R$$

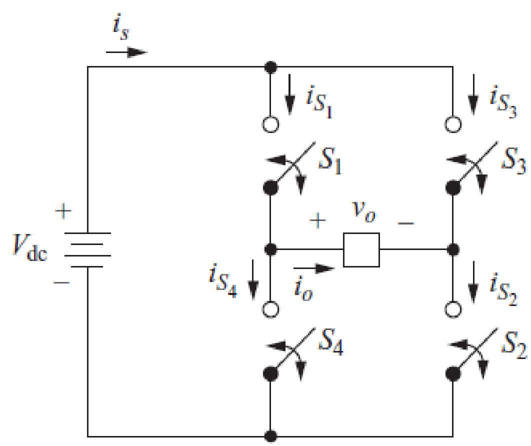
$$= \frac{(48/2)^2}{2.4}$$

$$= 240W$$

(c) The transistor current $I_p$ = 24/2.4 = 10 A
Because each of the transistor conducts for a 50% duty cycle, the average current of each transistor is $I_Q$ = 10/2 = 5 A.

## 2. Single-phase full-bridge inverter.

Four switches are used in the full-bridge inverter circuit as shown in fig. below. AC output is produced from a DC input by closing and opening the switches in an appropriate sequence. The output voltage $V_o$ can be $+V_{dc}$ and $-V_{dc}$ or zero, depending on which switches are closed as illusrated in table below .When the switch $S_1$ and $S_2$ are closed, and at the same time switch $S_3$ and $S_4$ are opened, the output voltage waveform is $+V_{dc}$ between the interval of 0 to T/2. Whereas, when the switch $S_3$ and $S_4$ are closed, and at the same time switch $S_1$ and $S_2$ are opened, the output voltage is $-V_{dc}$ in the interval of T/2 to T.

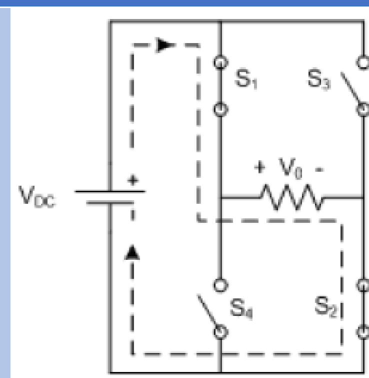
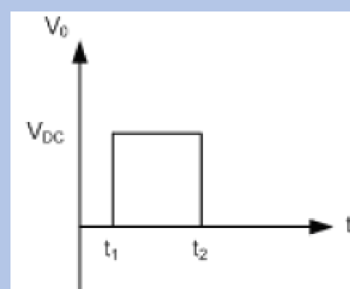
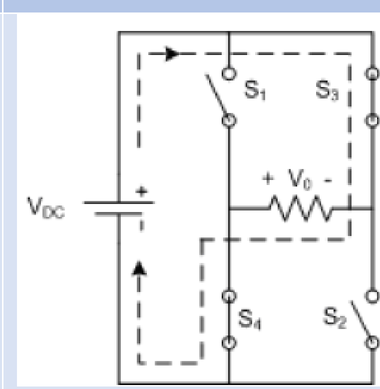
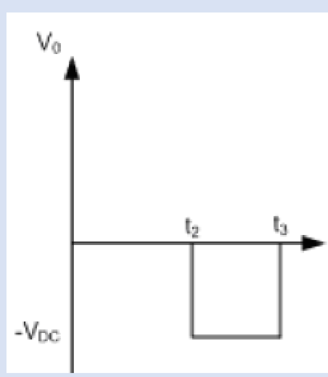



| Switches Closed | Output Voltage $v_o$ |
|---|---|
| $S_1$ and $S_2$ | $+V_{dc}$ |
| $S_3$ and $S_4$ | $-V_{dc}$ |
| $S_1$ and $S_3$ | 0 |
| $S_2$ and $S_4$ | 0 |

The periodic switching of the load voltage between $+V_{dc}$ and $-V_{dc}$ produces a two type of waves are: square-wave or sine-wave voltage across the load. The efficiency of a square wave is lower than that of a sine wave.To find the sinusoid output waveform, the square-wave waveform should be filtered.

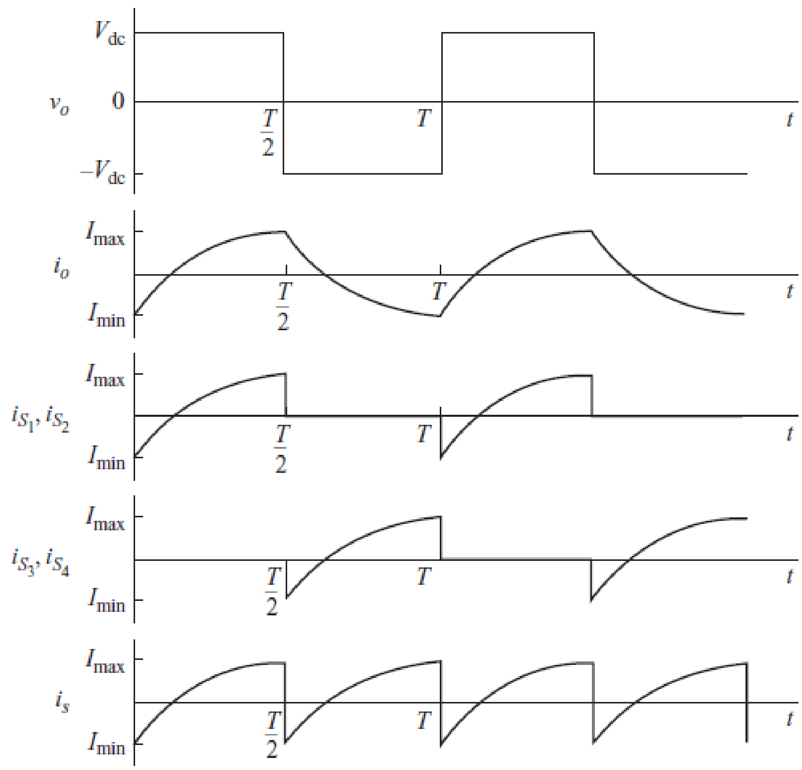
Notice that, the switch $S_1$ and $S_4$ should not be closed at the same time, similarly for switch $S_2$ and $S_3$. These because if that condition are occurs, a short circuit would exist across the dc source. In practice, the switches are not turned on and off instantaneously, there are switching transition times in the control of the switches.

