Lecture -2-

Three-phase Rectification

➢ Controlled Three -Phase Rectifiers

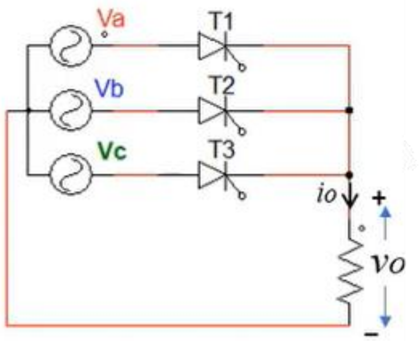
Features of Three -phase controlled rectifiers are:

- Operate from 3 phase ac supply voltage.
- The output can be controlled by substituting SCRs for diodes.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.
- Three-phase controlled rectifiers are extensively used in high-power variable speed industrial dc drives.

1) Half-Wave Rectifiers

– Three single-phase half-wave converters are connected together to form a three-phase half-wave converter.
– The thyristor will conduct (ON state), when the anode-to-cathode voltage is positive and a firing current pulse is applied to the gate terminal. Delaying the firing pulse by an angle α controls the load voltage.
– The firing angle α is measured from the crossing point between the phase supply voltages.
– The possible range for gating delay is between α = 0◦ and α = 180◦, but because of commutation problems in actual situations, the maximum firing angle is limited to around 150◦.

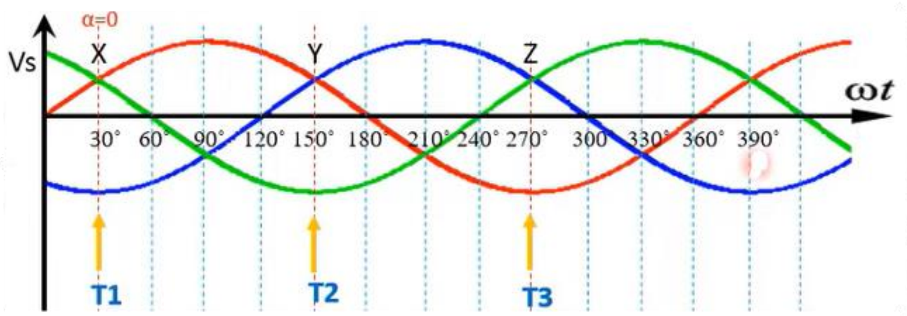
♦ ***__Case of resistive load__***



$\omega t = 30° \ to \ 150°$, T1 conduct

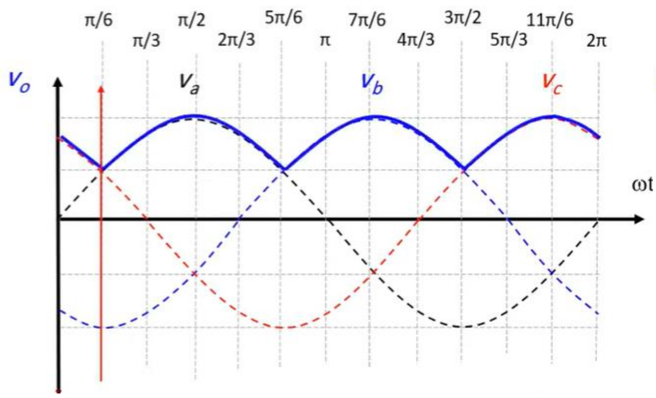
$\omega t = 150° \ to \ 270°$, T2 conduct

$\omega t = 270° \ to \ 390°$, T3 conduct

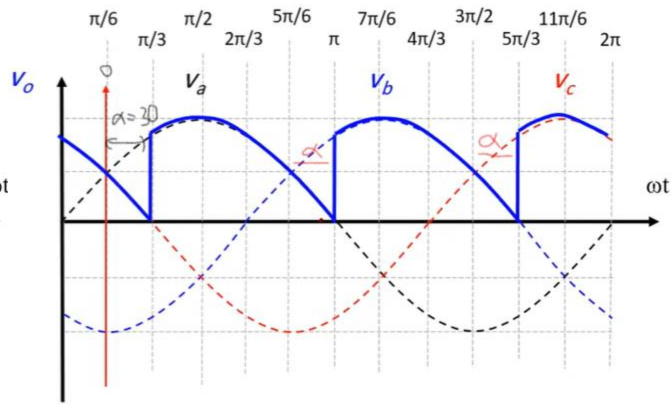


For Different Trigger Angles:

