Lecture -7-

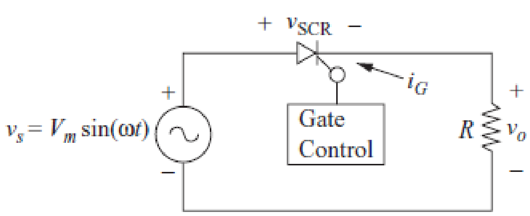
Single Phase Controlled Half Wave Rectifier

1) *Half Wave Controlled Rectifier with Resistive Load (R- Load).*

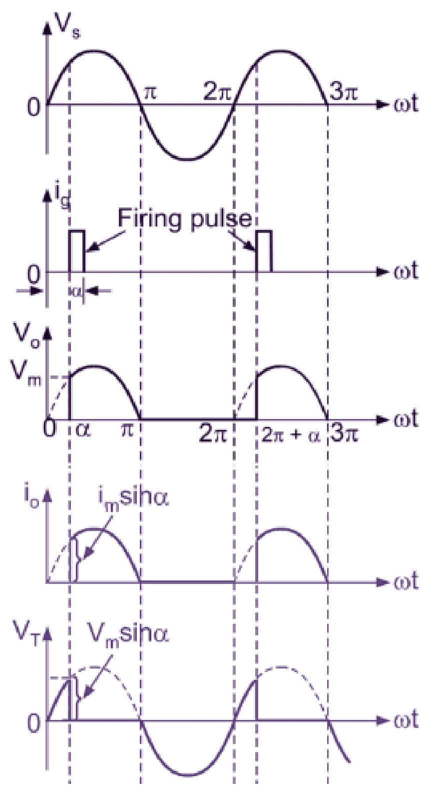
The half-wave rectifiers analyzed previously in this Lecture are classified as uncontrolled rectifiers. Once the source and load parameters are established, the DC level of the output and the power transferred to the load are fixed quantities. Away to control the output of a half-wave rectifier is to use an SCR1 instead of a diode. Figures below show a basic controlled half-wave rectifier with a resistive load and output waveforms. Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($V_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.

Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control. Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.



(a)



(b)

- If a gate signal is applied to the SCR at $\omega t = \alpha$, where α is the delay (firing or triggering) angle. The average (dc) voltage across the load resistor and the average (dc) current are:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

$$I_o = \frac{V_o}{R} = \frac{V_m}{2\pi R}(1 + \cos\alpha)$$

- The rms voltage across the resistor and the rms current are computed from

$$V_{orms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} v_s^2(\omega t)\, d\omega t} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} (V_m sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{2\pi}\int_{\alpha}^{\pi} \frac{1}{2}(1 - cos2\omega t)\, d\omega t}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

since : $(\sin\omega t)^2 = \frac{1}{2}(1 - cos2\omega t)$

$$I_{orms} = \frac{V_{orms}}{R}$$

- The power absorbed by the resistor (a.c. power) is

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R}$$

Example: The single-phase half wave rectifier has a purely resistive load of R and the delay angle is $\alpha = \pi/2$, determine: Vo, Io, $Vrms$, $Irms$.

$$V_o = \frac{V_m}{2\pi}(1 + \cos\alpha) = \frac{V_m}{2\pi}\left(1 + \cos\frac{\pi}{2}\right) = 0.16\, Vm \qquad v$$

$$I_o = \frac{V_o}{R} = 0.16\frac{Vm}{R} \qquad A$$

$$Vo,rms = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} = 0.35\, Vm$$

$$I_{orms} = \frac{V_{orms}}{R} = 0.35\, Vm/R$$

Example:

Design a circuit to produce an average voltage of 40 V across a 100 Ω load resistor from a 120-*V rms* 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

$$V_o = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

$$\frac{2\pi}{V_m} V_o = (1 + \cos\alpha)$$

$$(\frac{2\pi}{V_m}) V_o - 1 = \cos\alpha$$

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right]$$

$$= \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

$$Vrms = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

Load power is

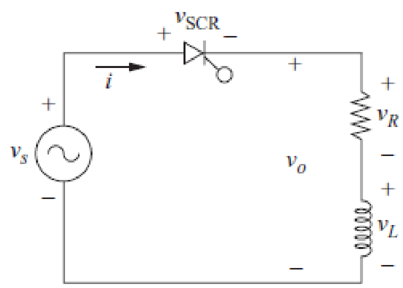
$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

