Lecture -6-

Single Phase Full-wave Rectifier with RL-Source load

Another general industrial load may be modeled as a series resistance, inductance, and a dc voltage source, as shown in Fig. A dc motor drive circuit and a battery charger are applications for this model.

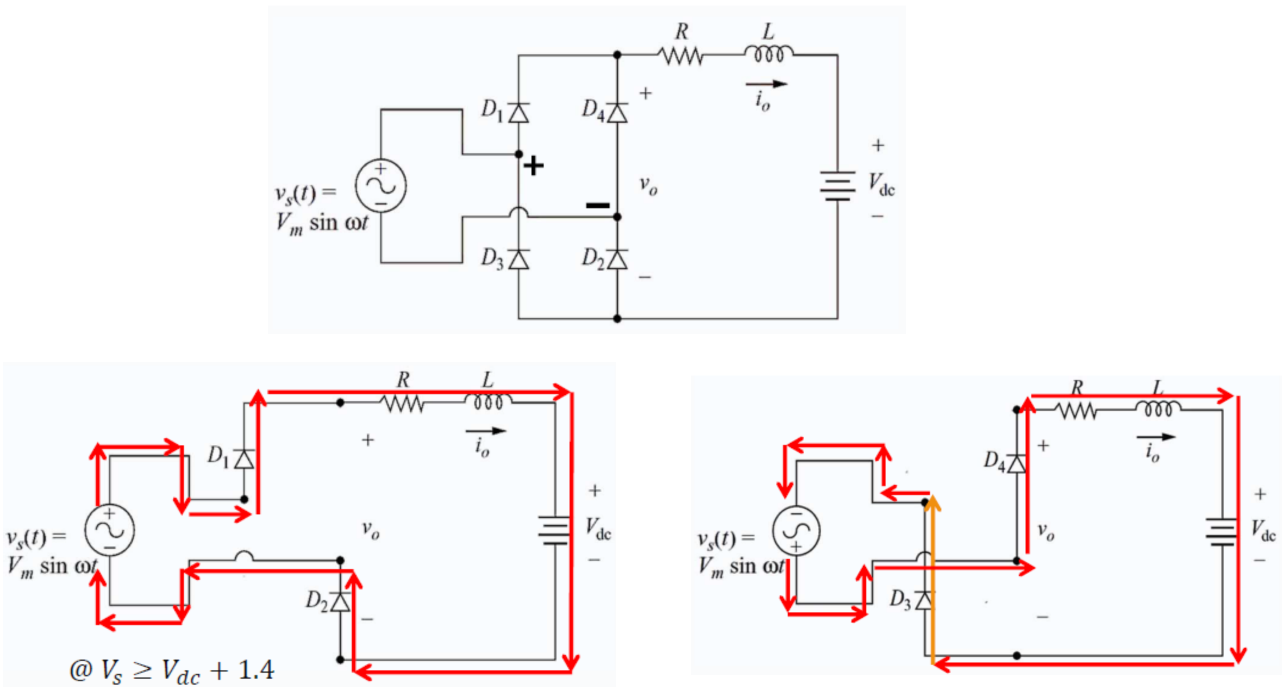
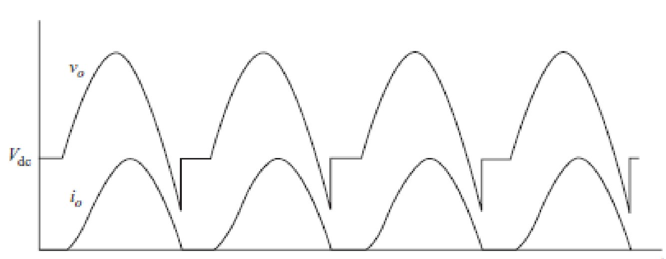


Figure A: Rectifier with *RL*-source load;

There are two possible modes of operation for this circuit:

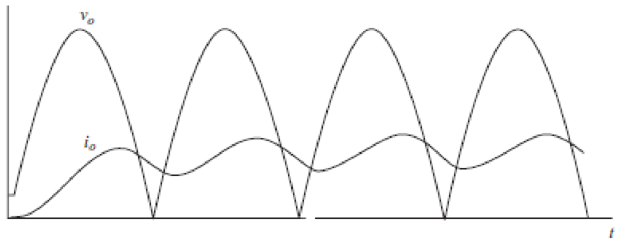
1- Discontinuous current

The load current returns to zero during every period. Discontinuous current is analyzed like the half-wave rectifier. The load voltage is not a full-wave rectified sine wave for this case, so the Fourier series does not apply.



2- Continuous current

The load current is always positive reaches the steady-state after a few periods. the voltage across the load is a full-wave rectified sine wave. The only modification to the analysis that was done for an *RL* load is in the dc term of the Fourier series.



The load voltage can be expressed as

$$V_o = \frac{2Vm}{\pi} - V_{dc}$$

$$I_0 = \frac{\frac{2V_m}{\pi} - V_{dc}}{R}$$

$$v_o(t) = V_o + \sum_{n=2,4,6,..}^{\infty} V_n \cos(n\omega t + \pi)$$

$$V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) , \qquad n = 2,4,6,..$$

$$I_n = \frac{V_n}{Z_n} , \quad Z_n = |R + jn\omega L| = \sqrt{R^2 + (n\omega L)^2} \quad \text{where } \omega = 2\pi f \quad rad/sec$$

$$I_{o,rms} = \sqrt{\sum_n I_{n,rms}^2} = \sqrt{I_0^2 + \sum_{n=2,4,6,..;} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

The power dissipated in the resistor can be computed from the rms current

$$P_R = I_{o,rms}^2 R$$

the power absorbed by the battery can be computed from the average current :

$$P_{dc} = I_0 V_{dc}$$

The total power supplied by the source (absorbed or dissipated by the load):

$$P_S = P_L = P_R + P_{dc} = I_{o,rms}^2 R + I_0 V_{dc}$$

The power factor can be computed from :

$$PF = \frac{P}{S} = \frac{P_L}{V_{s,rms} I_{s,rms}} = \frac{I_{o,rms}^2 R + I_0 V_{dc}}{V_{s,rms} I_{s,rms}}$$

Example:

A full-wave rectifier has an Ac source with 120 V rms at 60 Hz, R = 2 Ω, L = 10 mH, Vdc = 80 V. Determine (a) the power absorbed by the dc source, (b) the power absorbed by the resistor, and (c) the power factor.

a) $V_m = 120\ x\ \sqrt{2} = 169.7$ V

$$V_o = \frac{2V_m}{\pi} - V_{dc} = \frac{2\ x\ 169.7}{\pi} - 80 = 28\ V$$

$$I_0 = \frac{V_0}{R} = 14\ A$$

$$P_{dc} = I_0 V_{dc} = 14x80 = 1120\ W$$

b) Ac terms can be computed from ➔ $V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$

$$V_2 = \frac{2(169.7)}{\pi}\left(\frac{1}{2-1} - \frac{1}{2+1}\right) = 72\ V$$

$$V_4 = \frac{2(169.7)}{\pi}\left(\frac{1}{4-1} - \frac{1}{4+1}\right) = 14.4\ V \qquad , V_6 = 6.2V$$

$$I_n = \frac{V_n}{Z_n}\ , \quad Z_n = |R + jn\omega L| = \sqrt{R^2 + (n\omega L)^2} \qquad \text{where } \omega = 2\pi f = 377\ rad/sec$$

$$I_2 = \frac{V_2}{Z_2} = \frac{72}{\sqrt{2^2 + (2x377x0.01)^2}} = 9.23\ A$$

$$I_4 = \frac{V_4}{Z_4} = \frac{14.2}{\sqrt{2^2 + (4x377x0.01)^2}} = 0.9\ A \quad , \qquad I_6 = 0.27\ A$$

