Lecture -8-

Single Phase Controlled Full-wave Rectifier

- A versatile method of controlling the output of a full-wave rectifier is to substitute controlled switches such as thyristors (SCRs) for the diodes. Output is controlled by adjusting the delay angle of each SCR, resulting in an output voltage that is adjustable over a limited range.

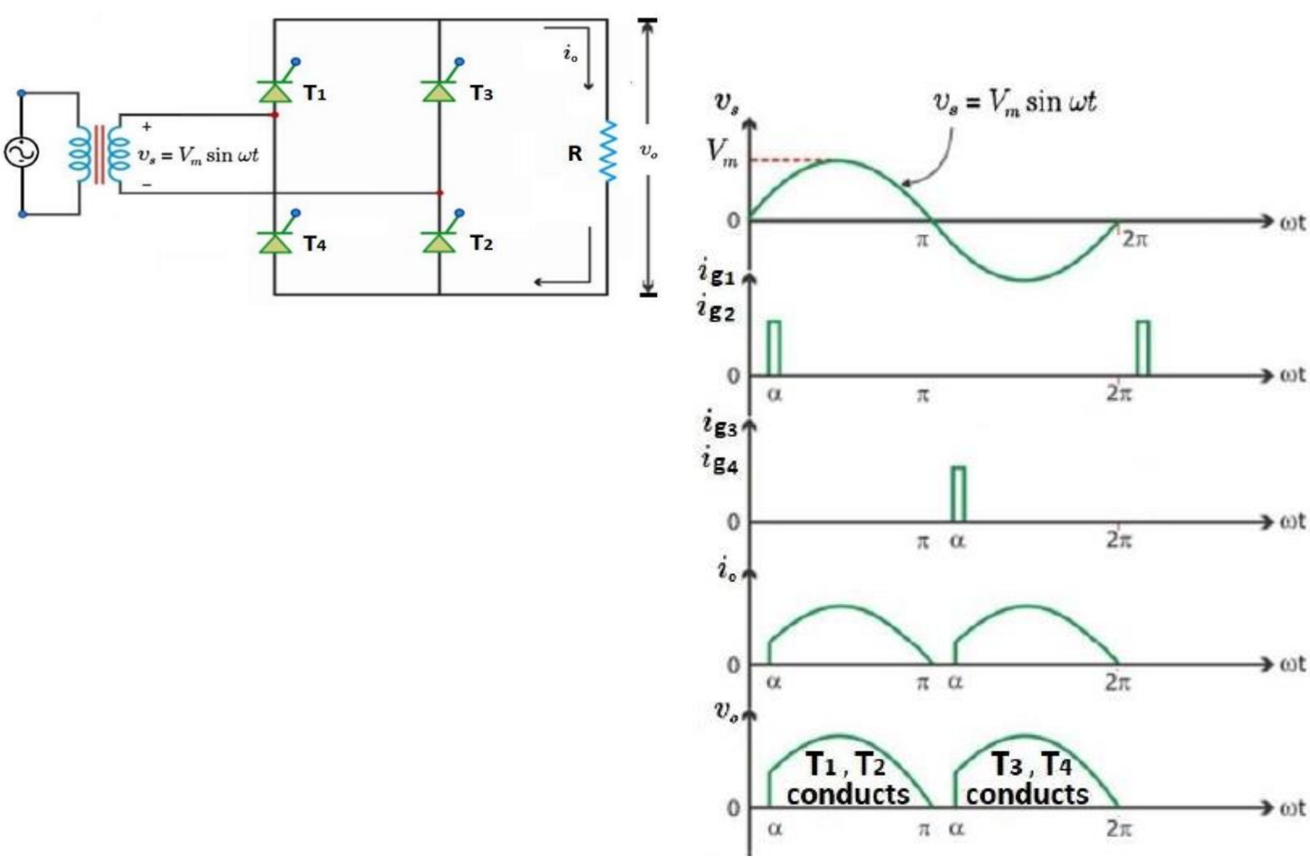
- T_1 and T_2 must be fired at the same time so will become forward-biased during the positive half cycle of the source voltage *vs* to allow the conduction of current. but will not conduct until gate signals are applied. Alternatively, T_3 and T_4 also will become forward-biased during the negative half cycle of the source voltage.

- The delay angle (α) is the interval between the forward biasing of the SCR and the gate signal application. If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes.