**There are two modes for operation the single-phase full-bridge square-wave inverter.**

| mode | period | Connected switches | equivalent cct. | Vo | Io |
|---|---|---|---|---|---|
| **I** | $T/2 = \pi$<br>$(0 - \pi)$ | $S_1, S_2$ | <br> | $-V_s + V_o = 0$<br>$V_o = V_s$ | $V_s / R$ |
| **II** | $T = 2\pi$<br>$(\pi - 2\pi)$ | $S_3, S_4$ | <br> | $-V_s - V_o = 0$<br>$V_o = -V_s$ | $- V_s / R$ |

Overlap of switch “on” times will result in a short circuit, sometimes called a *shoot-through fault*, across the dc source. The time allowed for switching is called *blanking time*.

The output waveform in Figure below shows the load voltage and current of the full wave bridge inverter.




The current waveform in the load depends on the load components. For the resistive load, the current waveform matches the shape of the output voltage. An inductive load will have a current that has more of a sinusoidal quality than the voltage because of the filtering property of the inductance. An inductive load presents some considerations in designing the switches in the full-bridge circuit because the switch currents must be bidirectional.

**Advantages of Full Bridge Inverter**

- Minimal fluctuation of voltage in the circuit.
- Suitable for High Power Applications
- Energy-Efficient.

**Disadvantages of Full Bridge Inverter**

- The efficiency of the full bridge inverter ( 95% ) is less than the half-bridge inverter (99%).
- Losses and noise are high, so it requires more switching elements.

**In steady state, the current waveforms described as**

$$\boxed{i_o(t) = \begin{cases} \dfrac{V_{\text{dc}}}{R} + \left(I_{\min} - \dfrac{V_{\text{dc}}}{R}\right)e^{-t/\tau} & \text{for } 0 < t < \dfrac{T}{2} \\ \dfrac{-V_{\text{dc}}}{R} + \left(I_{\max} + \dfrac{V_{\text{dc}}}{R}\right) - e^{(t-T/2)/\tau} & \text{for } \dfrac{T}{2} < t < T \end{cases}}$$

$$\boxed{I_{\max} = -I_{\min} = \frac{V_{\text{dc}}}{R}\left(\frac{1 - e^{-T/2\tau}}{1 + e^{-T/2\tau}}\right)}$$

$$V_o = V_{dc}$$

$$V_{o,rms} = \frac{4V_{dc}}{\sqrt{2}\,\pi} = 0.9\,V_{dc} \quad = 90\%\,V_{dc}$$

$$I_{o,rms} = \frac{V_{o,rms}}{Z}$$

where,

$$|Z| = \sqrt{R^2 + (\omega_o L)^2}$$

The rms value of current is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,d(t)} = \sqrt{\frac{2}{T}\int_0^{T/2} \left[\frac{V_{\text{dc}}}{R} + \left(I_{\min} - \frac{V_{\text{dc}}}{R}\right)e^{-t/\tau}\right]^2 dt}$$

If the switches are ideal, the power supplied by the source must be the same as absorbed by the load. Power from a dc source is determined from

$$P_{\text{dc}} = V_{\text{dc}} I_s$$

Power absorbed by the load can be determined from

$$P = I_{\text{rms}}^2 R$$

**Example 1 :**

The full-bridge inverter has a switching sequence that produces a square wave voltage across a series *RL* load. The switching frequency is 60 Hz, *V*dc =100 V, *R*=10 Ω , and *L* = 25 mH. Determine (*a*) an expression for load current, (*b*) the power absorbed by the load, and (*c*) the average current in the dc source.

■ **Solution**

(*a*) From the parameters given,

$$T = 1/f = 1/60 = 0.0167 \text{ s}$$

$$\tau = L/R = 0.025/10 = 0.0025 \text{ s}$$

$$T/2\tau = 3.33$$

to determine the maximum and minimum current.

$$\boxed{I_{\max} = -I_{\min} = \frac{V_{dc}}{R}\left(\frac{1 - e^{-T/2\tau}}{1 + e^{-T/2\tau}}\right)}$$

$$I_{\max} = -I_{\min} = \frac{100}{10}\left(\frac{1 - e^{-3.33}}{1 + e^{-3.33}}\right) = 9.31 \text{ A}$$

$$\boxed{i_o(t) = \begin{cases} \dfrac{V_{dc}}{R} + \left(I_{\min} - \dfrac{V_{dc}}{R}\right)e^{-t/\tau} & \text{for } 0 < t < \dfrac{T}{2} \\[2ex] \dfrac{-V_{dc}}{R} + \left(I_{\max} + \dfrac{V_{dc}}{R}\right) - e^{(t-T/2)/\tau} & \text{for } \dfrac{T}{2} < t < T \end{cases}}$$

$$i_o(t) = \frac{100}{10} + \left(-9.31 - \frac{100}{10}\right)e^{-t/0.0025}$$

$$= 10 - 19.31e^{-t/0.0025} \qquad 0 \le t \le \frac{1}{120}$$

$$\frac{T}{2} = \frac{1/60}{2} = \frac{1}{120}$$

$$i_o(t) = -\frac{100}{10} + \left(9.31 + \frac{100}{10}\right)e^{-(t-0.0167/2)/0.0025}$$

$$= -10 + 19.31e^{-(t-0.00835)0.0025} \qquad \frac{1}{120} \le t \le \frac{1}{60}$$

(*b*) Power absorbed by the load is computed from $P = I_{rms}^2 R$

$$I_{rms} = \sqrt{\frac{1}{120}\int_0^{1/120} [(10 - 19.31)e^{-t/0.0025}]^2 dt} = 6.64 \text{ A}$$

$P = I_{\text{rms}}^2 R = (6.64)^2(10) = 441$ W

Or by using,

$$V_{o,rms} = \frac{4V_{dc}}{\sqrt{2}\ \pi} = 0.9\ V_{dc} = 90\ v$$

$$I_{o,rms} = \frac{V_{o,rms}}{Z}$$

where,

$$|Z| = \sqrt{R^2 + (\omega_o L)^2}$$

$$I_{o,rms} = \frac{90}{13.7} = 6.573\ A$$

Power absorbed by the load is

$P = I_{\text{rms}}^2 R$ = *(6.573) ^2 *10=432.04 w*

(c) the Average source current can also be computed by equating source and load power, assuming a lossless converter.