A. For $\alpha \leq 30^o$



α=0



α=30

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} v_O.d(\omega t)\right]$$

$$v_O = v_{an} = V_m \sin\omega t \text{ for } \omega t = \left(30^0+\alpha\right) \text{ to } \left(150^0+\alpha\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\left(-\cos\omega t\right)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

Note from the trigonometric relationship

$$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}\right)\cos(\alpha) + \sin\left(\frac{5\pi}{6}\right)\sin(\alpha) + \cos\left(\frac{\pi}{6}\right).\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos(150^0)\cos(\alpha) + \sin(150^0)\sin(\alpha) + \cos(30^0).\cos(\alpha) - \sin(30^0)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos(180^0 - 30^0)\cos(\alpha) + \sin(180^0 - 30^0)\sin(\alpha) + \cos(30^0).\cos(\alpha) - \sin(30^0)\sin(\alpha)\right]$$

Note: $\cos(180^0 - 30^0) = -\cos(30^0)$

$\sin(180^0 - 30^0) = \sin(30^0)$

Therefore

$$V_{dc} = \frac{3V_m}{2\pi}\left[+\cos(30^0)\cos(\alpha) + \sin(30^0)\sin(\alpha) + \cos(30^0).\cos(\alpha) - \sin(30^0)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos(30^0)\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2 \times \frac{\sqrt{3}}{2}\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos(\alpha)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha)$$

Where

$V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage for a 3-phase star connected transformer.

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos(\alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{2\pi R}\cos\alpha$$

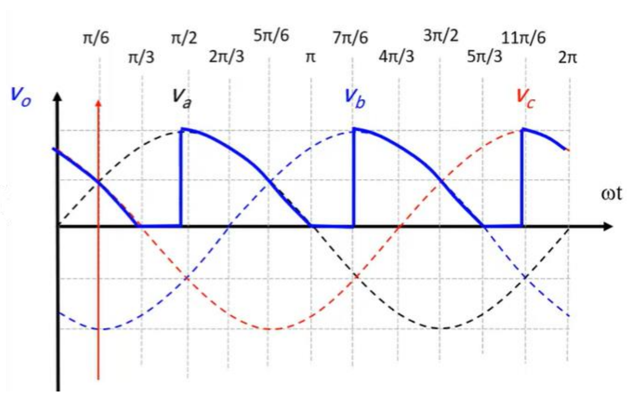
$$V_{rms} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}(V_m \sin\omega t)^2 d\omega t} = \sqrt{3}V_m\sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi}\cos 2\alpha}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{3}V_m}{R}\sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi}\cos 2\alpha}$$

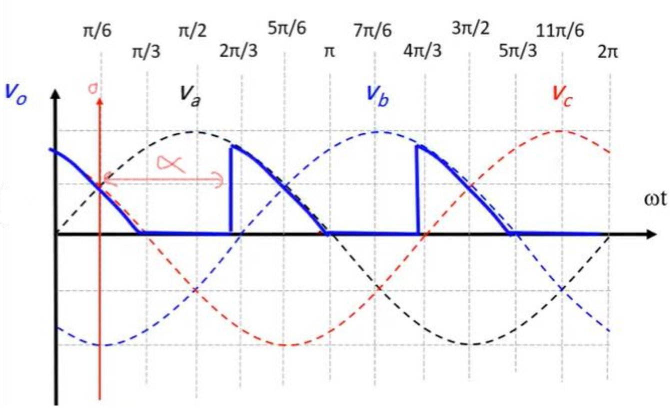
(at Load)

B. For $\alpha > 30^o$

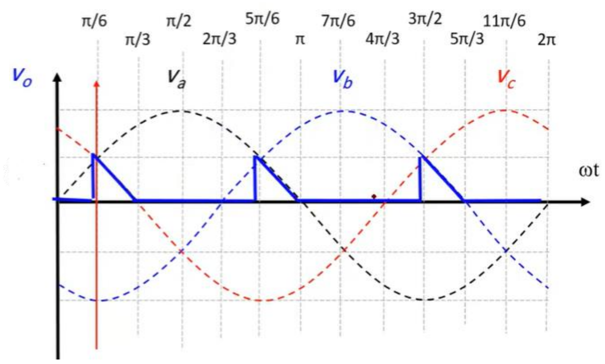
T_1 is triggered at $\omega t = \left(30^0 + \alpha\right)$ and T_1 conducts up to $\omega t = 180^0 = \pi$ radians. When the phase supply voltage v_{an} decreases to zero at $\omega t = \pi$, the load current falls to zero and the thyristor T_1 turns off. Thus T_1 conducts from $\omega t = \left(30^0 + \alpha\right)$ to $\left(180^0\right)$.



