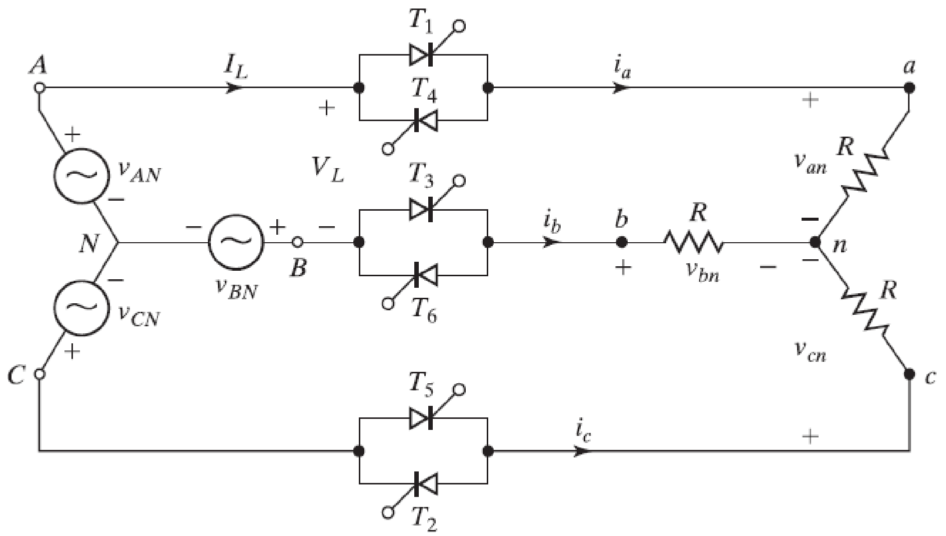
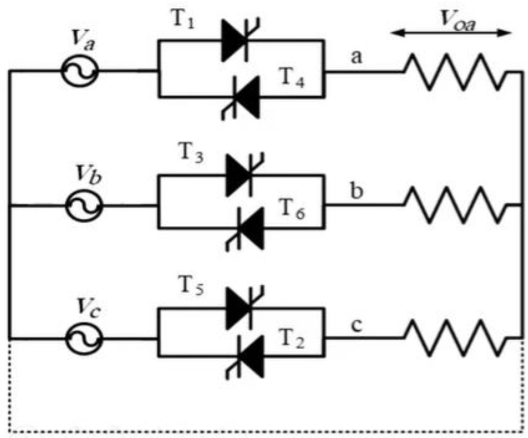
Lecture -4-

2. Three -phase ac Voltage Controller

A three-phase voltage controller with a *Y*-connected resistive load is shown in Fig. below.



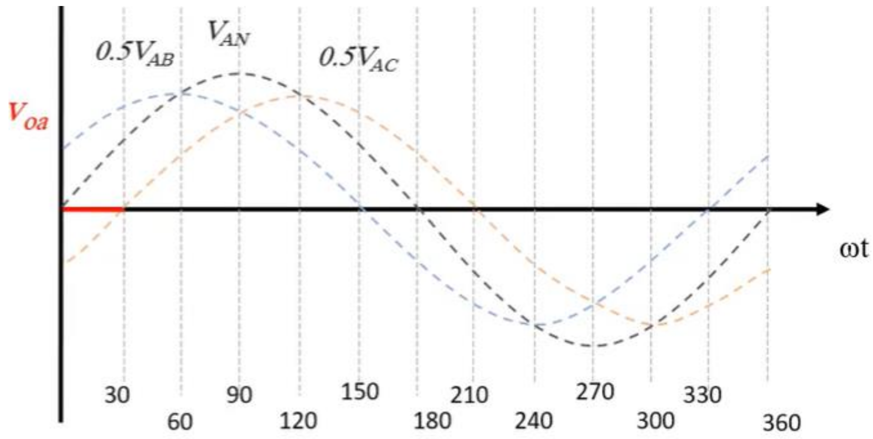
- The power delivered to the load is controlled by the delay angle α on each SCR.
- The six SCRs are turned on in the sequence 1-2-3-4-5-6, at 60^o intervals.
- Gate signals are maintained throughout the possible conduction angle.
- The instantaneous voltage across each load phase is determined by which SCRs conduct.
- At any instant, three SCRs, two SCRs, or no SCRs are on.
- The instantaneous load voltages are either a line-to-neutral voltage (***three on***), one-half of a line-to-line voltage (***two on***), or zero (***none on***).
- When three SCRs are on (one in each phase), The voltage across each phase of the load is the corresponding(equal) line-to-neutral voltage. For example, if T_1, T_2 and T_6 are on, $v_{an}= V_{AN}$, $v_{bn}= V_{BN}$ and $v_{cn}= V_{CN}$.
- When two SCRs are on, the line-to-line voltage of those two phases is equally divided between the two load resistors that are connected. For example, if only T_1 and T_2 are on, $v_{an}= V_{AC}/2$, $v_{cn}= V_{CA}/2$, and $v_{bn}= 0$.