$I_s = \frac{P_{\text{dc}}}{V_{\text{dc}}} =$ *432.04/100=4.32 A*

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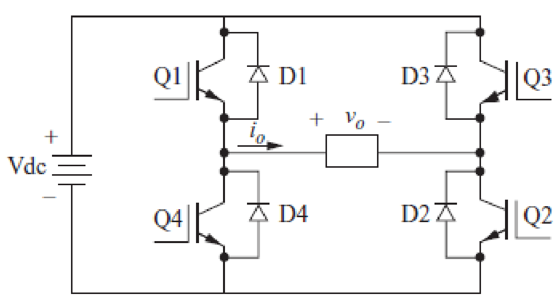
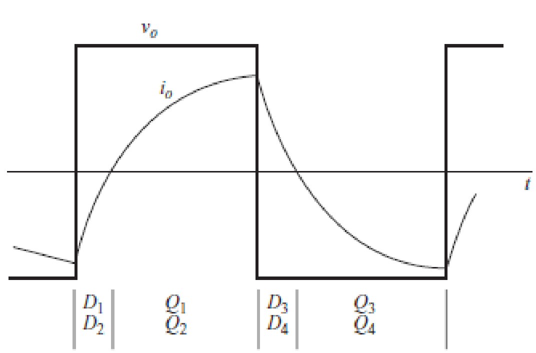
The figure below shows the full-bridge inverter with switches implemented as insulated gate bipolar transistors (IGBTs) with feedback diodes.



|   | Half Bridge Inverter | Full Bridge Inverter |
|---|---|---|
| 1 | The efficiency is high in half-bridge inverter | the efficiency is less than half in half-bridge inverter |
| 2 | The peak voltage is half of the DC supply voltage | The peak is the same as the DC supply voltage |
| 3 | The half-bridge inverter contains two switches | The full-bridge inverter contains four switches |
| 4 | The output voltage is $Vo = V_{dc}/2$ | The output voltage is $Vo = V_{dc}$ |
| 5 | rms voltage is $V = 0.45\ V_{dc}$ | rms voltage is $V = 0.9\ V_{dc}$ |
| 6 | Noise is Lower than Full Bridge Inverter | High Noise |

## ➢ Fourier Series Analysis

***Fourier Series Solution for the Square-Wave Inverter***

The Fourier series method is often the most practical way to analyze load current and to compute power absorbed in a load, especially when the load is more complex than a simple resistive or *RL* load. A useful approach for inverter analysis is to express the output voltage and load current in terms of a Fourier series. With no dc component in the output.

$$v_o(t) = \sum_{n=1}^{\infty} V_n \sin(n\omega_0 t + \theta_n)$$

$$i_o(t) = \sum_{n=1}^{\infty} I_n \sin(n\omega_0 t + \phi_n)$$

Power absorbed by a load with a series resistance is determined from $I^2_{\text{rms}}R$, where the rms current can be determined from the rms currents at each of the components in the Fourier series by

$$I_{\text{rms}} = \sqrt{\sum_{n=1}^{\infty} I^2_{n,\text{rms}}} = \sqrt{\sum_{n=1}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2}$$

where

$$I_n = \frac{V_n}{Z_n}$$

and $Z_n$ is the load impedance at harmonic $n$.

Equivalently, the power absorbed in the load resistor can be determined for each frequency in the Fourier series. Total power can be determined from

$$P = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} I_{n,\text{rms}}^2 R$$

where $I_{n,\text{rms}}$ is $I_n/\sqrt{2}$.

In the case of the square wave, the Fourier series contains the odd harmonics and can be represented as

$$v_o(t) = \sum_{n\,\text{odd}} \frac{4V_{\text{dc}}}{n\pi} \sin n\omega_0 t$$

**Example 2 :**

The full-bridge inverter has a switching sequence that produces a square wave voltage across a series *RL* load. The switching frequency is 60 Hz, *V*dc =100 V, *R*=10 Ω , and *L* = 25 mH. Determine: (a) the amplitudes of the Fourier series terms for the square wave load voltage, (b) the amplitudes of the Fourier series terms for load current, and (c) the power absorbed by the load.

**■ Solution**

**The load voltage is represented as the Fourier series**

$$v_o(t) = \sum_{n\,\text{odd}} \frac{4V_{\text{dc}}}{n\pi} \sin n\omega_0 t$$

The amplitude of each voltage term is

$$V_n = \frac{4V_{dc}}{n\pi}$$

$$V_1 = \frac{4 * 100}{\pi} = 127.3 \ v$$

$$V_3 = \frac{4 * 100}{3\,\pi} = 42.4 \ v$$

$$V_5 = \frac{4 * 100}{5\pi} = 25.4 \ v$$

**The amplitude of each current term is determined from**

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega_0 L)^2}}$$

$$Z_1 = \sqrt{100 + (377 * 0.025)^2} = 13.7\,\Omega$$

$$Z_3 = \sqrt{100 + (3 * 377 * 0.025)^2} = 29.99 = 30 \ \Omega$$

$$Z_5 = \sqrt{100 + (5 * 377 * 0.025)^2} = 48.1 \ \ \Omega$$

$$I_1 = \frac{V_1}{Z_1} = \frac{127.3}{13.7} = 9.28 \quad A$$

$$I_3 = \frac{V_3}{Z_3} = \frac{42.4}{30} = 1.4 \quad A$$

$$I_5 = \frac{V_5}{Z_5} = \frac{25.4}{48.1} = 0.5 \quad A$$

**Power at each frequency is determined from**

$$P_n = I_{n,\mathrm{rms}}^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R$$