n	V_n	Z_n	I_n
0	28	2	14
2	72	7.8	9.23
4	14.4	15.2	0.9
6	6.2	22.7	0.27
⋮	⋮	⋮	⋮

The power absorbed by the load is determined from : $P_R = I_{o,rms}^2 R$

$$I_{o,rms} = \sqrt{\sum_n I_{n,rms}^2} = \sqrt{(14)^2 + \left(\frac{9.23}{\sqrt{2}}\right)^2 + \left(\frac{0.9}{\sqrt{2}}\right)^2 + \cdots} \approx \mathbf{15.46\ A}$$

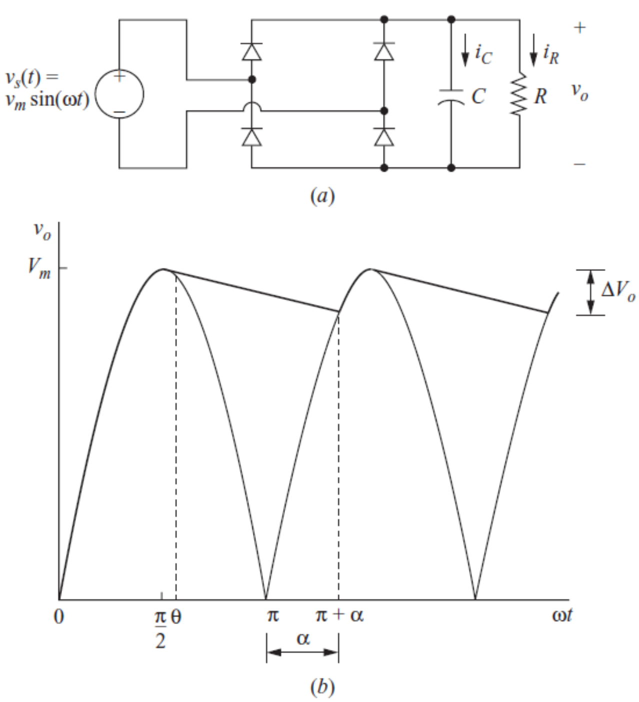
$$P_R = I_{o,rms}^2 R = (15.46)^2\ x\ 2 = \mathbf{478\ W}$$

c) $P_L = P_R + P_{dc} = 1120 + 478 = 1598\ W$

$$PF = \frac{P}{S} = \frac{P_L}{V_{s,rms}\ I_{s,rms}} = \frac{1598}{120x\ 15.46} = 0.86$$

➢ *Single-Phase full Wave Rectifier with A Capacitor Filter*

Placing a large capacitor in parallel with a resistive load in a bridge rectifier can produce an output voltage that is essentially dc. The analysis is very much like that of the half-wave rectifier with a capacitance filter. In the full-wave circuit, the time that the capacitor discharges are smaller than that for the half-wave circuit because of the rectified sine wave in the second half of each period. The output voltage ripple for the full-wave rectifier is approximately one-half that of the half-wave rectifier. The peak output voltage will be less in the full-wave circuit because there are two diode voltage drops rather than one.



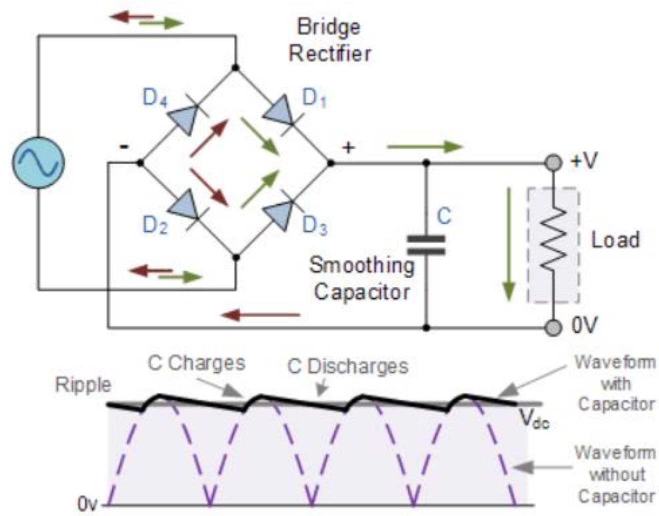


Figure (*a*) Full-wave rectifier with capacitance filter;(*b*) Source and output voltage.