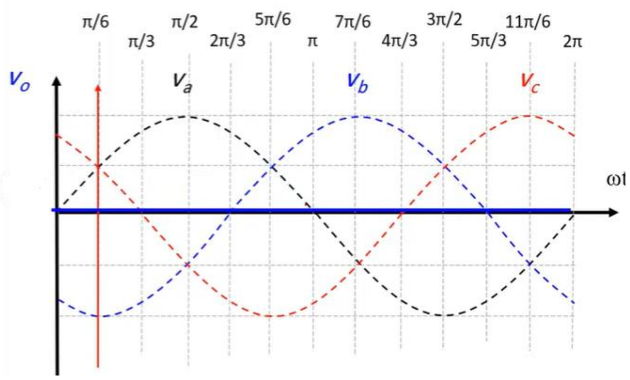
α =60



α =90



α =120



α =150

- For $\alpha > 30°$ the load current is discontinuous for R load only, but in this case $\alpha > 150°$ is not possible

$$V_{dc} = \frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \, sin\omega t \; d\omega t$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^0}^{180^0} V_m \sin \omega t . d\left(\omega t\right) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha+30^0}^{180^0}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos 180^0 + \cos\left(\alpha + 30^0\right)\right]$$

Since $\qquad \cos 180^0 = -1,$

We get $\qquad V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + 30^0\right)\right]$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3V_m}{2\pi R}\left(1 + \cos\left(\frac{\pi}{6} + \alpha\right)\right)$$

$$V_{rms} = V_m\sqrt{\frac{3}{4\pi}\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{\pi}{3} + 2\alpha\right)\right)}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R}\sqrt{\frac{3}{4\pi}\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{\pi}{3} + 2\alpha\right)\right)}$$

(at Load)

$$\boldsymbol{I_{T,avg} = I_{s,avg} = \frac{I_{o,avg}}{3}}$$

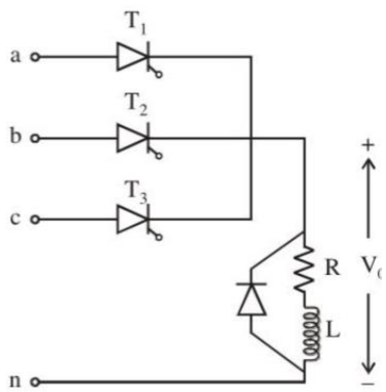
$$\boldsymbol{I_{s,rms} = I_{T,rms} = \frac{I_{o,rms}}{\sqrt{3}}}$$

$$\boldsymbol{P_o = R_{.} * {I_{o,rms}}^2}$$

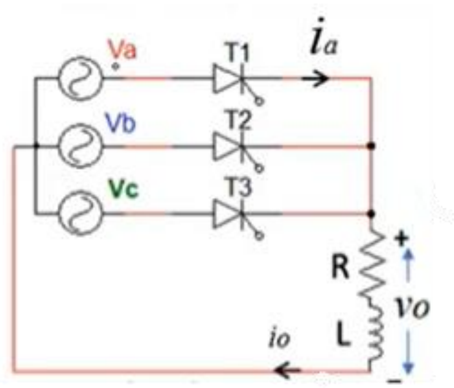
$$\boldsymbol{S = 3\, V_{s,rms} * I_{s,rms}} \qquad (apparent\ power)$$

$$\boldsymbol{PF = \frac{P_o}{S}}$$

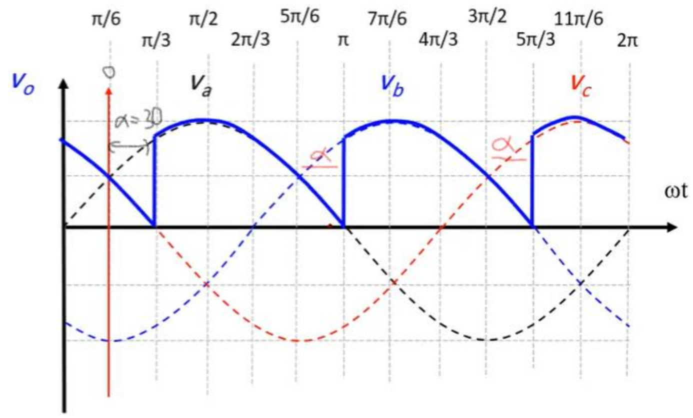
The expression of average or dc output voltage of a 3-phase half wave converter with resistive load can be applied to RL-load with Fwd.