  The discussion follows generally applies to both bridge and center-tapped rectifiers

1) Full-wave Controlled Rectifier with Resistive Load (R- Load).

Controlled full-wave rectifiers with resistive load circuit and output waveforms are shown in Fig.below.



The average component of this waveform is determined from

$$V_{dc}=V_o=\frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t)=\frac{V_m}{\pi}(1+\cos\alpha)$$

$$I_{dc}=\frac{V_{dc}}{R}=\frac{V_m}{\pi R}(1+cos\alpha)$$

$$V_{RMS(Load)}=\sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}[V_m\sin(\omega t)]^2\,d\omega t}=V_m\sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\sin(2\alpha)}{4\pi}}$$

The rms current in the source is the same as the rms current in the load.

$$I_{rms}=\sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)}$$

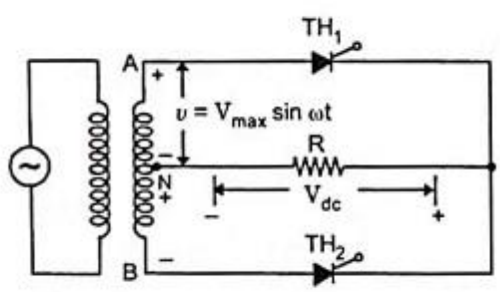
$$=\frac{V_m}{R}\sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\sin(2\alpha)}{4\pi}}$$

$$I_{RMS(Load)}=\frac{V_{RMS(Load)}}{R}$$

The rms current in the source is the same as the rms current in the load.

$(P=I_{rms}^2 * R)$, $P_o=\frac{{V_{RMS}}^2}{R}$ is used to determine the power in a resistive load.

- For the center-tapped transformer rectifier, T_1 is forward-biased when vs is positive, and T_2 is forward-biased when vs is negative, but each will not conduct until it receives a gate signal.



Example:

The full-wave controlled bridge rectifier has an ac input of 120 Vrms at 60 Hz and a 20Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source volt-amperes.

Solution:

The average output voltage is determined from

$$V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40°\right) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)}$$

$$= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

$\alpha = 40° = 0.698 \ rad$

$$I_{orms} = \frac{120\sqrt{2}}{20}\sqrt{\left[\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin 2 * 0.698}{4\pi}\right]} = 5.8 \ A$$

$$P = I^2_{\text{rms}} R = (5.80)^2 (20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

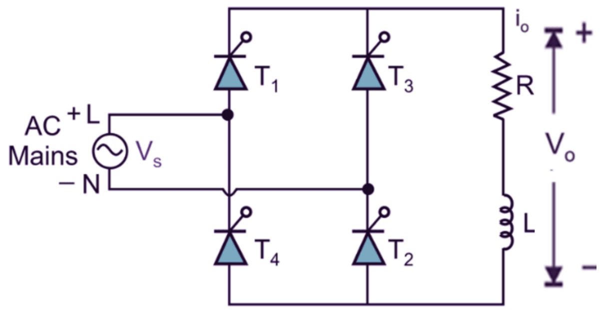
$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

2) *Full -wave Controlled Rectifier with (RL- Load).*

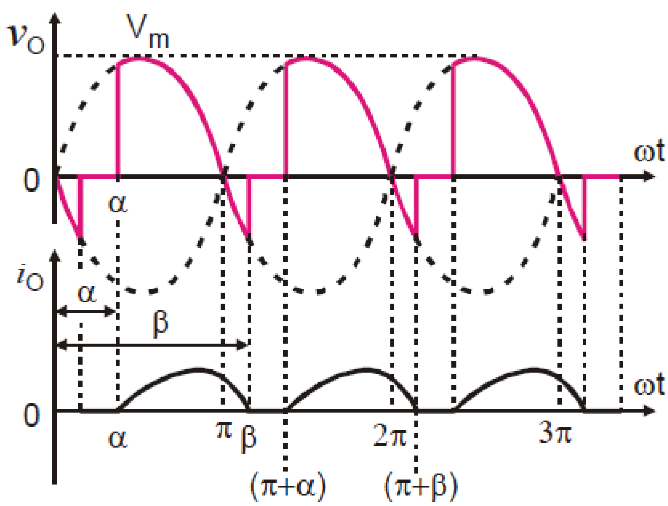
A single-phase bridge rectifier feeding an RL load is shown in Fig below. The output waveforms for this circuit depend on the values of the inductance L and the firing angle α . So, it can be continuous or discontinuous, and a separate analysis is required for each.