$$P_1 = \left(\frac{I_1}{\sqrt{2}}\right)^2 * R = \left(\frac{9.28}{\sqrt{2}}\right)^2 * 10 = 430.5 \quad w$$

$$P_3 = \left(\frac{I_3}{\sqrt{2}}\right)^2 * R = \left(\frac{1.4}{\sqrt{2}}\right)^2 * 10 = 9.8 \quad w$$

$$P_5 = \left(\frac{I_5}{\sqrt{2}}\right)^2 * R = \left(\frac{0.5}{\sqrt{2}}\right)^2 * 10 = 1.25 \quad w$$

| n | *fn* (Hz) | *Vn* (v) | *Zn* (Ω) | *In* (A) | $P_n$ (w) |
|---|---|---|---|---|---|
| **1** | 60 | 127.3 | 13.7 | 9.28 | 430.5 |
| **3** | 180 | 42.4 | 30 | 1.4 | 9.8 |
| **5** | 300 | 25.4 | 48.1 | 0.5 | 1.4 |

As the harmonic number *n* increases, the amplitude of the Fourier voltage component decreases and the magnitude of the corresponding impedance increases, both resulting in small currents for higher-order harmonics. Therefore, only the first few terms of the series are of practical interest. Note how the current and power terms become small for all but the first few frequencies.

$$P = \sum P_n = 430.5 + 9.8 + 1.4 + \cdots \quad = 441.5 \quad w$$

**Example 3:**

A square full wave bridge inverter has a dc source of 125 V, an output frequency of 60 Hz, and an RL series load with R = 20 Ω and L= 25 mH. Determine (a) an expression for load current, (b) rms load current, and (c) average source current.

$$a)\quad \frac{V_{dc}}{R}=\frac{125}{20}=6.25\ A.;\quad \tau=\frac{L}{R}=\frac{25\,mH}{20\Omega}=1.25\,ms;\quad \frac{T}{2\tau}=\frac{1/60}{1.25\,ms}=13.33$$

T=1/f

$$\boxed{I_{\max}=-I_{\min}=\frac{V_{dc}}{R}\left(\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right)}$$

$$I_{\max}=6.25\left(\frac{1-e^{-13.33}}{1+e^{-13.33}}\right)=6.25\ A.$$

$$I_{\min}=-I_{\max}=-6.25\ A.$$

$$\boxed{i_o(t)=\begin{cases}\frac{V_{dc}}{R}+\left(I_{\min}-\frac{V_{dc}}{R}\right)e^{-t/\tau} & \text{for } 0<t<\frac{T}{2}\\ \frac{-V_{dc}}{R}+\left(I_{\max}+\frac{V_{dc}}{R}\right)-e^{(t-T/2)/\tau} & \text{for } \frac{T}{2}<t<T\end{cases}}$$

$$i_o=\begin{cases}6.25-12.5e^{-t/.00125} & for\ \ 0\le t\le 8.33\,ms\\ -6.25+12.5e^{-(t-1/120)/.00125} & for\ \ 8.33\ ms\le t\le 16.7\ ms\end{cases}$$

$b)$ Using the first half-period,

$$I_{rms}=\sqrt{\frac{1}{120}\int_0^{1/120}\left(6.25-12.5e^{-t/.00125}\right)^2 dt}=5.45\ A.$$

Or $\quad Z=\sqrt{(R)^2+(wL)^2}=\sqrt{(20)^2+(2\pi*60*0.025)^2}=22\ \Omega$

$$I_{o,rms}=\frac{V_{o,rms}}{Z}=\frac{0.9*125}{22}=5.11\ \ A$$

$c)\ P=I_{o,rms}{}^2*R=5.11^2*20=522.24\ \ w$

$$I_s=\frac{P}{V_{dc}}=\frac{522.24}{125}=4.17\ \ A$$

## ➢ Total Harmonic Distortion

Since the objective of the inverter is to converts DC power into AC power requiring at the load, it is useful to describe the quality of the ac output voltage or current. The quality of a non-sinusoidal wave can be expressed in terms of total harmonic distortion (THD). Assuming no dc component in the output,

The THD of the load voltage is expressed as,

$$\boxed{THD_V = \frac{\sqrt{(V_{dc})^2 - \left(V_{o,rms}\right)^2}}{V_{o,rms}}}$$

The current THD is

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty}(I_{n,rms})^2}}{I_{1,rms}} \qquad THD_i = \frac{\sqrt{\left(\frac{I_3}{\sqrt{2}}\right)^2 + \left(\frac{I_5}{\sqrt{2}}\right)^2 + \left(\frac{I_n}{\sqrt{2}}\right)^2}}{\frac{I_1}{\sqrt{2}}}$$

$$I_{o,rms} = I_{1,rms}$$

Where n is the harmonics number.

**Example 4:**

Determine the total harmonic distortion of the load voltage and the load current for the square-wave inverter in the previous Example 2 ?