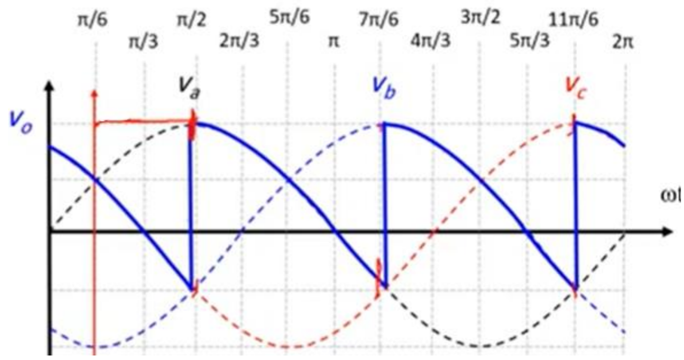
♦ ***Case of Rl- Load***



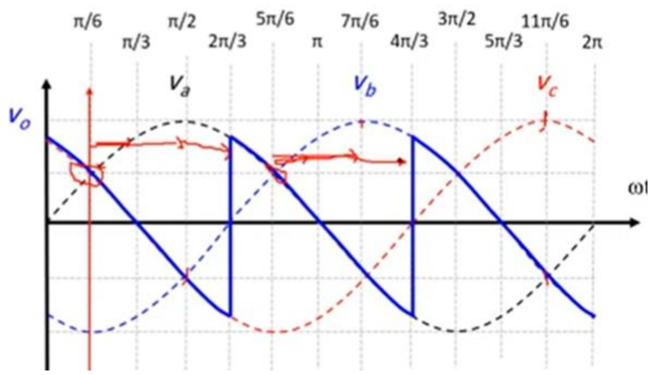
For Different Trigger Angles:



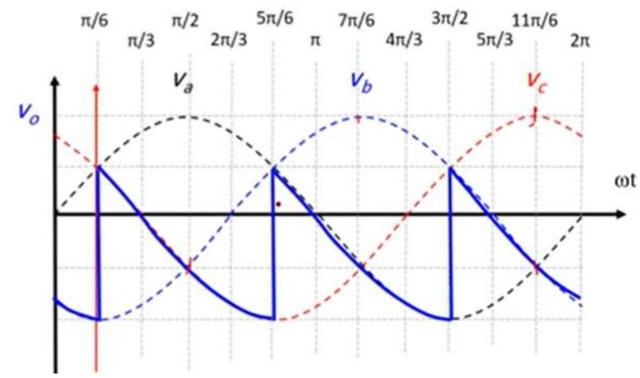
α = 30



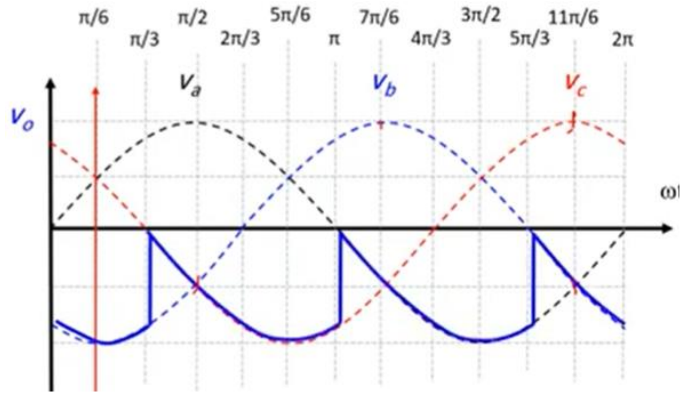
α = 60



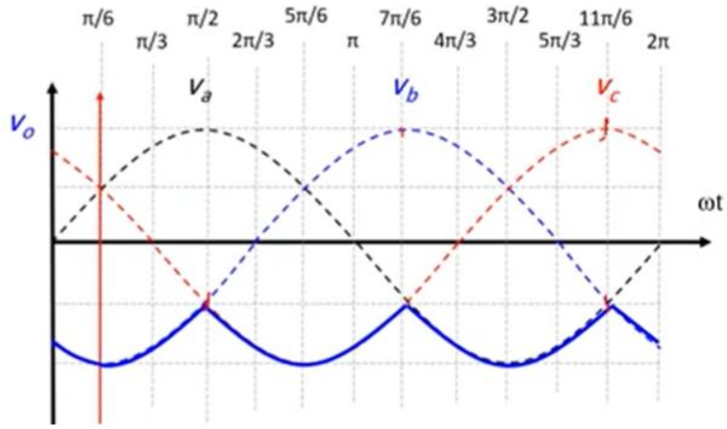
α = 90



α =120



α = 150



α = 180

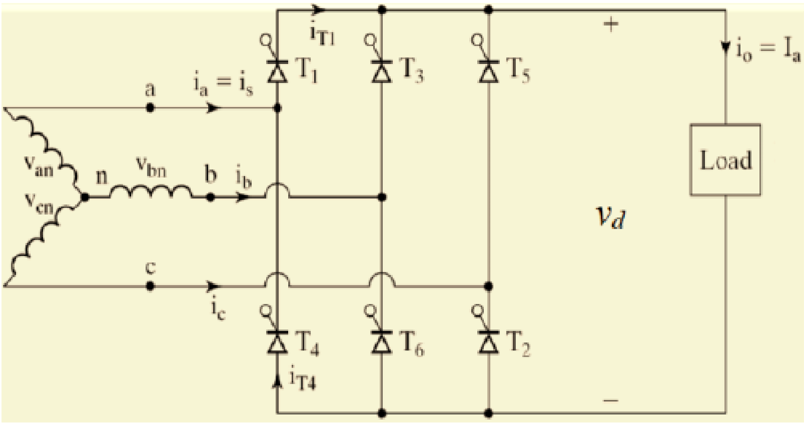
$\alpha < 90$	*Vdc is positive*
$\alpha = 90$	*Vdc is zero*
$\alpha > 90$	*Vdc is negative*

$$V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin \omega t \, d(\omega t) = \frac{3\sqrt{3}V_m}{2\pi} \cos \alpha$$

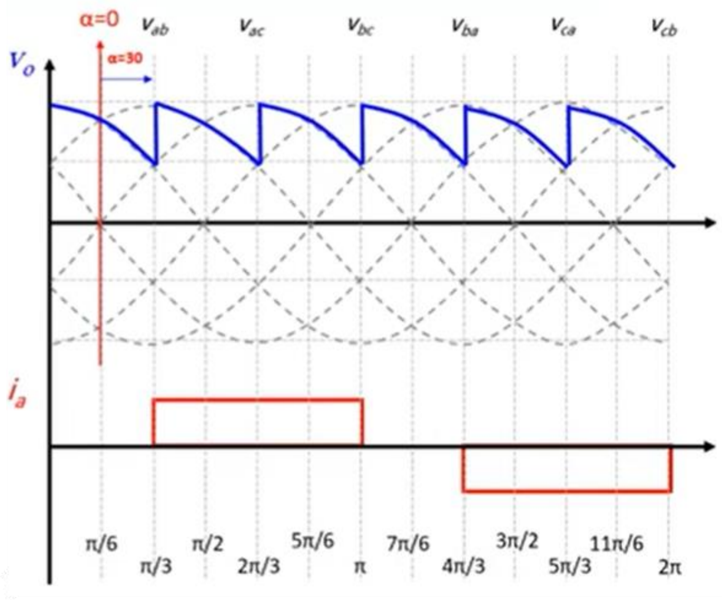
$$\boldsymbol{I_{DC} = I_{avg} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}Vm}{2\pi R} \cos\ \alpha \cong I_{o,rms}}$$

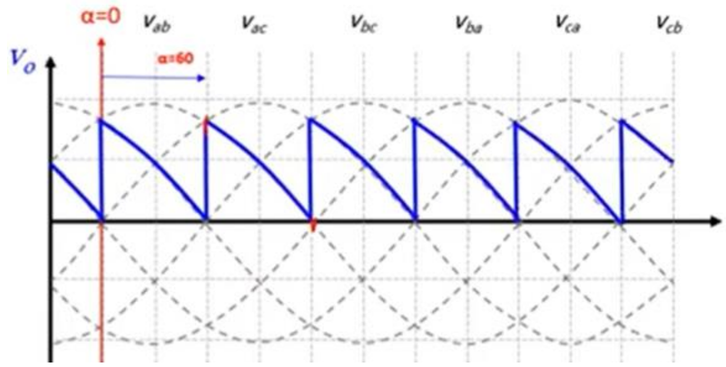
2) Full-Wave Rectifiers

Three-phase converters are extensively used in industrial applications up to the 120-kW level. The output of the three-phase rectifier can be controlled by substituting six SCRs for diodes connected in the form of a full wave bridge configuration. All six thyristors are turned on at an appropriate time by applying suitable gate trigger signals. This circuit is known as a three-phase full wave bridge or as a ***six-pulse rectifier.***