(i) Case of RL load with small L / R ratio:

In this case the current will be **Discontinuous** as shown below, mode of operation is

- At ωt=0 with zero load current, SCRs T_1 and T_2 in the bridge rectifier will be forward-biased and T_3 and T_4 will be reverse-biased as the source voltage becomes positive.
- At ωt=α, turning T_1 and T_2 by applied the gate signal. With T_1 and T_2 on, the load voltage is equal to the source voltage.
- At ωt= β, ($\alpha + \pi > \beta$) the current remains at zero until $\omega t = \pi + \alpha$ when gate signals are applied to T_3 and T_4 which are then forward-biased and begin to conduct.



Analysis of the controlled full-wave rectifier operating in the discontinuous current mode is identical to that of the controlled half-wave rectifier except that the period for the output is π rather than 2π rad.

So, the current function is

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

The average (dc) output voltage is

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\beta} V_m sin\omega t\, dt\omega t = \frac{V_m}{\pi}(cos\alpha - cos\beta)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}(cos\alpha - cos\beta)$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta}(V_m sin\omega t)^2\, d\omega t} = \sqrt{\frac{V_m{}^2}{2\pi}(\beta - \alpha - \frac{1}{2}\boldsymbol{sin2\beta} + \frac{1}{2}\boldsymbol{sin2}\alpha)}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{V_m{}^2}{2\pi}(\beta - \alpha - \frac{1}{2}\boldsymbol{sin2\beta} + \frac{1}{2}\boldsymbol{sin2}\alpha)}$$

Example:
A controlled full-wave bridge rectifier has a source of 120 Vrms at 60Hz, R=10Ω, L=20mH, and α=60°. Determine (a) an expression for load current, (b) the average load current, and (c) the power absorbed by the load. β=3.78 rad (216°).
Solution:
From the parameters given,

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60^\circ = 1.047 \text{ rad}$$

(a) Substituting into Eq.

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

$$i_o(wt) = \frac{120\sqrt{2}}{12.5}[\ \sin(wt - 0.646) - \sin(1.047 - 0.646)\ e^{-(wt-1.047)/0.754}\ ]$$

$$i_o(wt) = 13.57\,[\ \sin(wt - 0.646) - 0.39\, e^{-\frac{wt-1.047}{0.754}}\ ]$$

$$i_o(wt) = 13.57\ \sin(wt - 0.646) - 5.292\, e^{-\frac{wt-1.047}{0.754}} \qquad A \qquad \text{for } \alpha \le \text{wt} \le \beta$$

- For $\beta = 3.78$ rad, (216°).
  Since $\pi + \alpha = 3.14 + 1.047 = 4.18 > \beta$
  Or $\pi + \alpha = 180 + 60 = 240 > \beta$
  the current is discontinuous, and the above expression for current is valid.

(b) Average load current is determined from

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}(cos\alpha - cos\beta)$$

$$I_o = \frac{120\sqrt{2}}{\pi(10)}\ (\cos 1.047 - \cos 3.78) = 7.04\ A$$

(c) the power absorbed by the load occurs in the resistor and is computed from

$$P = (I_{rms})^2 * R$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{{V_m}^2}{2\pi}\left(\beta - \alpha - \frac{1}{2}\boldsymbol{sin2}\beta + \frac{1}{2}\boldsymbol{sin2}\alpha\right)}$$

$$I_{orms} = \frac{1}{12.5}\sqrt{\frac{\left(120\sqrt{2}\right)^2}{2\pi}[\ 3.78 - 1.047 - \frac{1}{2}\sin(2*3.78) + \frac{1}{2}\sin(2*1.047)\ ]}$$

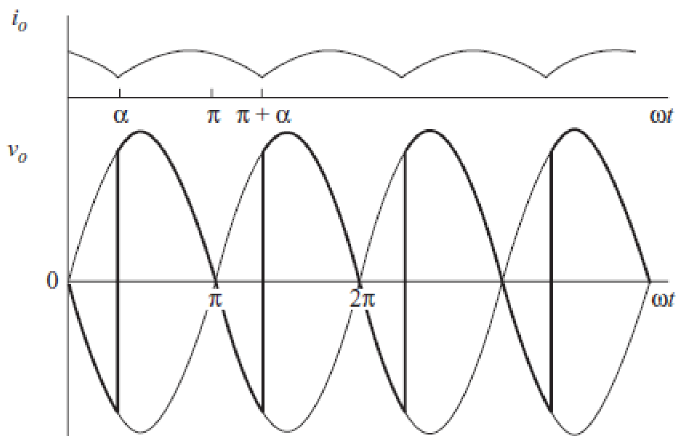
$$I_{orms} = \frac{1}{12.5}\sqrt{4583.66[2.73 - 0.47 + 0.43]} = \frac{1}{12.5}[111.86] = 8.94 \quad A$$

$$P = I_{rms} * R = \ (8.94)^2 * 10 = \ 799.23\ \ w$$

(ii) Case of RL load with large L / R ratio:

In this case the current will be **Continuous** as shown below

If the inductance value is large, the output current will be continuous. The SCRs T_1 and T_2 will continue to conduct until the other pair of SCRs T_3 and T_4 is fired. Since the polarity of the input voltage is already reversed, the firing of reverse biases T_1 and T_2, and turns them off at $\omega t=\pi+\alpha$. The load current shifts from the pair T_1 and T_2 to the pair T_3 and T_4.



$\boldsymbol{\alpha \leq tan^{-1}\left(\frac{wL}{R}\right)}$ → **Continuous current**

The dc (average) value is

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha}$$

A method for determining the ac terms of output voltage and current for the **continuous current** case is to use the Fourier series. The Fourier series of the voltage waveform for continuous-current case expressed in general form as

$$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

The amplitudes of the ac terms are calculated from

$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

The Fourier series for current is determined by superposition as was done for the uncontrolled rectifier.

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

$$I_o = \frac{V_o}{R} \qquad \text{and} \qquad I_n = \frac{V_n}{Z_n} \qquad Z_n = |R + jn\omega_0 L| = \sqrt{R^2 + (n\omega_0 L)^2}$$

As the harmonic number increases, the impedance for the inductance increases. Therefore, it may be necessary to solve for only a few terms of the series to be able to calculate the rms current. If the inductor is large, the ac terms will become small, and the current is essentially dc.

Figure below shows the relationship between normalized harmonic content of the output voltage and delay angle.

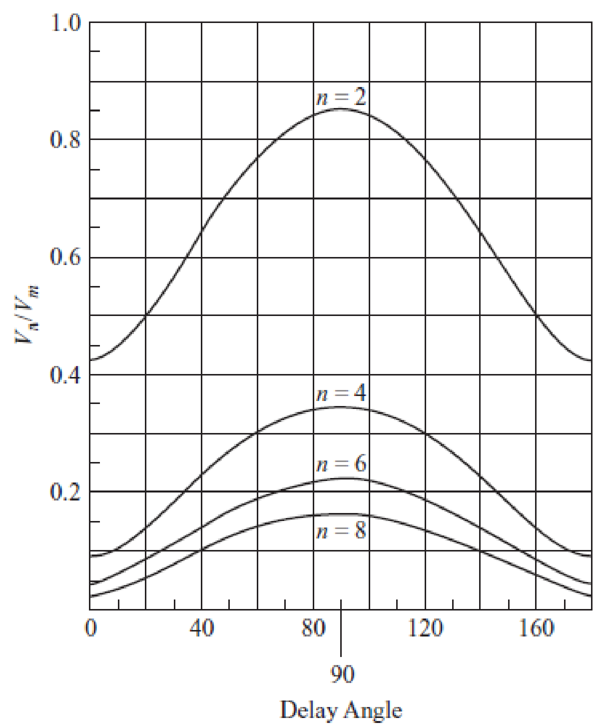


Figure. Output harmonic voltages as a function of delay angle for a single-phase controlled rectifier.

Example:

A controlled full-wave bridge rectifier has a source of 120Vrms at 60Hz, R=10Ω, L=100mH, and α=60o. Determine (a) Verify that the load current is continuous. (b) the dc (average) current, and (c) the power absorbed by the load.

Solution//

(a) to verify that the current is continuous.

$$\boldsymbol{\alpha \le tan^{-1}\left(\frac{wL}{R}\right)} \rightarrow \textbf{Continuous current}$$

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75°$$

$$\alpha = 60° < 75° \qquad \therefore \text{ continuous current}$$

(b) the dc (average) current

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}(120)}{\pi}\cos(60°) = 54.0 \text{ V}$$

$$I_o = \frac{54}{10} \quad 5.4 \quad A$$

(c) the power absorbed by the load.