The analysis proceeds exactly as for the half-wave rectifier. The output voltage is a positive sine function when one of the diode pairs is conducting and is a decaying exponential otherwise. Assuming ideal diodes,

$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin \theta) e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases}$$

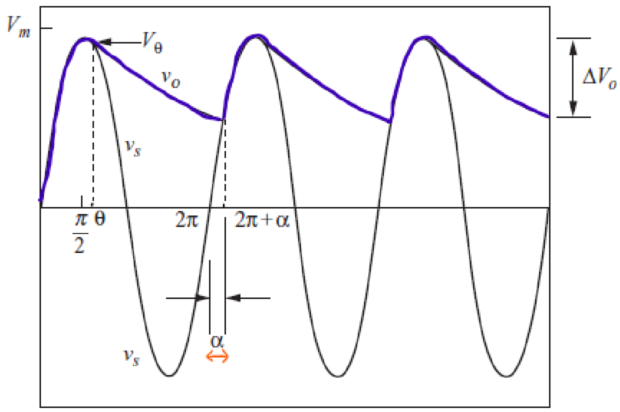
where is the angle where the diodes become reverse biased, which is the same as that for the half-wave rectifier

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

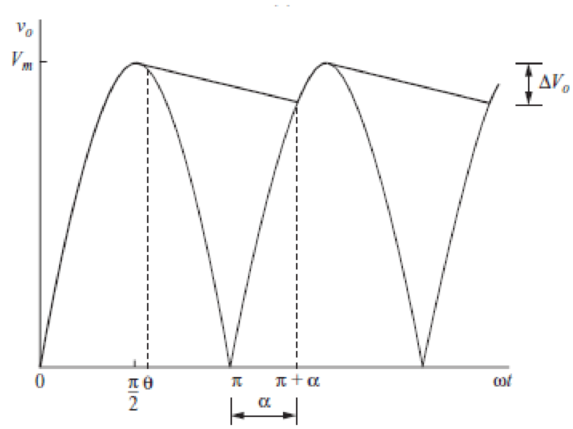
The peak-to-peak voltage variation, or ripple, is the difference between maximum and minimum voltages.

$$\boxed{\Delta V_o = V_m - |V_m \sin(\pi + \alpha)| = V_m(1 - \sin\alpha)}$$

This is the same as for voltage variation in the half-wave rectifier in fig (a). below, but α is larger for the full-wave rectifier and the ripple is smaller for a given load as fig. (b).



(a)



(b)

at **[ωt = π + α], and**

$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin\theta) e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases}$$

Where $\theta \approx \pi/2 \qquad \alpha \approx \pi/2$

The change in output voltage when the diode is off,

$$v_o(\pi + \alpha) = V_m e^{-(\pi + \pi/2 - \pi/2)/\omega RC} = V_m e^{-\pi/\omega RC}$$

The ripple voltage for the full-wave rectifier with a capacitor filter can then be approximated as

$$\Delta V_o \approx V_m(1 - e^{-\pi/\omega RC})$$

Furthermore, the exponential in the above equation can be approximated by the series expansion

$$e^{-\pi/\omega RC} \approx 1 - \frac{\pi}{\omega RC}$$

Substituting for the exponential in the approximation, the peak-to-peak ripple is

$$\boxed{\Delta V_o \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}}$$

Note that the approximate peak-to-peak ripple voltage for the full-wave rectifier is one-half that of the half-wave rectifier.

