Lecture -5-

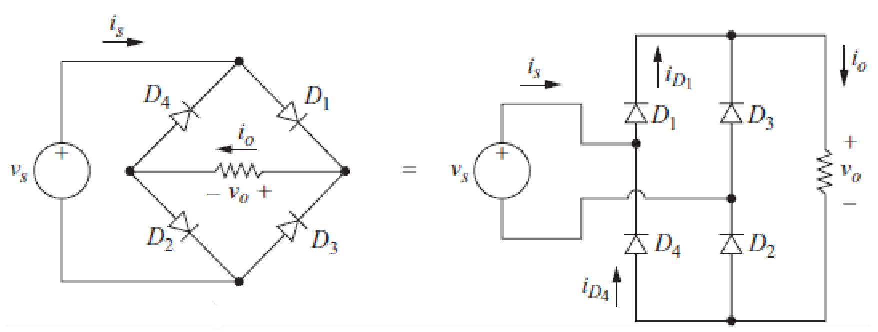
Single Phase Uncontrolled Full -Wave Rectifier

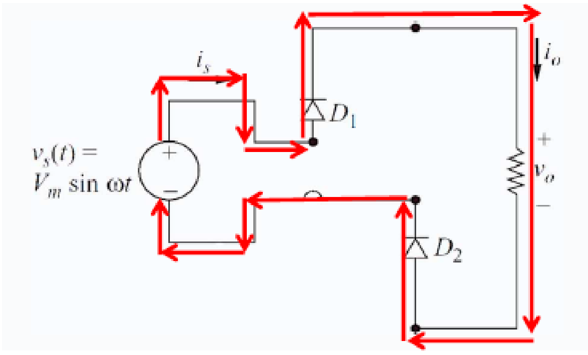
There are two types of single-phase full-wave rectifiers, namely, full-wave rectifiers with center-tapped transformers and bridge rectifiers.

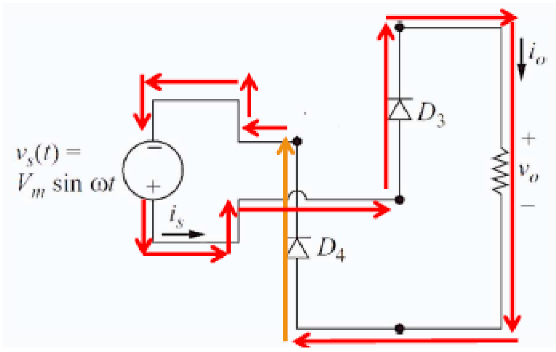
➢ *full-wave rectifier with a Bridge Rectifier:*

For the bridge rectifier of Fig. 4-1, these are some basic observations:

1. Diodes $D1$ and $D2$ conduct together, and $D3$ and $D4$ conduct together. Kirchhoff's voltage law around the loop containing the source, $D1$, and $D3$ shows that $D1$ and $D3$ cannot be on at the same time. Similarly,$D2$ and D cannot conduct simultaneously. The load current can be positive or zero but can never be negative.
2. The voltage across the load is $+vs$ when $D1$ and $D2$ are on. The voltage across the load is $-vs$ when $D3$ and $D4$ are on.
3. The maximum voltage across a reverse-biased diode is the peak value of the source. This can be shown by Kirchhoff's voltage law around the loop containing the source, $D1$, and $D3$. With $D1$ on, the voltage across $D3$ is $-vs$.
4. The current entering the bridge from the source is $iD1$ to $iD4$, which is symmetric about zero. Therefore, the average source current is zero.
5. The rms source current is the same as the rms load current. The source current is the same as the load current for one-half of the source period and is the negative of the load current for the other half. The squares of the load and source currents are the same, so the rms currents are equal.
6. The fundamental frequency of the output voltage is 2ω, where ω is the frequency of the ac input since twoperiods of the output occur for every period of the input. The Fourier series of the output consists of a dc term and the even harmonics of the source frequency.