The amplitudes of the ac terms are calculated from

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

$$a_2 = \frac{2 * 120\sqrt{2}}{\pi}\left[\frac{\cos(2+1)*60°}{2+1} - \frac{\cos(2-1)*60°}{2-1}\right] = -90.0$$

$$b_2 = \frac{2 * 120\sqrt{2}}{\pi}\left[\frac{\sin(2+1)*60°}{2+1} - \frac{\sin(2-1)*60°}{2-1}\right] = -93.5$$

$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$V_2 = \sqrt{(-90)^2 + (-93.5)^2} = 129.8 \quad v$$

$$I_n = \frac{V_n}{Z_n}$$

$$Z_n = |R + jn\omega_0 L| = \sqrt{R^2 + (n\omega_0 L)^2}$$

$$Z_2 = \sqrt{(10)^2 + (2 * 377 * 0.1)^2} = 76.0$$

$$I_2 = \frac{V_2}{Z_2} = \frac{129.8}{76} = 1.7 \quad A$$

n	a_n	b_n	V_n	Z_n	I_n
0 (dc)	—	—	54.0	10	5.40
2	−90	−93.5	129.8	76.0	1.71
4	46.8	−18.7	50.4	151.1	0.33
6	−3.19	32.0	32.2	226.4	0.14

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

$$I_{\text{rms}} = \sqrt{(5.40)^2 + \left(\frac{1.71}{\sqrt{2}}\right)^2 + \left(\frac{0.33}{\sqrt{2}}\right)^2 + \left(\frac{0.14}{\sqrt{2}}\right)^2 + \cdots} \approx 5.54 \text{ A}$$

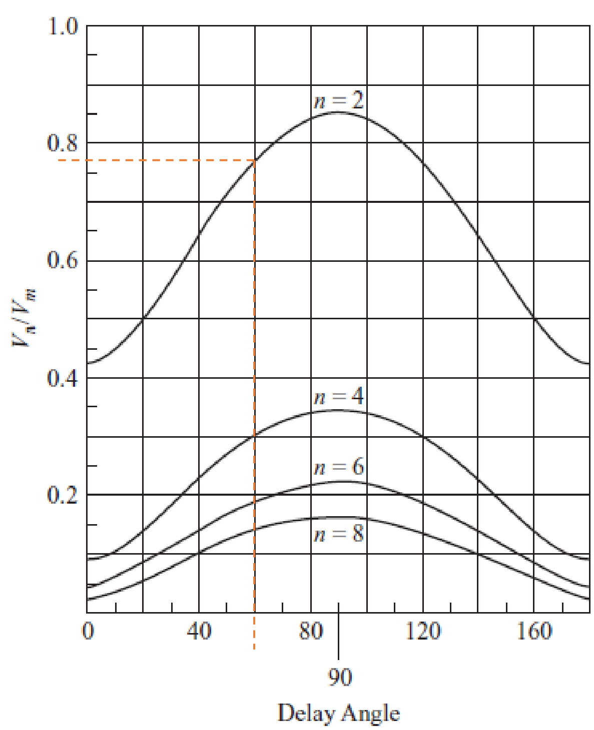
Power is computed from $I_{\text{rms}}^2 R$.

$$P = (5.54)^2(10) = 307 \text{ W}$$

****also can solve by using fig.**

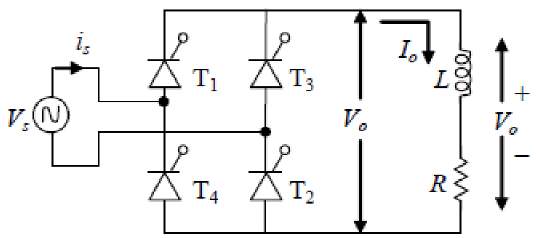
$$\frac{V_n}{V_m} = 0.76$$

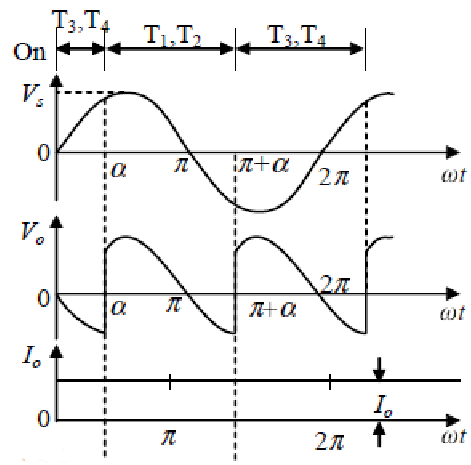
$$V_n = 0.76 * 120\sqrt{2} = 128.9$$



(iii) Case of RL load with Highly Inductive Load, L>>R

If the value of inductance is very large then the output current will be constant and ripple free. The voltage and current waveforms for highly inductive load are depicted shown in Fig. below. The SCRs T_1 and T_2 conduct from α to $\pi+\alpha$ and the SCRs T_3 and T_4 from $\pi+\alpha$ to $2\pi+\alpha$ and so on, each pair conducting for period π.