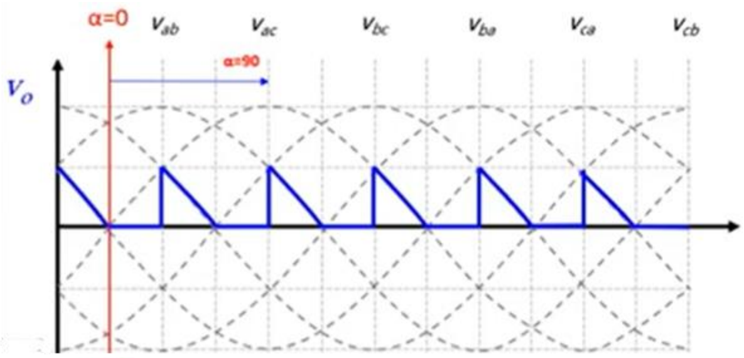
- **The three thyristors (T_1,T_3 andT_5) will not work together at the same time or two of them also will not work together at the same time.**
- **The three thyristors (T_2,T_4 andT_6) will not work together at the same time or two of them also will not work together at the same time.**
- **(T_1 and T_4), (T_3 and T_6) or (T_5 and T_2) will not work together at the same time.**
- **Each thyristor is triggered at an interval of $2\pi/3$.**
- **Each thyristors pair ((T_6&T_1), (T_1&T_2), (T_2&T_3), (T_3&T_4), (T_4&T_5), (T_5&T_6)) is triggered at an interval of $\pi/3$.**
- **The frequency of output ripple voltage is $6f_S$.**