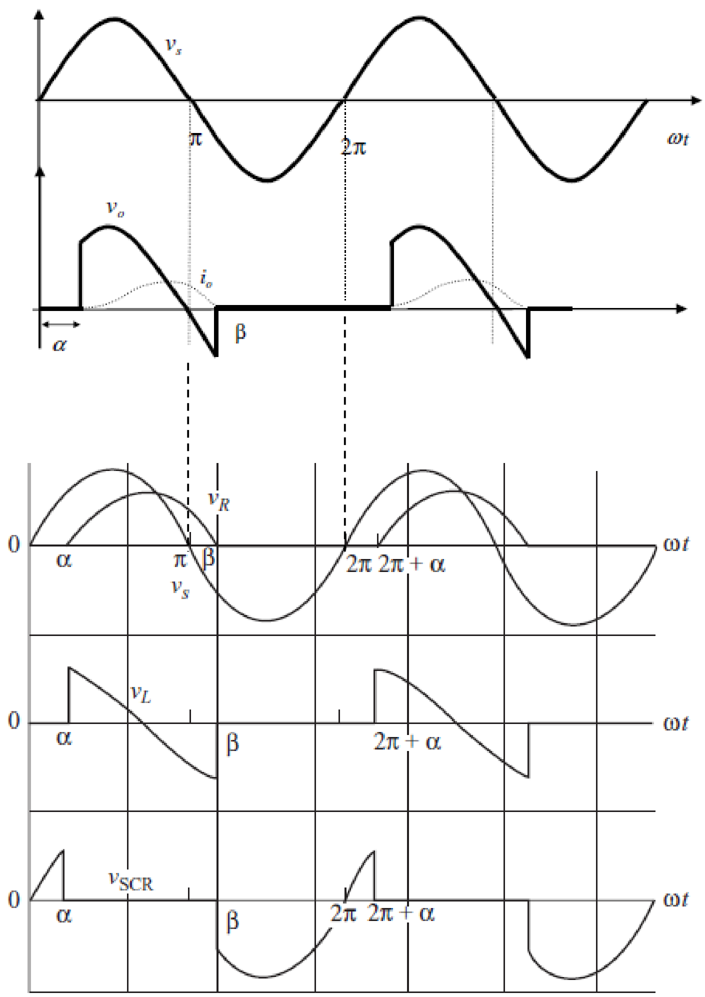
The power factor of the circuit is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

2) *Half Wave Controlled Rectifier with (RL- Load).*

A controlled half-wave rectifier with an *RL* load is shown in Fig.





The analysis of this circuit is similar to that of the uncontrolled rectifier. The current is the sum of the forced and natural responses,

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant A is determined from the initial condition $i(\alpha) = 0$:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right] \cdot e^{\alpha/\omega\tau}$$

Substituting for A and simplifying,

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}}$$

where $\quad Z = \sqrt{R^2 + (\omega L)^2} \quad$ and $\quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \tau = \frac{L}{R}$

The average value of the load voltage can be calculated as follows:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

$$I_o = \frac{V_o}{R} = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta)$$

The *rms* value of the load voltage *Vorms* can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} v_s^2(\omega t)d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta}(V_m sin\omega t)^2\, d\omega t} = \sqrt{\frac{(V_m)^2}{2\pi}\int_{\alpha}^{\beta}\frac{1}{2}(1 - cos2\omega t)d\omega t}$$

$$= \frac{V_m}{2}\sqrt{\frac{1}{\pi}[(\beta - \alpha) - \frac{1}{2}(sin2\beta - sin2\alpha)]}$$

Therefore, the *rms* value of the load current *Iorms* is

$$I_{orms} = \frac{V_{orms}}{Z}$$

Example:

For the circuit of controlled half-wave rectifier with RL Load, the source is 120Vrms at 60 Hz, R=20Ω, L=0.04H, and the delay angle is 45°. Determine (a) an expression for i(ωt), (b) the average current (c) the power absorbed by the load, and (d) the power factor. β=3.79 rad, 217°.

