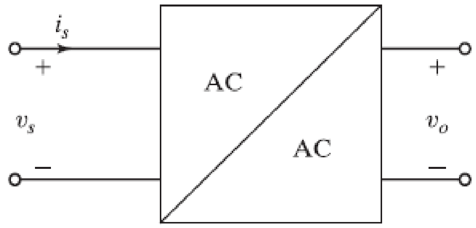
Lecture -3-

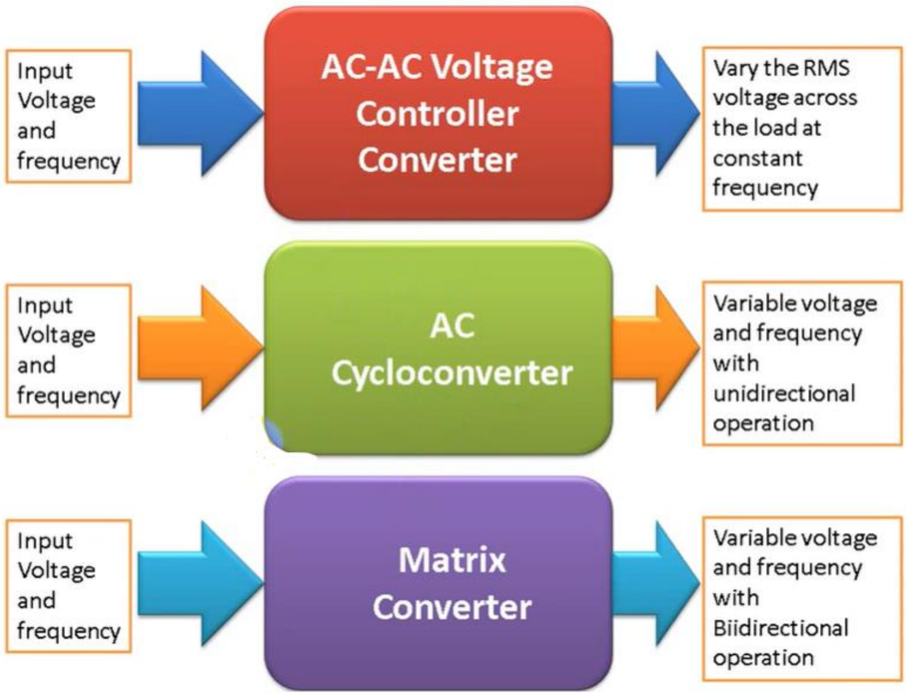
AC to AC Converters

– AC/AC converters, in general form, are power electronic devices that convert one input alternating voltage (AC) to another ac system at output load with waveforms of different amplitude, frequency, and phase (vary the RMS value).



– AC/AC converters can be categorized into Three topologies:

1. AC-AC Voltage Controller Converter (AC Choppers)
2. AC Cycloconverter
3. Matrix Converter
   → Indirect Matrix Converter (AC-DC-AC Converter)
   → Direct Matrix Converter



- ## Ac Voltage Controller:

The AC voltage controller is a converter that controls the voltage, current, and average power delivered to an ac load from an ac source. Electronic switches connect and disconnect the source and the load at regular intervals. ac voltage controller used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output. The RMS value of the ac output voltage and the ac power flow to the load is controlled by varying (adjusting) the trigger angle ‘α’.

The ac voltage controllers are classified into two types based on the type of input ac supply applied to the circuit.

1) **Single phase ac voltage controllers.**
   - Half wave ac voltage controller (uni-directional controller).
   - Full wave ac voltage controller (bi-directional controller).

2) **Three phase ac voltage controllers.**
   - Half wave ac voltage controller (uni-directional controller).
   - Full wave ac voltage controller (bi-directional controller).

- ## Applications of Ac Voltage Controllers

1. Lighting / Illumination control in ac power circuits.
2. Induction heating.
3. Industrial heating & Domestic heating.
4. Transformer tap changing.
5. Speed control of induction motors
6. AC magnet controls.

- **There are two basic techniques for controlling the ac output voltage:**

  i. On-Off control (or integral cycle control)
  ii. Phase control

i. <u>On-Off control:</u> is a suitable technique for systems with a large time constant, such as temperature control systems. The load power can be controlled by connecting the ac supply (source) to the load for a few complete cycles then disconnecting the source from the load for another number of cycles and repeating the switching cycle. The relative duration of the on and off periods, i.e. the duty cycle d is adjusted so that the average power delivered to the load meets some particular objectives. Integral cycle control is not suitable for loads with a short time constant. in Fig. below the typical pattern. In ideal circumstances, the average power to the load can be controlled from 0% through 100%.