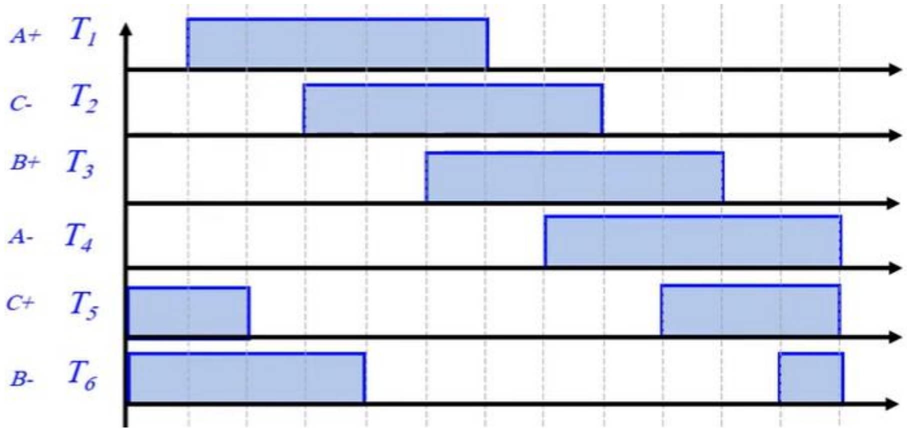


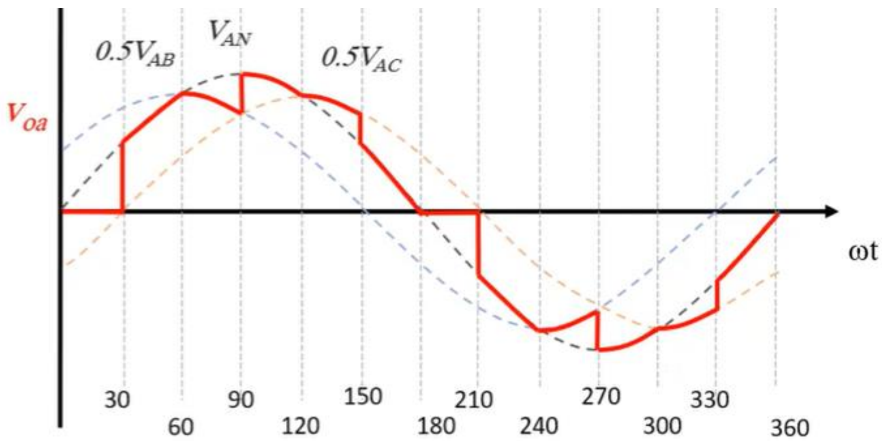
→ The following are the ranges of α that produce particular types of load voltages with an example for each:

1) For $0 \le \alpha < 60$, (Mode 2/3)