■ **Solution**

(*a*) From the parameters given,

$$V_m = 120\sqrt{2} = 169.7\ \text{V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [20^2 + (377*0.04)^2]^{0.5} = 25.0\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.04)/20) = 0.646\ \text{rad}$$
$$\omega\tau = \omega L/R = 377*0.04/20 = 0.754$$
$$\alpha = 45° = 0.785\ \text{rad}$$

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for}\,\alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}}$$

$$i(\omega t) = 6.78\sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\quad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

(*b*) Average current is determined from

$$I_o = \frac{V_o}{R} = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta)$$

$$I_o = \frac{120\sqrt{2}}{2\pi(20)}(\cos 0.785 - \cos 3.79) = 2.03\quad A$$

(*c*) The power absorbed by the load is computed from

$$P = (I_{rms})^2 * R$$

$$V_{orms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}[(\beta - \alpha) - \frac{1}{2}(sin2\beta - sin2\alpha)]}$$

$$V_{orms} = \frac{120\sqrt{2}}{2}\sqrt{\frac{1}{\pi}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2*3.79 - \sin 2*0.785)\right]}$$

$$V_{orms} = 84.85\sqrt{\frac{1}{\pi}\left[3 - \frac{1}{2}(0.96 - 0.99)\right]}$$

$$V_{orms} = 84.85\,(0.98) = 83.15\quad v$$

$$I_{orms} = \frac{V_{orms}}{Z} = \frac{83.15}{25} = 3.3 \quad A$$

$$P = (I_{rms})^2 * R = (3.3\,)^2 * 20 = 217.8 \quad w$$

(d) the power factor

$$PF = \frac{P}{S} = \frac{P}{V_{s,rms} * I_{rms}} = \frac{217.8}{120 * 3.3} = 0.55$$

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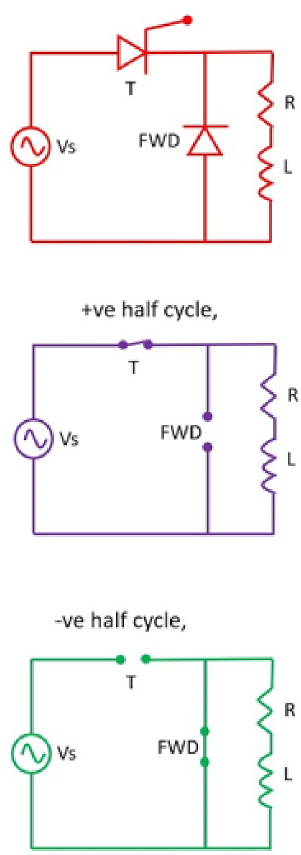
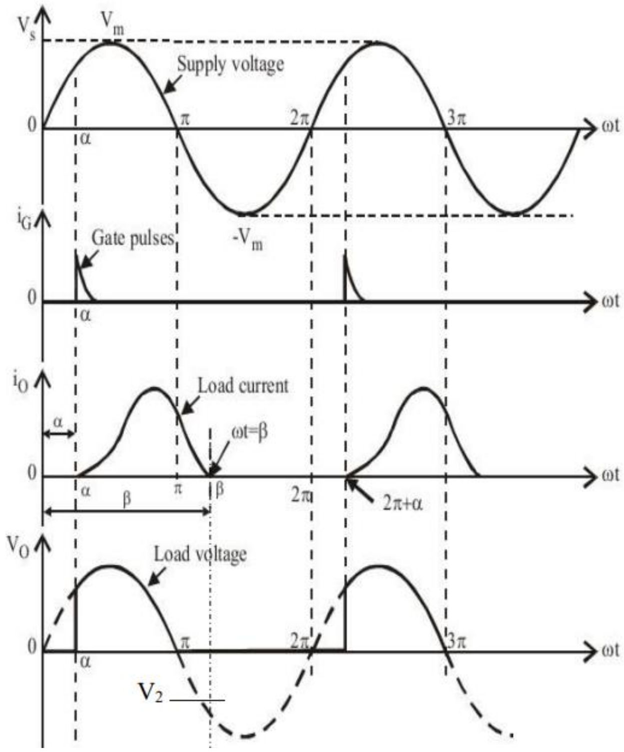
3) *Half-wave Controlled Rectifier with Freewheeling Diode(FWD).*

To overcome the impact of the inductive load at the output voltage (the average output voltage reduces), a freewheeling diode is used in parallel to the load that freewheels the energy stored in the inductor during the negative cycle of the input voltage as shown in Fig.below.