Capacitor current is described by the same equations as for the half-wave rectifier.

$$i_C(\omega t) = \begin{cases} -\left(\dfrac{V_m \sin\theta}{R}\right) e^{-(\omega t - \theta)/\omega RC} & \text{for } \theta \le \omega t \le \pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } \pi + \alpha \le \omega t \le \pi + \theta \quad \text{(diode on)} \end{cases}$$

Peak capacitor current occurs when the diode turns on at **[$\omega t = \pi + \alpha$],**

$$I_{C,\text{peak}} = \omega C V_m \cos(\pi + \alpha) = \omega C V_m \cos\alpha$$

for the half-wave rectifier, the peak diode current is much larger than the average diode current and Eq. below applies.

$$\boxed{I_{D,\text{ peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)}$$

the average diode current is half of average load current

$$I_D = \frac{I_o}{2}$$

The average source current is zero.

Example:

The full-wave rectifier of Fig. 4-6 has a 120-V rms 60 Hz source and a load resistance of 200 Ω. Determine the filter capacitance required to limit the peak to-peak output voltage ripple to 1 percent of the dc output. Determine the peak and average diode currents.

$$\Delta V \approx \frac{V_m}{2fRC}; \quad V_o \approx V_m 120\sqrt{2} = 169.7\ V.; \quad 0.01V_o \approx 1.7\ V.$$

$$C = \frac{V_m}{2fR\Delta V_o} = \frac{169.7}{2(60)(200)(1.7)} = 4160\ \mu F.$$

$$I_D = \frac{I_o}{2} = \frac{V_o}{2R} \approx \frac{169.7}{2(200)} = 0.43\ A.$$

$I_{D,peak}$: *from Eq.*

$$\alpha = \sin^{-1}\left(1 - \frac{\Delta V_o}{V_m}\right) = \sin^{-1}\left(1 - \frac{1.7}{169.7}\right) = 81.9^\circ$$

From Eq. $\quad I_{D,peak} = V_m\left(\omega C\cos\alpha + \frac{\sin\alpha}{R}\right)$

$$= 120\sqrt{2}\left(377(8.32)(10)^{-3}\cos 81.9^\circ + \frac{\sin 81.9^\circ}{200}\right) = 38.5\ A.$$

Example:

The full-wave rectifier with capacitance filter has a 120 rms V source at 60 Hz, R =500 Ω, and C =100 μF. (a) Determine the peak-to-peak voltage variation of the output (b) Determine the value of capacitance that would reduce the output voltage ripple to 1 percent of the dc value. $\alpha = 1.06\text{ rad} = 60.6^\circ$

■ **Solution**

$$V_m = 120\sqrt{2} = 169.7\text{ V}$$

(*a*) Peak-to-peak output voltage is described by

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7[1 - \sin(1.06)] = 22\text{ V}$$

(b) With the ripple limited to 1 percent, the output voltage will be held close to *Vm* and the approximation of Eq.

$$\frac{\Delta V_o}{V_m} = 0.01 \approx \frac{1}{2fRC}$$

$$C \approx \frac{1}{2fR(\Delta V_o/V_m)} = \frac{1}{(2)(60)(500)(0.01)} = 1670\ \mu\text{F}$$

Example:

A half-wave rectifier has a 120 V rms source at 60 Hz, $R = 500\ \Omega$. The capacitance required for a 1 percent ripple in output voltage was determined to be 3333 μF. Determine the capacitance required for a 1 percent ripple if a full-wave rectifier is used instead. Determine the peak diode currents for each circuit. Discuss the advantages and disadvantages of each circuit.

$C \approx 3333/2 = 1667\ \mu F$. Peak diode currents are the same. Fullwave circuit has advantages of zero average source current, smaller capacitor, and average diode current ½ that for the halfwave. The halfwave circuit has fewer diodes, and has only one diode voltage drop rather than two.