α=30°

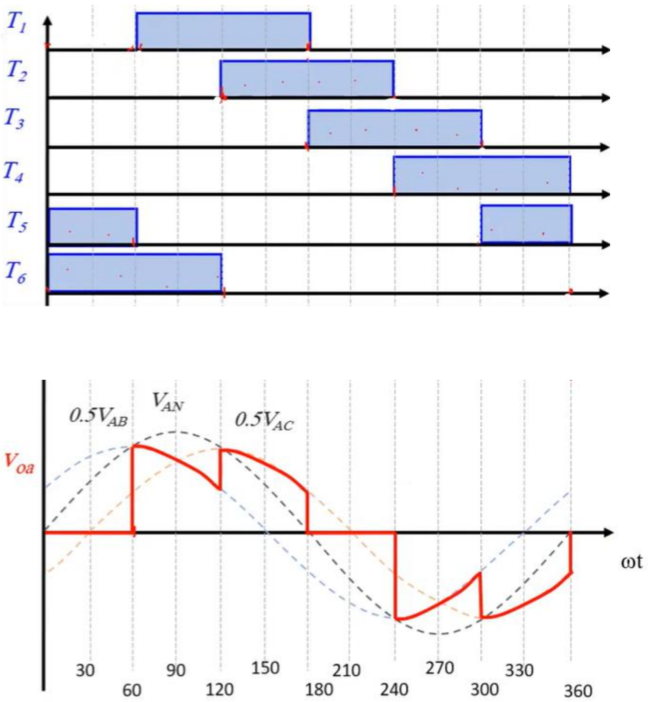




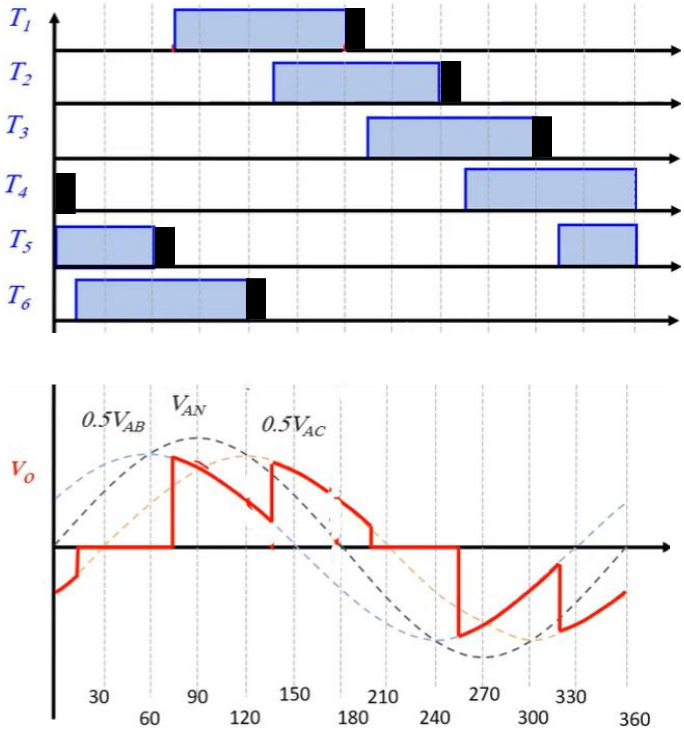


2) For $60 \le \alpha \le 90$, (Mode 2/2)