For The full-bridge inverter The output voltage is $Vo = V_{dc}=100\ v$

$$V_{o,rms} = \frac{4V_{dc}}{\sqrt{2}\ \pi} = 0.9\ V_{dc} = 90\ v$$

**The THD for voltage is**

$$THD_V = \frac{\sqrt{(V_{dc})^2 - \left(V_{o,rms}\right)^2}}{V_{o,rms}} = \frac{\sqrt{(100)^2 - (90)^2}}{90} = 0.484 = 48.4\ \%$$

$$I_{o,rms} = \frac{V_{o,rms}}{Z} = \frac{90}{13.7} = 6.56\ \ A$$

***OR/*** $\quad I_{o,rms} = I_{1,rms} = \frac{I_1}{\sqrt{2}} = \frac{9.28}{\sqrt{2}} = 6.56\ \ A$

**The current THD is**

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty}(I_{n,rms})^2}}{I_{1,rms}} \qquad THD_i = \frac{\sqrt{\left(\frac{I_3}{\sqrt{2}}\right)^2 + \left(\frac{I_5}{\sqrt{2}}\right)^2 + \left(\frac{I_n}{\sqrt{2}}\right)^2}}{\frac{I_1}{\sqrt{2}}}$$

$$THD_i = \frac{\sqrt{\left(\frac{1.4}{\sqrt{2}}\right)^2 + \left(\frac{0.5}{\sqrt{2}}\right)^2}}{\frac{9.28}{\sqrt{2}}} = 0.16 = 16\ \%$$

---

**Example 5:**

A full-bridge inverter has an RL load with R = 15 Ω and L= 10 mH. The inverter output frequency is 400 Hz. (a) Determine the value of the dc source required to establish a load current rms that has a fundamental frequency component of 8 A. (b) Determine the THD of the load current.

Sol/

$$a) \qquad Z_1 = \sqrt{15^2 + \left[2\pi(400)(0.01)\right]^2} = 29.3\ \Omega$$

$$V_{o,rms} = I_{o,rms} * Z = 8 * 29.3 = 234.4\ V$$

$$V_{o,rms} = \frac{4V_{dc}}{\sqrt{2}\ \pi}$$

$$V_{dc} = \frac{\sqrt{2}\ \pi * V_{o,rms}}{4} = \frac{\sqrt{2}\ \pi * 234.4}{4} = 260.35\ V$$

$$V_n = \frac{4V_{dc}}{n\pi}; \qquad Z_n = \sqrt{R^2 + (2\pi 400 L)^2}\ ; \quad I_n = \frac{V_n}{Z_n}$$

$$V_1 = \frac{4 * 260.35}{\pi} = 331.4\ v$$

$$V_3 = \frac{4 * 260.35}{3\ \pi} = 110.5\ v$$

$$V_5 = \frac{4 * 260.35}{5\pi} = 66.3\ v$$

**The amplitude of each current term is determined from**

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega_0 L)^2}}$$

$$Z_1 = \sqrt{225 + (2\pi * 400 * 0.01)^2} = 29.3\Omega$$

$$Z_3 = \sqrt{225 + (3 * 2\pi * 400 * 0.01)^2} = 76.87\ \Omega$$

$$Z_5 = \sqrt{225 + (5 * 2\pi * 400 * 0.01)^2} = 126.5\ \Omega$$

$$I_1 = \frac{V_1}{Z_1} = \frac{331.4}{29.3} = 11.3\ A$$

$$I_3 = \frac{V_3}{Z_3} = \frac{110.5}{76.87} = 1.43\ A$$

$$I_5 = \frac{V_5}{Z_5} = \frac{66.3}{126.5} = 0.52\ A$$

| n | ***fn*** **(Hz)** | ***Vn*** **(v)** | ***Zn*** **(Ω)** | ***In*** **(A)** |
|---|---|---|---|---|
| **1** | 400 | 331.4 | 29.3 | 11.3 |
| **3** | 1200 | 110.5 | 76.87 | 1.43 |
| **5** | 2000 | 66.3 | 126.5 | 0.52 |

(b) Determine the THD of the load current.

**The current THD is**

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty} (I_{n,rms})^2}}{I_{1,rms}} \qquad THD_i = \frac{\sqrt{\left(\frac{I_3}{\sqrt{2}}\right)^2 + \left(\frac{I_5}{\sqrt{2}}\right)^2 + \left(\frac{I_n}{\sqrt{2}}\right)^2}}{\frac{I_1}{\sqrt{2}}}$$

$$I_{o,rms} = I_{1,rms} = \frac{I_1}{\sqrt{2}} = 8\ A$$

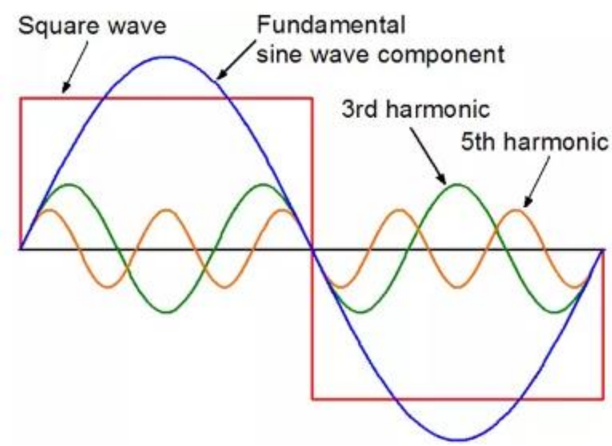
$$THD_i = \frac{\sqrt{\left(\frac{1.43}{\sqrt{2}}\right)^2 + \left(\frac{0.52}{\sqrt{2}}\right)^2}}{8} = 0.16 = 16\ \%$$