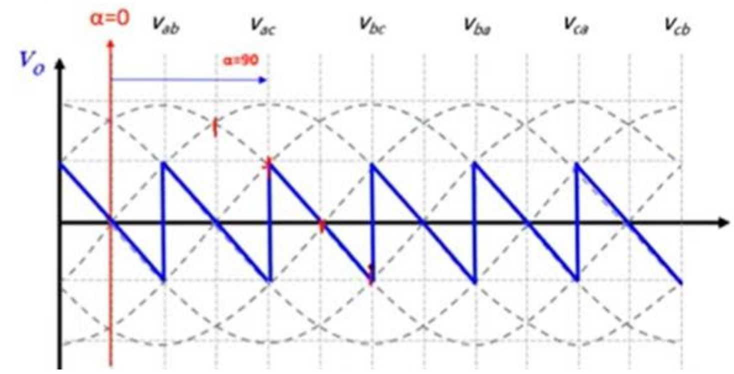
Same for R and RL load

$0 \leq \alpha \leq 60$



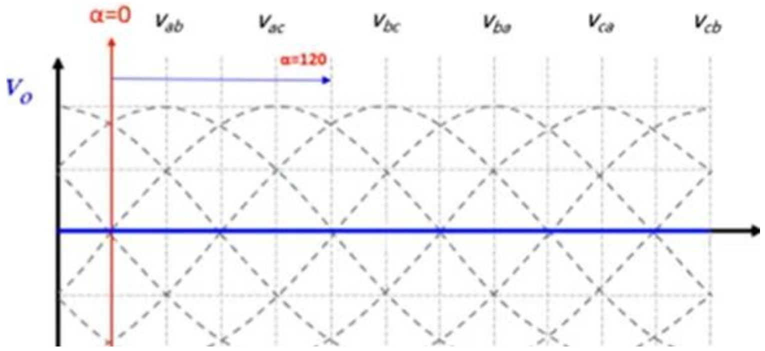
for R- load

α=90



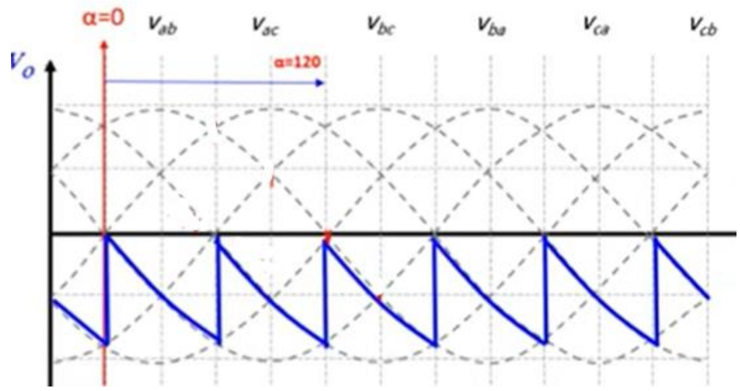
for RL- load

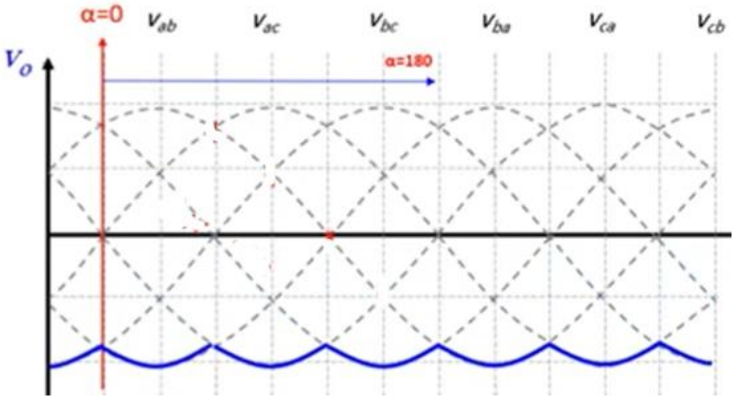
α=90



for R- load

α = 120





for RL- load

α = 120

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha = \frac{3V_{mL}}{\pi}\cos\alpha$$

Where $V_{mL} = \sqrt{3}V_m$ = Max. line-to-line supply voltage

the average output voltage is reduced as the delay angle α increases.

$$Vrms = \sqrt{3}V_m\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}cos2\alpha}$$

$$Irms = Vrms\ /\mathrm{R}$$

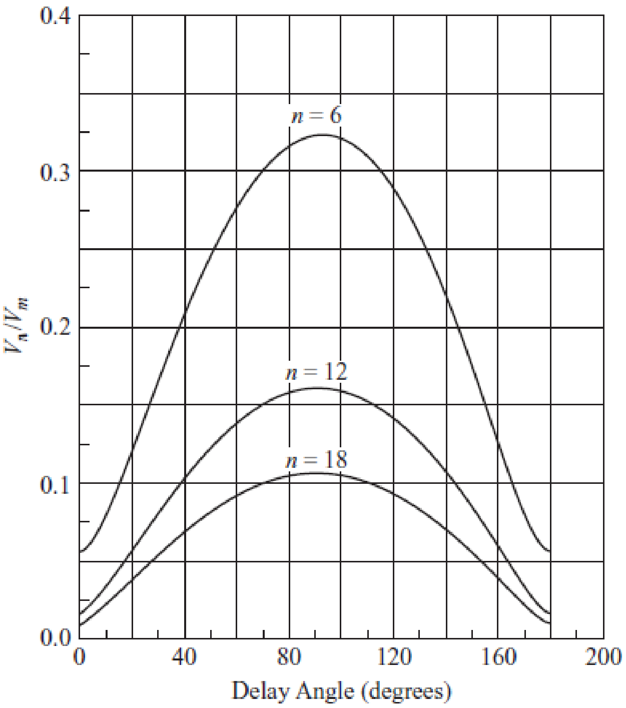
$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{\sqrt{3}V_m}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}cos2\alpha}$$

Special case: resistive load α>60º

$$V_{dc} = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})d\omega t = \frac{3\sqrt{3}V_m}{\pi}\cos(\frac{\pi}{3} + \alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{\pi R}cos(\frac{\pi}{3} + \alpha)$$

Harmonics for the output voltage remain of order 6k, but the amplitudes are functions of α. The figure shows the first three normalized harmonic amplitudes.



Example

The three-phase controlled rectifier of Fig. 4-20*a* is supplied from a 4160-V rms line-to-line 60-Hz source. The load is a 120-Ω resistor. (*a*) Determine the delay angle required to produce an average load current of 25 A. (*b*) Estimate the amplitudes of the voltage harmonics V_6, V_{12}, and V_{18}.

Solution:

$$a)\ V_o = I_o R = (25)(120) = 3000\ V.$$

$$\alpha = \cos^{-1}\left(\frac{\pi V_o}{3V_m}\right) = \cos^{-1}\left(\frac{\pi 3000}{3\sqrt{2}(4160)}\right) = 57.7^\circ$$

$$b)\ From\ Fig. \qquad \frac{V_6}{V_m} \approx 0.28 \ \Rightarrow\ V_6 = 0.28\sqrt{2}(4160) = 1640\ V.$$

$$\frac{V_{12}}{V_m} \approx 0.135 \ \Rightarrow\ V_{12} = 794\ V.$$

$$\frac{V_{18}}{V_m} \approx 0.09 \ \Rightarrow\ V_{18} = 525\ V.$$

Example:

The six-pulse controlled three-phase converter is supplied from a 480-V rms line-to-line 60-Hz three-phase source. The delay angle is 35°, and the load is a series *RL* combination with $R = 50\ \Omega$ and $L = 50$ mH. Determine (*a*) the average current in the load, (*b*) the amplitude of the sixth harmonic current, and (*c*) the rms current in each line from the ac source.