α=60°

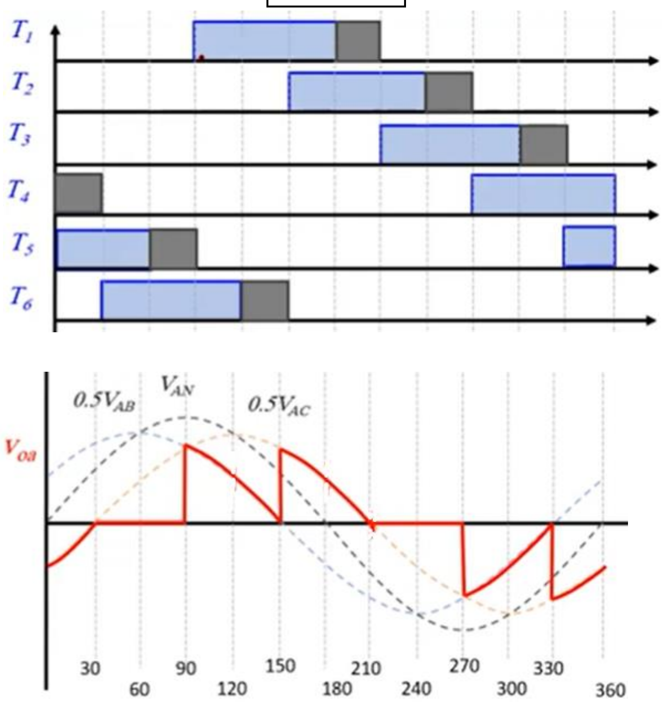


α=75°



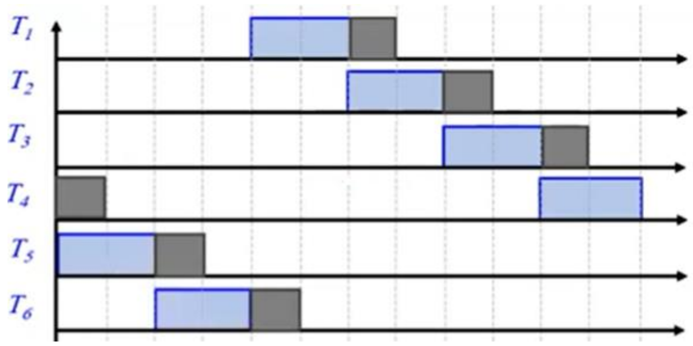
Note/ For α=75° , after T_1 turned off, the T_2 will turn on alone and this is impossible, so must increase the interval turning of T1 by 15° . and so on….

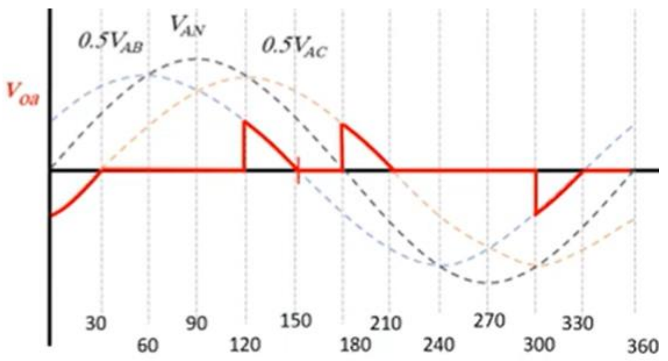
α=90°



3) For 90 < α < 150 , (Mode 0/2)

$\alpha=120^{o}$





For α >150^{o} , there is no time interval when an SCR is forward-biased while a gate signal is applied. So, the output voltage is zero for this condition.

$$V_{o,rms} = V_{s,rms(ph)}\sqrt{1 - \frac{3\alpha}{2\pi} + \frac{3}{4\pi}\sin(2\alpha)} \qquad 0 \le \alpha < 60$$

$$V_{o,rms} = V_{s,rms(ph)}\sqrt{\frac{1}{2} + \frac{9}{8\pi}\sin(2\alpha) + \frac{3\sqrt{3}}{8\pi}\cos(2\alpha)} \qquad 60 \le \alpha < 90$$

$$V_{o,rms} = V_{s,rms(ph)}\sqrt{\frac{5}{4} - \frac{3\alpha}{2\pi} + \frac{3}{8\pi}\sin(2\alpha) + \frac{3\sqrt{3}}{8\pi}\cos(2\alpha)} \qquad 90 \le \alpha < 150$$

The rms output/supply current $$I_{o,rms} = \frac{V_{o,rms}}{R} = I_{s,rms}$$

The output power $$P_o = 3I_{o,rms}{}^2 R = \frac{3V_{o,rms}{}^2}{R}$$

The apparent power $$S = 3V_{s,rms(phase)} I_{s,rms}$$

The supply power factor $$pf = \frac{P_o}{S} = \frac{V_o}{V_s}$$

Example/

The three-phase full-wave controller supplies a Y-connected resistive load of $R = 10\ \Omega$ and the line-to-line input voltage is 208 V (rms), 60 Hz. The delay angle is $\alpha = \pi/3$. Determine (a) the rms output phase voltage V_o, (b) the input PF,