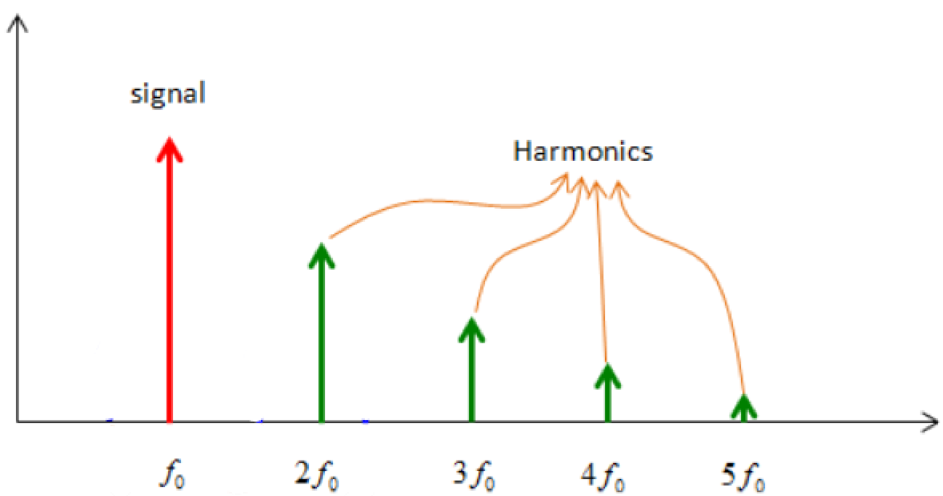
$$THD_i = \frac{\sqrt{1.02 + 0.135}}{8} = 0.134 = 13.4\ \%$$

## What is a harmonic?

A harmonic is a wave or signal whose frequency is multiple of the Fundamental Frequency of the wave signal. These frequency multiples add up to the fundamental frequency, producing a complex periodic waveform with specific characteristics.

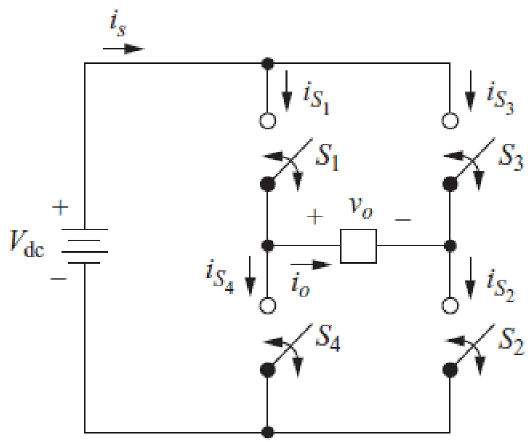
In other words, Harmonics are defined as an unwanted higher frequency component that is an integer multiple of the fundamental frequency, It can be expressed as a 2f, 3f, 4f, 5f………,nf, etc. Harmonics create a distortion in the fundamental waveform. Harmonics usually have a lower amplitude (volume) than the fundamental frequency.

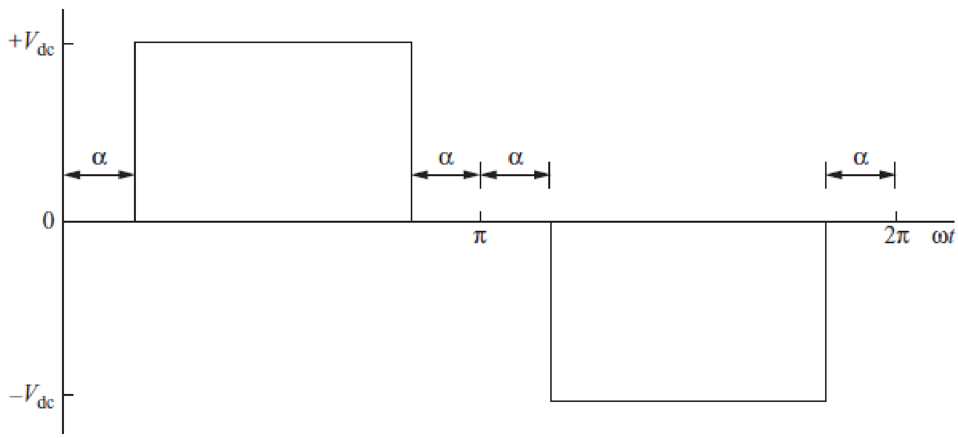


- ## Amplitude and harmonic control

The objective of this strategy is to minimize both the harmonic distortion and the switching losses in the inverter. The output voltage of the full-bridge inverter can be controlled by adjusting the interval of α on each side of the pulse where the output is zero. The Figure shows the interval of α when the output is zero at each side of the pulse.




In the beginning, the integral sine wave was valid at the output of (*0-π*) and (*π-2π*). the harmonic control method is to adjust the value of $\alpha$ on both sides of the positive half-cycle and negative half-cycle in the same way to maintain the sine wave shape.

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc}^2\, d(\omega t)} = V_{dc}\sqrt{1-\frac{2\alpha}{\pi}}$$

The Fourier series of the waveform is expressed as

$$v_o(t) = \sum_{n\ \text{odd}} V_n \sin(n\omega_0 t)$$

Taking advantage of half-wave symmetry, the amplitudes are

$$V_n = \frac{2}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc}\sin(n\omega_0 t)\,d(\omega_0 t) = \frac{4V_{dc}}{n\pi}\cos(n\alpha)$$

Harmonic content can also be controlled by adjusting $\alpha$. If $\alpha = 30°$, for example, $V_3 = 0$. This is significant because the third harmonic can be eliminated from the output voltage and current. Other harmonics can be eliminated by choosing a value of $\alpha$ which makes the cosine term in Eq.Vn to go to zero. Hence, minimize THD by using amplitude and harmonic control technique for the same circuit without adding any external component.

The amplitude of the fundamental frequency (n=1) is controllable by adjusting the angle $\alpha$ ***is***

$$V_1 = \left(\frac{4V_{DC}}{\pi}\right)\cos\alpha$$

Harmonic $n$ is eliminated at $\alpha$ if *cos* $n\alpha = 0$
or

$$\boxed{\alpha = \frac{90°}{n}}$$

**Example 6:**

The inverter has a resistive load of 10 Ω and an inductive load of 25 mH connected in series with the fundamental frequency current amplitude of 9.27A at 60 Hz.. The THD of the inverter is not more than 10%. If at the beginning of designing the inverter, the THD of the current is 16.7% which does not meet the specification, find the voltage amplitude at the fundamental frequency, the required DC input supply, and the new THD of the current.
Sol/

$$Z_1 = \sqrt{100 + (377 * 0.025)^2} = 13.7\ \ \Omega$$

$$V_{o,rms} = I_{o,rms} * Z = 9.27 * 13.7 = 127\ \ V\ \ = V_1$$

The THD can be reduced by eliminate the third harmonic which is n=3

$$\boxed{\alpha = \frac{90°}{n}}$$

$$\alpha = \frac{90}{3} = 30$$

$$V_n = \frac{4\,V_{dc}}{n\,\pi}\cos(n\alpha)$$

$$V_1 = \left(\frac{4V_{dc}}{\pi}\right)\cos\,\alpha$$

Therefore, the required DC input voltage is

$$V_{dc} = \frac{\pi * 127}{4\cos 30} = \ 115.17\,V$$

$$V_3 = \frac{4 * 115.17}{3\,\pi}\cos(3 * 30) = 0$$

$$V_5 = \frac{4 * 115.17}{5\,\pi}\cos(5 * 30) = 25.5\,v \qquad\qquad and\ so\ on\ ....$$