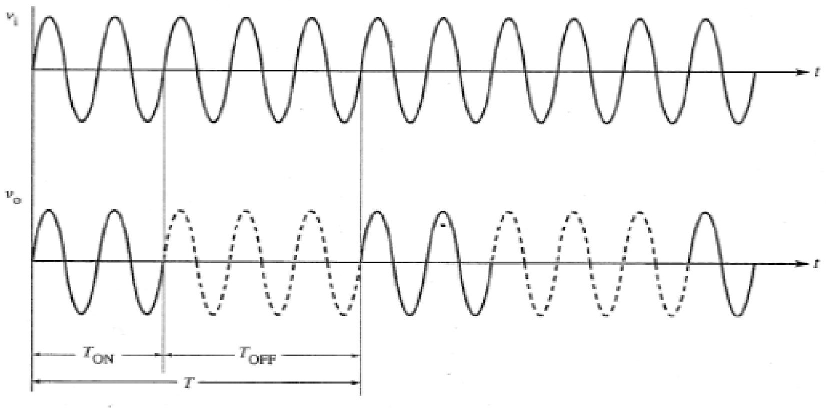


Figure : Integral Cycle Control

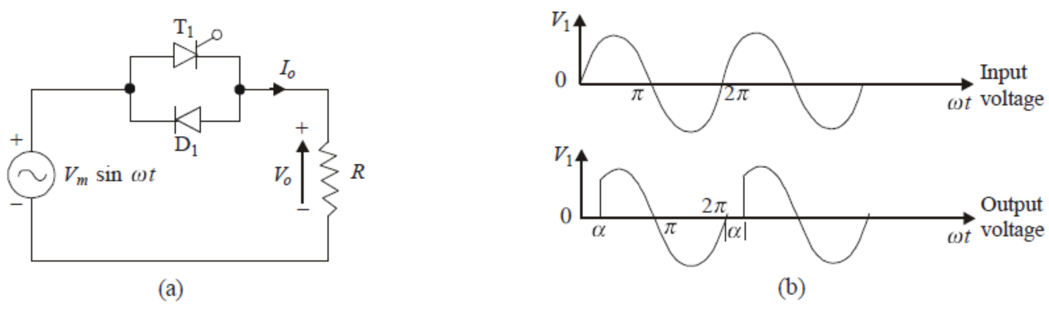
ii. **<u>Phase control:</u>** the switch connects the load to the source for a part of each cycle of input voltage. That is the ac supply voltage is chopped using Thyristors /Triac during a part of each input cycle. The thyristor switch is turned on for a part of every half cycle, so that input supply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load. By controlling the phase angle or the trigger angle 'a' (delay angle), the output RMS voltage across the load can be controlled.

1. *Single-phase ac Voltage Controller*

➢ Half wave ac voltage controller (uni-directional controller).

In the uni-directional voltage controllers (*a*), phase control is applied only during positive-half cycle of the input supply using a thyristor T_1. By assuming T_1 as an ideal thyristor switch it can be considered as a closed switch when it is turned-on during the period wt=α to π radians. When the input supply voltage decreases to zero at wt= π, for a resistive load the load current also falls to zero at wt= π and hence the thyristor T_1 turns off.

The supply voltage reverses (negative-half cycle), phase control is not applied because a diode D_1 is used and becomes forward biased, hence turns ON and conducts. The load current flows in the opposite direction during wt= π to 2π when D_1 is ON and the output voltage follows the negative half cycle of input supply. Fig. (b) illustrates the output waveform obtained from the unidirectional controller.



Disadvantages of single phase half wave ac voltage controller.

1. The output waveforms Fig. (b) obtained from the unidirectional controller illustrate that the positive half cycle is not identical to the negative half cycle, this results in the insertion of a DC component in the supply and the load circuit. This is not desirable for both the supply and the load; hence, unidirectional voltage controllers are limited to very small power applications.
2. The half wave ac voltage controller using a single thyristor and a single diode provides control on the thyristor only in one half cycle of the input supply. Hence ac power flow to the load can be controlled only in one half cycle.
3. Half-wave ac voltage controller gives a limited range of RMS output voltage control. Because the RMS value of ac output voltage can be varied from a maximum of 100% of Vs at a trigger angle $\alpha = 0$ to a low of 70.7% of Vs at $\alpha = \pi$.