Solution

$V_L = 208$ V, $V_s = V_L/\sqrt{3} = 208/\sqrt{3} = 120$ V, $\alpha = \pi/3$, and $R = 10\ \Omega$.

$$a)\ \ V_{o,rms} = V_{s,rms(ph)}\sqrt{\frac{1}{2} + \frac{9}{8\pi}\sin(2\alpha) + \frac{3\sqrt{3}}{8\pi}\cos(2\alpha)} \qquad 60 \le \alpha < 90$$

$$V_{o,rms} = 120 * \sqrt{\frac{1}{2} + \frac{9}{8\pi}\sin(2*60) + \frac{3\sqrt{3}}{8\pi}\cos(2*60)} = 101.1\ \ V$$

$$b)\qquad I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{101.1}{10} = 10.11\ A = \ I_{s,rms}$$

$$PF = \frac{P}{S} = \frac{3 * \ I^2_{o,rms} * R}{3 * V_{s,rms} * I_{s,rms}}$$

$$PF = \frac{3 * (10.11)^2 * 10}{3 * 120 * 10.11} = \frac{303.3}{360} = 0.84$$

Find for $\alpha = 45$? ans: $I_{o,rms} = 7.58\ A$,

Summary

- Voltage controllers use electronic switches to connect and disconnect a load to an ac source at regular intervals. This type of circuit is classified as an ac-ac converter.

- Voltage controllers are used in applications such as single-phase light-dimmer circuits, single-phase or three-phase induction motor control, and static VAR control.

- The delay angle for the thyristors controls the time interval for the switch being on and thereby controls the effective value of voltage at the load. The range of control for load voltage is between full ac source voltage and zero.

- An ac voltage controller can be designed to function in either the fully on or fully off mode. This application is used as a solid-state relay.

- The load and source current and voltage in ac voltage controller circuits may contain significant harmonics. For equal delay angles in the positive and negative half-cycles, the average source current is zero, and only odd harmonics exist.

Maximum RMS voltage will be applied to the load when $\alpha = 0$, in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage $= \frac{V_m}{\sqrt{2}}$. When α is increased the RMS load voltage decreases.

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi - 0) + \frac{\sin 2\times 0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi) + \frac{0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S$$

Q/ . *A single phase full wave ac voltage controller supplies an RL load. The input supply voltage is 230V, RMS at 50Hz. The load has L = 10mH, R = 10Ω, the delay angle of thyristors T_1 and T_2 are equal, where $\alpha_1 = \alpha_2 = \frac{\pi}{3}$. Determine*

b. *RMS output voltage.*
c. *The input power factor.*
Comment on the type of operation.

Given

$V_s = 230V$, $f = 50Hz$, $L = 10mH$, $R = 10\Omega$, $\alpha = 60^0$,

$\alpha = \alpha_1 = \alpha_2 = \frac{\pi}{3}$ radians, .

$$V_m = \sqrt{2}V_S = \sqrt{2} \times 230 = 325.2691193\ V$$

$$Z = \text{ Load Impedance} = \sqrt{R^2 + (\omega L)^2} = \sqrt{(10)^2 + (\omega L)^2}$$

$$\omega L = (2\pi fL) = (2\pi \times 50 \times 10 \times 10^{-3}) = \pi = 3.14159\Omega$$

$$Z = \sqrt{(10)^2 + (3.14159)^2} = \sqrt{109.8696} = 10.4818\Omega$$

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2} \times 230}{10.4818} = 31.03179\ A$$

Load Impedance Angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\phi = \tan^{-1}\left(\frac{\pi}{10}\right) = \tan^{-1}(0.314159) = 17.44059^0$$