Other harmonic voltages, and currents for these harmonics are determined from voltage amplitude and load impedance using the same technique as for the square-wave inverter. The results are summarized in Table

| $n$ | $f_n$ (Hz) | $V_n$ (V) | $Z_n$ (Ω) | $I_n$ (A) |
|---|---|---|---|---|
| 1 | 60 | 127 | 13.7 | 9.27 |
| 3 | 180 | 0 | 30.0 | 0 |
| 5 | 300 | 25.5 | 48.2 | 0.53 |
| 7 | 420 | 18.2 | 66.7 | 0.27 |
| 9 | 540 | 0 | 85.4 | 0 |
| 11 | 660 | 11.6 | 104 | 0.11 |

The THD of load current now is

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty}(I_{n,rms})^2}}{I_{1,rms}}$$

$$THD_i = \frac{\sqrt{\left(\frac{I_3}{\sqrt{2}}\right)^2 + \left(\frac{I_5}{\sqrt{2}}\right)^2 + \left(\frac{I_n}{\sqrt{2}}\right)^2}}{\frac{I_1}{\sqrt{2}}}$$

$$THD_i = 0.066 = 6.6\ \%$$

## *-Pulse Width Modulation (PWM)-*

- Pulse-width modulation (PWM) provides a way to decrease the total harmonic distortion (THD) * of load.
- A PWM inverter output, with some filtering can generally meet THD requirements more easily than the square wave switching scheme. The unfiltered PWM output will have a relatively high THD, but the harmonics will be at much higher frequencies than for a square wave, making filtering easier.
- Harmonic reduction can be achieved by filtering and/or using harmonic elimination techniques.
- The fundamental magnitude of the output voltage from an inverter can be controlled by controlling within the inverter itself so that no external controlled circuit is required.
- PWM techniques are characterized by constant amplitude pulses with different duty cycles for each period.
- There are several types of PWM techniques to control the inverter switching such as sinusoidal sampling, regular sampling, optimized PWM, harmonic elimination or minimization PWM and space-vector modulation (SVM).
- The different PWM techniques essentially differ in the harmonic content of their respective output voltages, thus, the choice of a PWM technique depends on the permissible harmonic content in the inverter output voltage.

→ **Advantages of the PWM control scheme:**

1. PWM minimizes the lower order harmonics, while the higher order harmonics can be eliminated using a filter.
2. Control of the output voltage amplitude.
3. No need to add any external components for controlling.

→ **Advantages of the PWM control scheme:**

1. More complex control circuits for the switches.
2. Increased losses due to more frequent switching.

---

Total Harmonic Distortion (THD): is a measure to determine the “quality” of a given waveform.

**When using PWM, several definitions should be stated:**

1. ***Reference signal***, sometimes called a modulating or control signal, which is a sinusoid in this case.
2. ***Carrier signal***, which is a triangular wave that controls the switching frequency.
3. ***Amplitude Modulation Ratio***, ($M_a$): also called as modulation index is defined as the ratio of amplitudes of the reference signal to the carrier signal as given in equation:

$$M_a = \frac{V_{m,\,reference}}{V_{m,\,\text{carrier}}} = \frac{V_{m,\,\sin e}}{V_{m,\,\text{tri}}}$$

4. ***Frequency Modulation Ratio***, ($M_f$) is defined as the ratio of the frequency of the carrier and reference signal as given in equation:

$$M_f = \frac{f_{carrier}}{f_{\text{reference}}} = \frac{f_{tri}}{f_{\sin e}}$$

$M_f$ is related to the harmonic frequency. When the carrier frequency increases, the frequency at which the harmonics occur will also increase.

---

## ➢ Bipolar Switching of SPWM ثنائي القطب

The principle of sinusoidal bipolar PWM is illustrated in Figure below. Fig. (a) shows a sinusoidal reference signal and a triangular carrier signal while Fig. (b) is the modulated output signal. When the instantaneous value of the reference signal is larger than the triangular carrier, the output is at +Vdc and when the reference is less than the carrier the output is at –Vdc.

$$v_o = +V_{\text{dc}} \quad \text{for } v_{\text{sine}} > v_{\text{tri}}$$

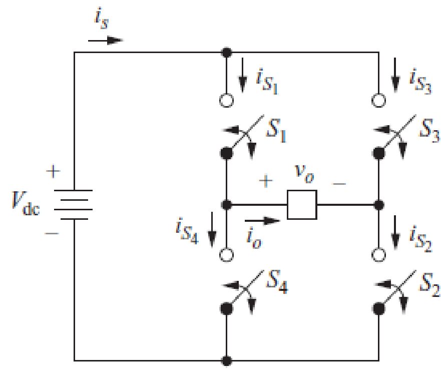
$$v_o = -V_{\text{dc}} \quad \text{for } v_{\text{sine}} < v_{\text{tri}}$$