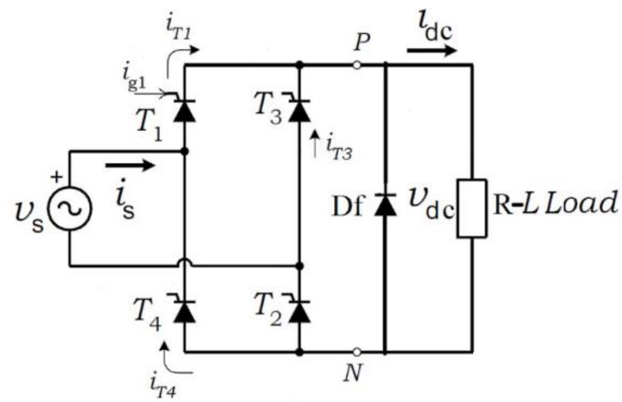
$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha}$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} (V_m sin\omega t)^2\, d\omega t} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = I_{dc} = I_o$$

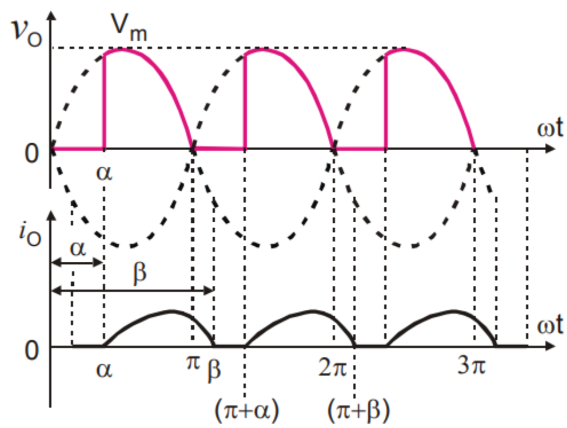
3) *Full -wave Controlled Rectifier with FWD.*

If a diode is connected across the load (R and large L), the circuit can operate as a rectifier because the diode prevents negative values of Vo from appearing across the load. Figure below shows the bridge rectifier circuit with the addition of a freewheeling diode (Df). The diode provides an extra path for the flow of load current. Three paths are now possible: T_1 and T_2 , T_3 and T_4, and the path through diode Df.



- **For Discontinuous Load Current Operation with FWD**

  The load current is discontinuous for low values of load inductance and for large values of trigger angles.



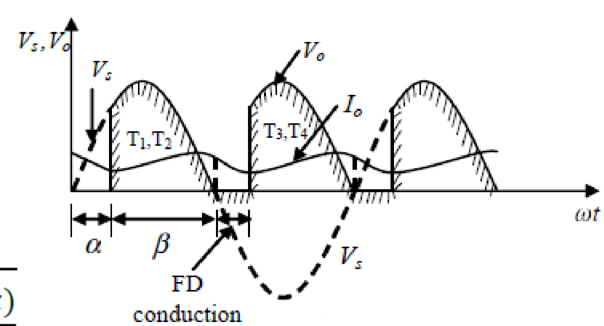
- **For Continuous Load Current Operation with FWD**

  The load current is continuous for large load inductance and for low trigger angles.