RMS Output Voltage

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

$$V_{O(RMS)} = 230\sqrt{\frac{1}{\pi}\left[\left(3.4456 - \frac{\pi}{3}\right) + \frac{\sin 2(60^0)}{2} - \frac{\sin 2(197.42^0)}{2}\right]}$$

$$V_{O(RMS)} = 230\sqrt{\frac{1}{\pi}[(2.39843) + 0.4330 - 0.285640]}$$

$$V_{O(RMS)} = 230 \times 0.9 = 207.0445\ \text{V}$$

Input Power Factor

$$PF = \frac{P_O}{V_S \times I_S}$$

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{207.0445}{10.4818} = 19.7527\ \text{A}$$

$$P_O = I_{O(RMS)}^2 \times R_L = (19.7527)^2 \times 10 = 3901.716 \text{ W}$$

$$V_S = 230V, \qquad I_S = I_{O(RMS)} = 19.7527$$

$$PF = \frac{P_O}{V_S \times I_S} = \frac{3901.716}{230 \times 19.7527} = 0.8588$$

Q/

A single phase full wave controller has an input voltage of 120 V (RMS) and a load resistance of 6 ohm. The firing angle of thyristor is $\pi/2$. Find

a. *RMS output voltage*
b. *Power output*
c. *Input power factor*
d. *Average and RMS thyristor current.*

Solution

$$\alpha = \frac{\pi}{2} = 90^0, \quad V_S = 120 \text{ V}, \quad R = 6\Omega$$

RMS Value of Output Voltage

$$V_O = V_S \left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_O = 120\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 180}{2}\right)\right]^{\frac{1}{2}}$$

$$V_O = 84.85 \text{ Volts}$$

RMS Output Current

$$I_O = \frac{V_O}{R} = \frac{84.85}{6} = 14.14 \text{ A}$$

Load Power

$$P_O = I_O^2 \times R$$

$$P_O = (14.14)^2 \times 6 = 1200 \text{ watts}$$

Input Current is same as Load Current

Therefore $I_S = I_O = 14.14$ Amps

Input Supply Volt-Amp $= V_S I_S = 120 \times 14.14 = 1696.8 \ VA$

Therefore

$$\text{Input Power Factor} = \frac{\text{Load Power}}{\text{Input Volt-Amp}} = \frac{1200}{1696.8} = 0.707$$

Average thyristor current $I_{T(Avg)}$

$$I_{T(Avg)} = \frac{1}{2\pi R}\int_{\alpha}^{\pi} V_m \sin\omega t . d(\omega t)$$

$$= \frac{V_m}{2\pi R}(1+\cos\alpha) \; ; \qquad \mathrm{V_m} = \sqrt{2}V_S$$

$$= \frac{\sqrt{2}\times 120}{2\pi\times 6}[1+\cos 90] = 4.5 \text{ A}$$

RMS thyristor current $I_{T(RMS)}$

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}\frac{V_m^2\sin^2\omega t}{R^2}d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{2\pi R^2}\int_{\alpha}^{\pi}\frac{(1-\cos 2\omega t)}{2}d(\omega t)}$$

$$= \frac{V_m}{2R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}V_S}{2R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}\times 120}{2\times 6}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 180}{2}\right)\right]^{\frac{1}{2}} = 10 \text{ Amps}$$

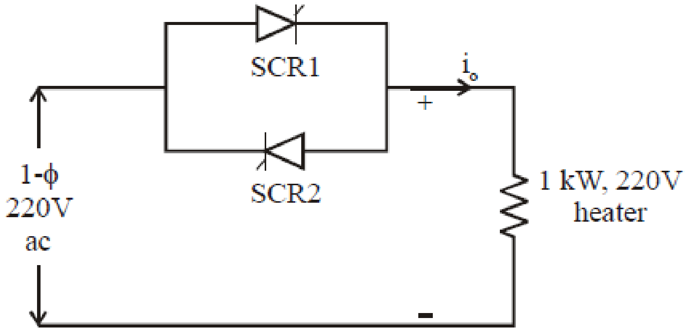
or

The rms current in each SCR is

$$I_{\mathrm{SCR,rms}} = \frac{I_{o,\mathrm{rms}}}{\sqrt{2}}$$

Q

Find the RMS and average current flowing through the heater shown in figure. The delay angle of both the SCRs is 45^0.



The load voltages for a three-phase voltage controller with an RL load are again characterized by being a line-to-neutral voltage, one-half of a line-to-line voltage, or zero. The analysis is much more difficult for an RL load than for a resistive load, and simulation provides results that would be extremely difficult to obtain analytically.