Sol/

$$a)\ V_o = \frac{3V_m}{\pi}\cos\alpha = \frac{3\sqrt{2}(480)}{\pi}\cos 35^\circ = 531\ V.$$

$$I_o = \frac{V_o}{R} = \frac{531}{50} = 10.6\ A.$$

$$b)\ \frac{V_6}{V_m} \approx 0.19 \ \Rightarrow\ V_6 = 0.19\sqrt{2}(480) = 130\ V.$$

$$Z_6 = |R + j6\omega_0 L| = |50 + j6(377)(0.05)| = 124\ \Omega$$

$$I_6 = \frac{V_6}{Z_6} = \frac{130}{124} = 1.05\ A.$$

$$I_{o,rms} \approx \sqrt{i_o^2 + \left(\frac{I_6}{\sqrt{2}}\right)^2} = \sqrt{10.6^2 + \left(\frac{1.05}{\sqrt{2}}\right)^2} = 10.65\ A.$$

$$I_{s,rms} = \left(\sqrt{\frac{2}{3}}\right) I_{o,rms} = \left(\sqrt{\frac{2}{3}}\right) 10.65 = 8.6\ A.$$

The six-pulse controlled three-phase converter of is supplied from a 480-V rms line-to-line 60-Hz three-phase source. The delay angle is 50°, and the load is a series RL combination with $R = 10\ \Omega$ and $L = 10$ mH. Determine (a) the average current in the load, (b) the amplitude of the sixth harmonic current, and (c) the rms current in each line from the ac source.

$$a)\ V_o = \frac{3V_m}{\pi}\cos\alpha = \frac{3\sqrt{2}(480)}{\pi}\cos 50^\circ = 417\ V.$$

$$I_o = \frac{V_o}{R} = \frac{417}{10} = 41.7\ A.$$

$$b)\ \frac{V_6}{V_m} \approx 0.25 \ \Rightarrow\ V_6 = 0.25\sqrt{2}(480) = 170\ V.$$

$$Z_6 = \left|R + j6\omega_0 L\right| = \left|10 + j6(377)(0.01)\right| = 24.7\ \Omega$$

$$I_6 = \frac{V_6}{Z_6} = \frac{170}{24.7} = 6.9\ A.$$

$$I_{o,rms} \approx \sqrt{i_o^2 + \left(\frac{I_6}{\sqrt{2}}\right)^2} = \sqrt{41.7^2 + \left(\frac{6.9}{\sqrt{2}}\right)^2} = 42.3\ A.$$

$$I_{s,rms} = \left(\sqrt{\frac{2}{3}}\right) I_{o,rms} = \left(\sqrt{\frac{2}{3}}\right) 41.7 = 34\ A.$$

The six-pulse controlled three-phase converter of is supplied form a 480-V rms line-to-line 60-Hz three-phase source. The load is a series *RL* combination with $R = 20\ \Omega$. (*a*) Determine the delay angle required for an average load current of 20 A. (*b*) Determine the value of *L* such that the first ac current term ($n = 6$) is less than 2 percent of the average current. (*c*) Verify your results with a PSpice simulation.

$$a)\ V_o = I_o R = (20)(20) = 400\ V.$$

$$a = \cos^{-1}\left(\frac{\pi V_o}{3V_m}\right) = \cos^{-1}\left(\frac{\pi 400}{3\sqrt{2}(480)}\right) = 52^\circ$$

$$b)\qquad \frac{V_6}{V_m} \approx 0.25 \Rightarrow V_6 = 0.25(\sqrt{2})(480) = 170\ V.$$

$$\sqrt{\left(\frac{I_6}{\sqrt{2}}\right)^2 + \left(\frac{I_{12}}{\sqrt{2}}\right)^2 + \left(\frac{I_{18}}{\sqrt{2}}\right)^2} < 0.02 I_o \quad or \quad \sqrt{I_6^2 + I_{12}^2 + I_{18}^2} < 0.02\sqrt{2} I_o$$

$$Z_6 = |R + j6\omega L|$$

$$\frac{V_6}{Z_6} = I_6 < 0.02 I_o = 0.02(20) = 0.4\ A.$$

$$Z_6 = \frac{V_6}{I_6} = \frac{170}{0.4} = 425\ \Omega = |R + j6\omega L| = |20 + j6(377)L|$$

$$6(377)L \approx 425$$

$$L = \frac{425}{6(377)} = 0.188\ H$$

$$L \approx 190\ mH$$