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \, sin\omega t \; d\omega t = \frac{V_m}{\pi}(1 + cos\alpha)$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}(1 + cos\alpha)$$

$$V_{RMS(Load)} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2 \, d\omega t} = V_m\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$



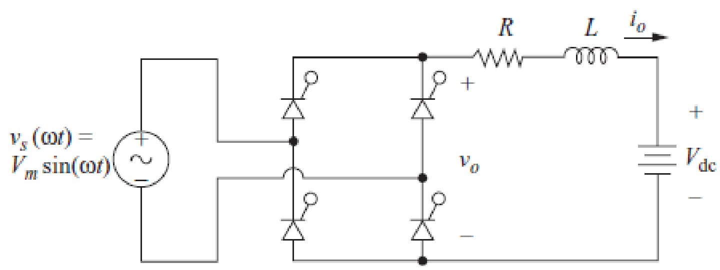
4) *Full -wave Controlled Rectifier with (RL-Source Load).*

The controlled rectifier with a load that is a series resistance, inductance, and dc voltage (Fig. below) is analyzed much like the uncontrolled rectifier. For the controlled rectifier, the SCRs may be turned on at any time that they are forward-biased, which is at an angle

$$\alpha \geq \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

The average bridge output voltage is

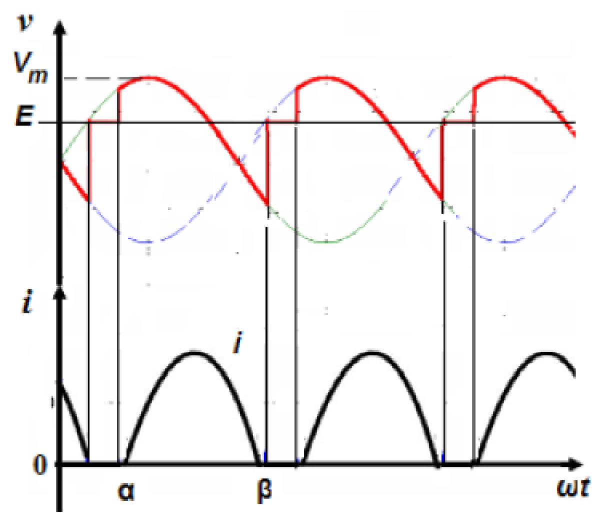
$$V_o = \frac{2V_m}{\pi}\cos\alpha$$



$$I_o = \frac{V_o - V_{dc}}{R}$$

$$P_{dc} = I_o V_{dc}$$

Power absorbed by the resistor in the load is $I_{rms}^2 R$. If the inductance is large and the load current has little ripple, power absorbed by the resistor is approximately $I_o^2 R$.



Example

Controlled Rectifier with *RL*-Source Load has an ac source of 240 V rms at 60 Hz, *V*dc =100 V, *R* =5 Ω, and an inductor large enough to cause continuous current. (*a*) Determine the delay angle α such that the power absorbed by the dc source is 1000 W. (*b*) Determine the value of inductance that will limit the peak-to-peak load current variation to 2 A.

Solution

(a) For the power in the 100-V dc source to be 1000 W

$$I = \frac{P}{V} = \frac{1000}{100} = 10\ A$$

$$I_o = \frac{V_o - V_{dc}}{R}$$

$$V_o = V_{dc} + I_o R = 100 + (10)(5) = 150\ \text{V}$$

The delay angle which will produce a 150 V dc output from the rectifier is determined from

$$V_o = \frac{2V_m}{\pi}\cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{V_o\pi}{2V_m}\right) = \cos^{-1}\left[\frac{(150)(\pi)}{2\sqrt{2}(240)}\right] = 46°$$

(*b*) Determine the value of inductance that will limit the peak-to-peak load current variation to 2 A.

Variation in load current is due to the ac terms in the Fourier series. The load current amplitude for each of the ac terms is

$$I_n = \frac{V_n}{Z_n}$$

where *Vn* is described

$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

Or from the graph. The impedance for the ac terms is

$$Z_n = |R + jn\omega_0 L| = \sqrt{R^2 + (n\omega_0 L)^2}$$

Since the decreasing amplitude of the voltage terms and the increasing magnitude of the impedance both contribute to decreasing ac currents as n increases, the peak-to-peak current variation will be estimated from the first ac term.

For $n = 2$,

V_n/V_m is estimated from Fig. as 0.68 for $\alpha = 46°$, making $V_2 = 0.68V_m = 0.68\,(240\sqrt{2}) = 230$ V. The peak-to-peak variation of 2 A corresponds to a 1-A zero-to-peak amplitude. The required load impedance for $n = 2$ is then

$$Z_2 = \frac{V_2}{I_2} = \frac{230\text{ V}}{1\text{ A}} = 230\ \Omega$$

The 5-Ω resistor is insignificant compared to the total 230-Ω required impedance, so $Z_n \approx n\omega L$. Solving for L,

$$L \approx \frac{Z_2}{2\omega} = \frac{230}{2(377)} = 0.31\text{ H}$$