The modes operation of the rectifier and waveforms shown below:



State	SCR	FWD	v_o	v_{T1}	i_{T1}	I_{FWD}
For $\omega t = 0 \rightarrow \alpha$	OFF	OFF	0	V_1	0	0
For $\omega t = \alpha \rightarrow \pi$	ON	OFF	v_S	0	$i_O(t)$	0
For $\omega t = \pi \rightarrow \beta$	OFF	ON	0	V_2	0	$i_O(t)$
For $\omega t = \beta \rightarrow 2\pi$	OFF	OFF	0	V_m	0	0

From a waveform figure, it can be seen that during the positive half cycle after the application of the gate signal (α), the output voltage across the load is the same as at the source but it is started at the gate signal, whereas for a negative half cycle the thyristor blocks all the input voltage across itself.

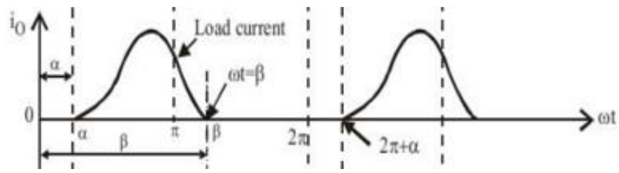
The output load current is the combination of the thyristor current (I_{scr}) when it is triggered in the positive half cycle and the freewheeling diode current (I_{fwd}) when the thyristor is reverse-biased and the energy stored in the inductor dissipates through the FWD.

I_{scr} is the current flowing through the thyristor when it is in the forward conduction mode. Its behavior is increasing as the inductive load stores energy gradually in its magnetic field.

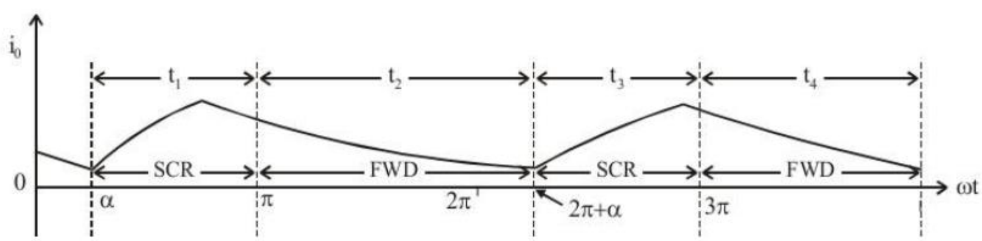
I_{fwd} is the freewheeling diode current during the reverse biasing of the thyristor and its behavior is decreasing as the energy stored in the inductor dissipates gradually through the FWD.

NOTE: The following points are to be noted

- If L value is not very large, the energy stored in its able to maintain the load current only up to $\omega t=\beta$, where $\pi < \beta < 2\pi$, where before the next gate pulse, the load current tends to become discontinuous.



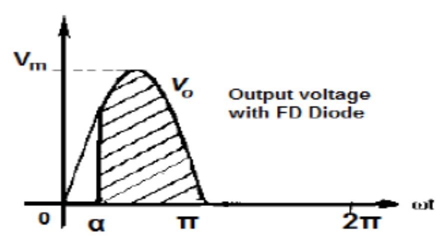
- If the value of L is very large, i_o does not decrease to zero during the freewheeling time interval and the ripple in i_o waveform decreases. So, the current will continuously flow through the load even when the SCR is OFF until the inductor is fully discharged as fig below. Thus the **Freewheeling Diode improves the waveforms of the load current.**



The avg output voltage after using the freewheeling diode determined as:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} v_s(\omega t)d\omega t = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m sin\omega t\, d\omega t$$

$$= \frac{V_m}{2\pi}(-cos\pi + cos\alpha) = \frac{V_m}{2\pi}(1 + cos\alpha)$$



$$I_o = \frac{V_o}{R}$$

$$Vrms = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$I_{orms} = \frac{V_{orms}}{R}$$