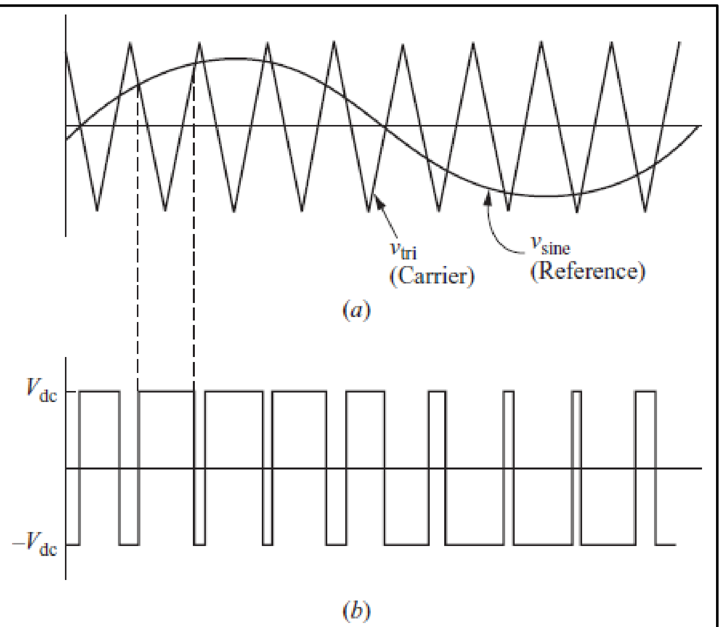
This version of PWM is *bipolar* because the output alternates between plus and minus the dc supply voltage.

If the bipolar scheme is implemented for the full-bridge inverter of Fig. below, the switching is determined by comparing the instantaneous reference and carrier signal as given by,

$$S_1 \text{ and } S_2 \text{ are on when } v_{\text{sine}} > v_{\text{tri}} \quad (v_o = +V_{\text{dc}})$$

$$S_3 \text{ and } S_4 \text{ are on when } v_{\text{sine}} < v_{\text{tri.}} \quad (v_o = -V_{\text{dc}})$$



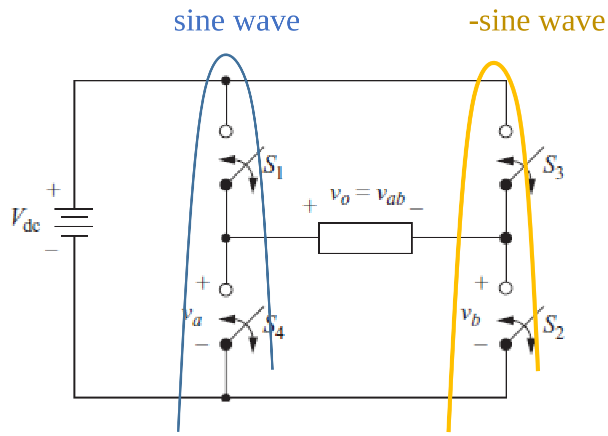




Bipolar pulse-width modulation. (*a*) Sinusoidal reference and triangular carrier; (*b*) Output is $+V_{dc}$ when $v_{sine} > v_{tri}$ and is $-V_{dc}$ when $v_{sine} < v_{tri}$.

## ➢ Unipolar Switching of SPWM احادي القطب

In unipolar switching scheme of PWM full-bridge inverter of Fig. below, the output is switched from either higher to zero or low to zero, rather than between high and low as in bipolar schemes. Two sinusoidal reference signal and one triangular carrier signal are used in this scheme. The switch controls for unipolar switching scheme given as,

$$
\begin{aligned}
&S_1 \text{ is on when } v_{sine} > v_{tri}\\
&S_2 \text{ is on when } -v_{sine} < v_{tri}\\
&S_3 \text{ is on when } -v_{sine} > v_{tri}\\
&S_4 \text{ is on when } v_{sine} < v_{tri}
\end{aligned}
$$




Note that switch pairs ($S1$, $S4$) and ($S2$, $S3$) are complementary when one switch in a pair is closed, the other is open. The voltages $va$ and $vb$ in Fig. (c) alternate between $+V$dc and zero. The output voltage $vo = vab = va - vb$ as shown inFig. (d).

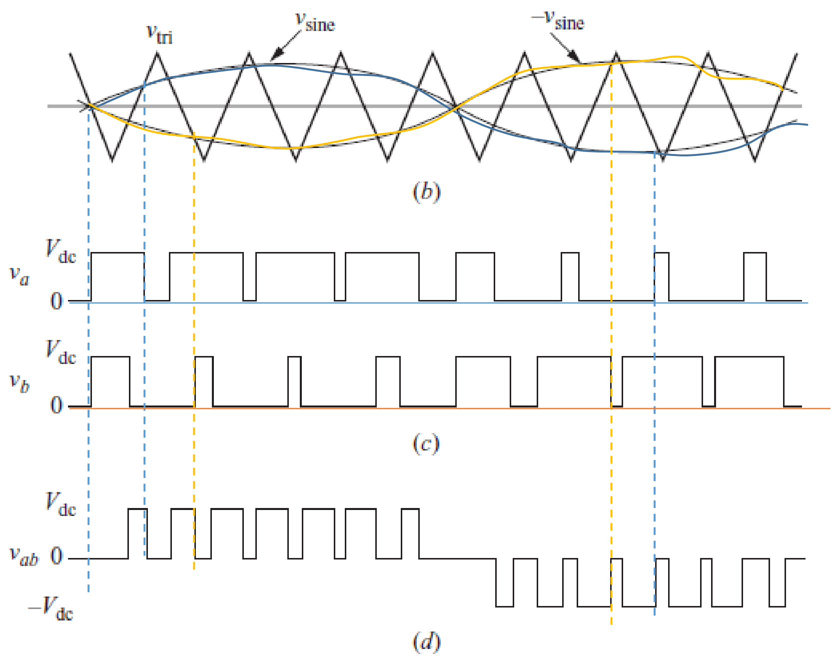



**Figure** (*a*) Full-bridge converter for unipolar PWM; (*b*) Reference and carrier signals; (*c*) Bridge voltages $v_a$ and $v_b$; (*d*) Output voltage.