These drawbacks of single phase half wave ac voltage controller can be overcome by using a single phase full wave ac voltage controller.

To Calculate the Average Value (Dc Value) Of Output Voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{2\pi} V_m \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{2\pi} \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos 2\pi + \cos\alpha\right] \qquad ; \quad \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi}\left[\cos\alpha - 1\right] \quad ; V_m = \sqrt{2}V_S$$

$$\text{Hence } V_{dc} = \frac{\sqrt{2}V_S}{2\pi}(\cos\alpha - 1)$$

To Derive an Expression for rms Output Voltage $V_{O(RMS)}$:

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m^{\,2}\sin^2\omega t.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\,2}}{2\pi}\left[\int_{\alpha}^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right).d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{4\pi}\left[\int_{\alpha}^{2\pi}(1-\cos 2\omega t).d(\omega t)\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\int_{\alpha}^{2\pi} d(\omega t) - \int_{\alpha}^{2\pi}\cos 2\omega t.d\omega t\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[(\omega t)\Big/_{\alpha}^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}\right]}$$

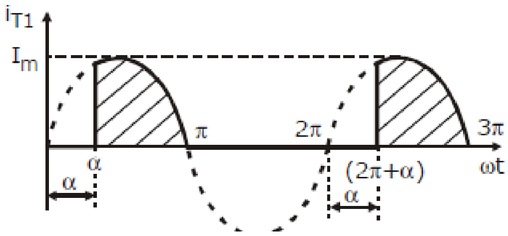
$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{(2\pi-\alpha) - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{(2\pi-\alpha) - \left\{\frac{\sin 4\pi}{2} - \frac{\sin 2\alpha}{2}\right\}} \qquad ;\sin 4\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{(2\pi-\alpha) + \frac{\sin 2\alpha}{2}}$$

$V_{i(RMS)} = V_S = \frac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage

Average Thyristor Currents



$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m \sin\omega t.d(\omega t)\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[\int_{\alpha}^{\pi}\sin\omega t.d(\omega t)\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[(-\cos\omega t)\Big/_{\alpha}^{\pi}\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos(\pi)+\cos\alpha\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[1+\cos\alpha\right]$$

Where, $I_m = \frac{V_m}{R_L}$ = Peak thyristor current = Peak load current.

RMS Thyristor Currents

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$I_{T(RMS)} = \sqrt{\frac{I_m^2}{2\pi}\left[\int_{\alpha}^{\pi} \frac{(1-\cos 2\omega t)}{2} . d(\omega t)\right]}$$

$$I_{T(RMS)} = \sqrt{\frac{I_m^2}{4\pi}\left[\int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t . d(\omega t)\right]}$$

$\pi + \alpha$

$$I_{T(RMS)} = I_m \sqrt{\frac{1}{4\pi}\left[(\omega t)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right]}$$

$$I_{T(RMS)} = I_m \sqrt{\frac{1}{4\pi}\left[(\pi - \alpha) - \left\{\frac{\sin 2\pi - \sin 2\alpha}{2}\right\}\right]}$$

$$I_{T(RMS)} = I_m \sqrt{\frac{1}{4\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

Example/

A single phase half-wave ac voltage controller has a load resistance $R = 50\Omega$; input ac supply voltage is 230 V rms at 50Hz. The input supply transformer has a turn's ratio of 1:1. If the thyristor T_1 is triggered at $\alpha = 60^\circ$. Calculate:

1) RMS output voltage.
2) Output power.
3) Input power factor.
4) Average dc load current
5) Average and RMS thyristor current.

solution

Given,

$V_p = 230V, RMS$ primary supply voltage.

$f =$ Input supply frequency = 50Hz.

$R_L = 50\Omega$

$\alpha = 60^0 = \frac{\pi}{3}$ radians.

$V_S =$ RMS secondary voltage.

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{1}{1} = 1$$

Therefore, Vs=230 v

Where, N_P = Number of turns in the primary winding.

N_S = Number of turns in the secondary winding.

1) RMS output voltage.

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = 218.4696\ V \approx 218.47\ V$$

2) Output power.

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L} = \frac{218.46966}{50} = 4.36939\ Amps$$

$$P_O = I_{O(RMS)}^2 \times R_L = (4.36939)^2 \times 50 = 954.5799\ Watts$$

$$P_O = 0.9545799\ KW$$

3) Input power factor.

$$PF = \frac{P_O}{V_S \times I_S}$$

V_S = RMS secondary supply voltage = 230V.

I_S = RMS secondary supply current = RMS load current.

$$\therefore I_S = I_{O(RMS)} = 4.36939\ Amps$$

$$\therefore PF = \frac{954.5799\ \text{W}}{(230 \times 4.36939)\ \text{W}} = 0.9498$$

4) Average dc load current

$$V_{O(dc)} = \frac{V_m}{2\pi}[\cos\alpha - 1]$$

$$V_{O(dc)} = \frac{\sqrt{2} \times 230}{2\pi}\left[\cos(60^0) - 1\right] = \frac{325.2691193}{2\pi}[0.5 - 1]$$

$$V_{O(dc)} = \frac{325.2691193}{2\pi}[-0.5] = -25.88409\ \text{Volts}$$

$$I_{O(dc)} = \frac{V_{O(dc)}}{R_L} = \frac{-25.884094}{50} = -0.51768\ \text{Amps}$$

5) Average and RMS thyristor current.

$$I_{T(Avg)} = \frac{I_m}{2\pi}[1+\cos\alpha]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi R_L}[1+\cos\alpha]$$

$$I_{T(Avg)} = \frac{\sqrt{2}\times 230}{2\pi\times 50}\left[1+\cos\left(60^0\right)\right]$$

$$I_{T(Avg)} = \frac{\sqrt{2}\times 230}{100\pi}[1+0.5]$$

$$I_{T(Avg)} = 1.5530 \text{ Amps}$$

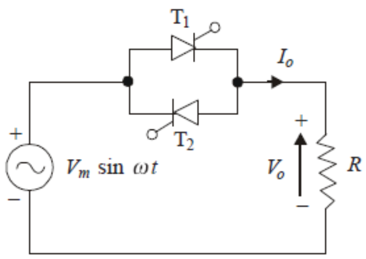
$$I_{T(RMS)} = I_m\sqrt{\frac{1}{4\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

$$I_{T(RMS)} = 2.91746 \text{ Amps}$$

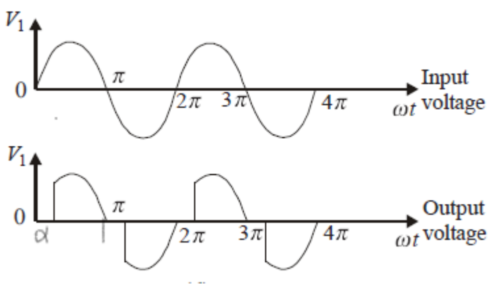
➢ Full wave ac voltage controller (bi-directional controller).

A basic bidirectional voltage controller in Fig. *(c)* consists of a pair of anti-parallel (inverse-parallel) thyristors (SCRs) to control both positive and negative half cycles. This SCR arrangement makes it possible to have current in either direction in the load, where the SCRs carry current in opposite directions. Atriac is equivalent to the antiparallel SCRs.

the waveform (d) reveals that positive and negative half cycles of the output voltage wave can be made symmetrical by choosing the firing angles α and $(\alpha+\pi)$ for the thyristors T1 and T2 respectively. Unlike the unidirectional voltage controller, this circuit does not introduce a direct component in the supply and the load circuits. This circuit is thus, more suited to practical applications.

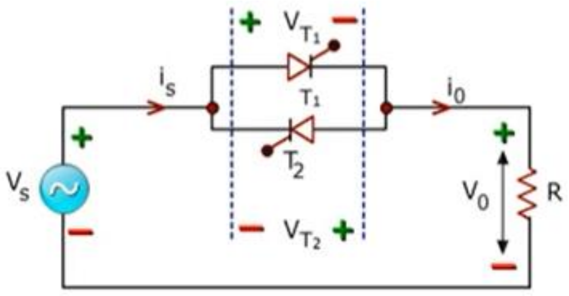


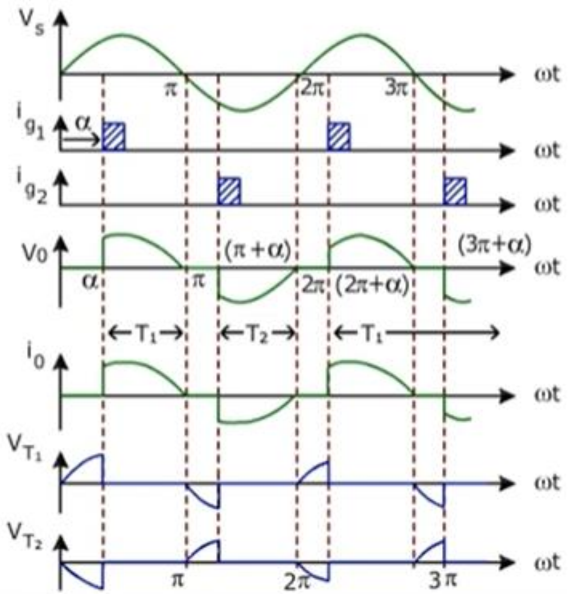
(c)



➢ ***with (R- Load)***

The figure shows a single-phase voltage controller connected to a resistive load R.





During positive half cycle, T_1 is triggered at a firing angle α and during negative half cycle T_2 is triggered at a firing angle $(\pi + \alpha)$. T_1 starts conducting at α and the source voltage is applied to the load from α to π. At π both V_o and I_o fall to zero. Immediately after π, T_1 is subjected to reverse bias and, therefore, it is turned off. During negative half cycle, T_2 is triggered at $(\pi + \alpha)$. T_2 starts conducting from $(\pi + \alpha)$ and source voltage is applied to the load from $(\pi + \alpha)$ to 2π. At 2π, both V_o, and I_o fall to zero. Immediately after 2π, T_2 is subjected to a reverse bias, and therefore, it is turned off. This cycle repeats with T_1 getting triggered at $(2\pi + \alpha)$ again. Load and source currents have the same waveform as the load and source are series connected.

Within each cycle the load current exists when one of the thyristors is on, i.e., for $\alpha < \omega t < \pi$ and $\pi + \alpha < \omega t < 2\pi$. In general, the angles of conduction of the thyristors do not have to be the same. The rms value of the load voltage or the load current is controlled by varying the angle of conduction α of the thyristors T1 and T2. For this reason, this principle of control is known as the phase method of converting an AC voltage.

$v_s(\omega t) = V_m \sin \omega t$

Output voltage is

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } \alpha < \omega t < \pi \text{ and } \alpha + \pi < \omega t < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

The rms load voltage is determined by taking advantage of positive and negative symmetry of the voltage waveform, necessitating evaluation of only a half period of the waveform:

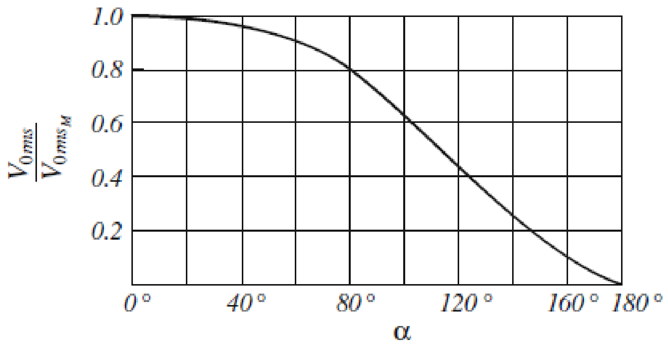
$$\boxed{V_{o,\text{rms}} = \sqrt{\frac{1}{\pi}\int_\alpha^\pi [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}}$$

$$I_{o,\text{rms}} = \frac{V_{o,\text{rms}}}{R}$$

The dependence of the rms value of the output voltage on the angle of conduction is the control characteristic of an AC/AC voltage converter in figure below. for $\alpha = 0$, the load voltage is a sine function equal to the rms value of the source voltage ($V_{o,rms} = V_m/\sqrt{2}$). If $\alpha = 180°$, both thyristors are off and the output voltage is equal to zero.

Note/ This chart used to find the value of α

– Find $\frac{V_{o,rms}}{V_{s,rms}}$
– Delay angle (degrees)



Normalized control characteristic of a phase AC/AC voltage converter

and the power factor of the load is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{V^2_{o,\text{rms}}/R}{V_{s,\text{rms}}(V_{o,\text{rms}}/R)} = \frac{V_{o,\text{rms}}}{V_{s,\text{rms}}}$$

$$= \frac{\frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{(\sin 2\alpha)}{2\pi}}}{V_m/\sqrt{2}}$$

$$\boxed{\text{pf} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}}$$

Example:

A single-phase ac voltage controller has a 120-V rms 60 Hz source. The load resistance is 15Ω . Determine (a) the delay angle required to deliver 500 W to the load, (b) the rms source current, (c) the rms and average currents in the SCRs, (d) the power factor.

Sol/

(*a*) The required rms voltage to deliver 500 W to a 15-Ω load is

$$P = \frac{V_{o,\text{rms}}^2}{R}$$

$$V_{o,\text{rms}} = \sqrt{PR} = \sqrt{(500)(15)} = 86.6 \text{ V}$$

the delay angle required can be found from the Normalized rms load voltage chart:

$$\frac{V_{o,rms}}{V_{s,rms}} = \frac{86.6}{120} = 0.72$$

so ; α approximately = 88.1° = 1.54 rad

(*b*) Source rms current is

$$I_{o,\text{rms}} = \frac{V_{o,\text{rms}}}{R} = \frac{86.6}{15} = 5.77 \text{ A}$$

(*c*) SCR currents are

$$I_{\text{SCR, rms}} = \frac{I_{\text{rms}}}{\sqrt{2}} = \frac{5.77}{\sqrt{2}} = 4.08 \text{ A}$$

$$I_{\text{SCR, avg}} = \frac{\sqrt{2}(120)}{2\pi(15)}\left[1 + \cos(88.1°)\right] = 1.86 \text{ A}$$

(*d*) The power factor is

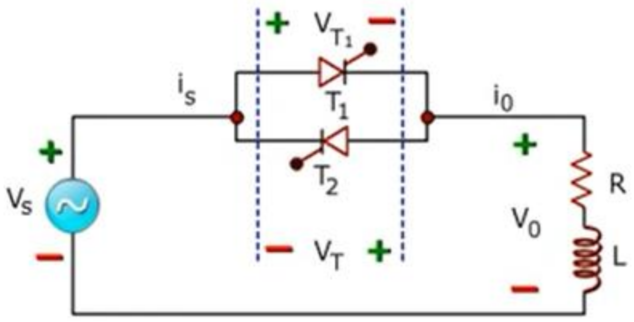
$$\text{pf} = \frac{P}{S} = \frac{500}{(120)(5.77)} = 0.72$$

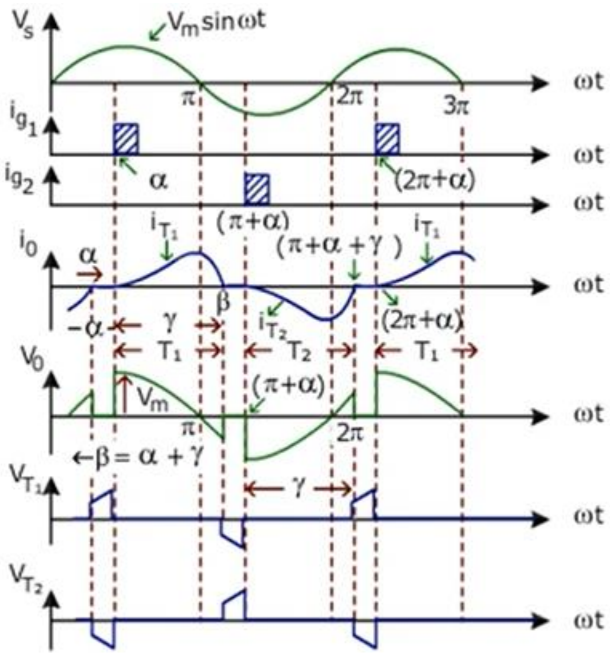
or from

$$\text{pf} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

➤ *with (RL- Load)*

The inductance changes the character of the load current and voltage. Compared to a purely resistive load. the inductance slows down the variations of the load current (Fig.below). For this reason, there exists a phase delay of the load current compared to the phase of the load voltage. In other words, the current i_o flows the load and the corresponding thyristor after the source voltage has crossed zero, reaching the zero value at an angle $\omega t = \beta$ in the next half-cycle of the input voltage Vi.





$$i_o(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where

$$Z = \sqrt{R^2 + (\omega L)^2}\,, \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The extinction angle β is the angle at which the current returns to zero, when $\omega t = \beta$,

In the interval between π and β when the source voltage is Negative and the load current is still positive, T_2 cannot be turned on because it is not forward-biased. The gate signal to T_2 must be delayed at least until the current in S1 reaches zero, at ωt = β. The delay angle is therefore at least β - π

$$\alpha \geq \beta - \pi$$

$$V_{o,rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} (Vm \sin wt)^2 \; d(wt)}$$

$$V_{o,rms} = \sqrt{\frac{Vm^2}{\pi}\int_{\alpha}^{\beta} \frac{1-\cos\,(2wt)}{2} \, .\, d(wt)}$$

$$V_{o,rms} = \sqrt{\frac{Vm^2}{2\pi}\left[\int_{\alpha}^{\beta} d(wt) - \int_{\alpha}^{\beta} \cos 2wt . d(wt)\right]}$$

$$V_{o,rms} = \frac{Vm}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\beta-\alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right\}}$$

An expression for rms load current is determined by recognizing that the square of the current waveform repeats every π rad. Using the definition of rms,

$$I_{o,\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o^2(\omega t)\, d(\omega t)}$$

OR

$$I_{o,rms} = \frac{V_{o,rms}}{Z}$$

Power absorbed by the load is determined from

$$P = I_{o,\text{rms}}^2 R$$

The rms current in each SCR is

$$I_{\text{SCR,rms}} = \frac{I_{o,\text{rms}}}{\sqrt{2}}$$

The average load current is zero, but each SCR carries one-half of the current waveform, making the average SCR current

$$I_{\text{SCR,avg}} = \frac{1}{2\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t)$$

Example:

For the single-phase voltage controller, the source is 120 V rms at 60 Hz, and the load is a series RL combination with R = 20 Ω and L = 50 mH. The delay angle α is 90°. Determine (a) an expression for load current for the first half-period, (b) the rms load current, (c) the rms SCR current, (d) the average SCR current, (e) the power delivered to the load, and (f) the power factor. $\beta = 3.83 \text{ rad} = 220°$

Sol/

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{(20)^2 + [(377)(0.05)]^2} = 27.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\frac{(377)(0.05)}{20} = 0.756 \text{ rad}$$

$$\omega\tau = \omega\left(\frac{L}{R}\right) = 377\left(\frac{0.05}{20}\right) = 0.943 \text{ rad}$$

$$\frac{V_m}{Z} = \frac{120\sqrt{2}}{27.5} = 6.18 \text{ A}$$

$$\alpha = 90° = 1.57 \text{ rad}$$

$$i_o(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

$$i_o(\omega t) = 6.18 \sin(\omega t - 0.756) - 23.8e^{-\omega t/0.943} \quad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

(b) the rms load current

$$V_{o,rms} = \frac{Vm}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

$$V_{o,rms} = \frac{120\sqrt{2}}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[\left(3.83 - \frac{\pi}{2}\right) + \frac{\sin(2*90)}{2} - \frac{\sin(2*220)}{2}\right]} = 90.07\ V$$

$$I_{o,rms} = \frac{V_{o,rms}}{Z} = \frac{90.07}{27.5} = 3.2\ A$$

(c) The rms current in each SCR is

$$I_{SCR,rms} = \frac{I_{o,rms}}{\sqrt{2}} = \frac{3.2}{\sqrt{2}} = 2.26\ A$$

(*d*) Average SCR current is

$$I_{\text{SCR, avg}} = \frac{1}{2\pi}\int_{1.57}^{3.83}\left[6.18\sin(\omega t - 0.756) - 23.8e^{-\omega t/0.943}\right]d(\omega t) = 1.04\text{ A}$$

(*e*) Power absorbed by the load is

$$P = I_{o,\text{rms}}^2 R$$

(*f*) Power factor is determined from *P*/*S*.

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}}$$

Q/ R-load

The single-phase ac voltage controller has a 120-V rms source at 60 Hz and a load resistance of 40 Ω. Determine the range of α so that the output power can be controlled from 200 to 400 W. Determine the range of power factor that will result.

Sol/

$$\text{For } P = 200W,\ V_{o,rms} = \sqrt{PR} = \sqrt{200(40)} = 89.4\ V$$

From chart

$$\alpha = 1.48\ rad = 85^\circ$$

$$pf = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}} = \frac{200}{(120)(89.4/40)} = 0.75 = 75\%.$$

$$\text{For } P = 400\ W,\ V_{o,rms} = \sqrt{PR} = \sqrt{400(40)} = 126\ V$$

Since 126 V > 120 V of the source, 400 W is not possible.

The maximum power available is $\frac{120^2}{40} = 360$ W. The pf is 1.0 for 360 W.