$v_s(t) =$
$V_m \sin \omega t$
i_s
D_1
D_2
i_o
v_o


$v_s(t) =$
$V_m \sin \omega t$
i_s
D_3
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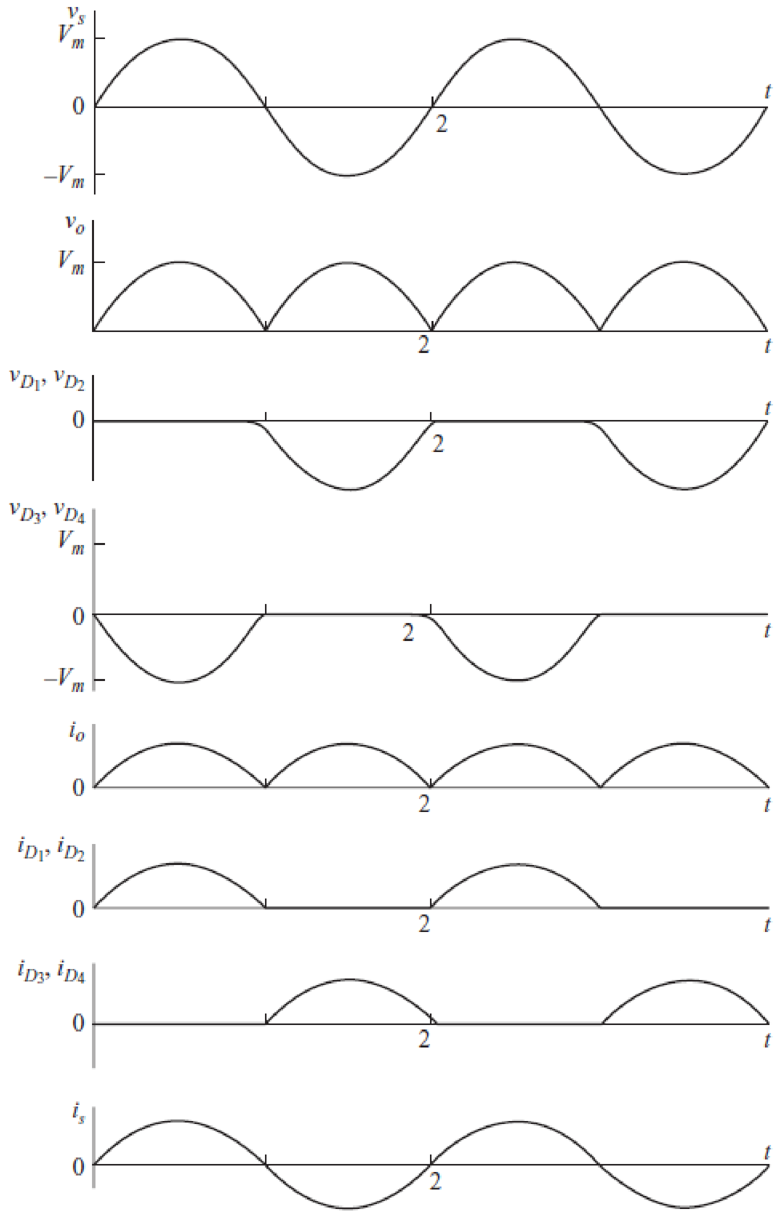
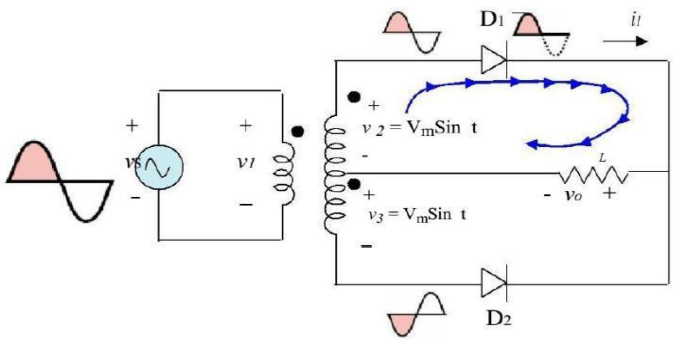
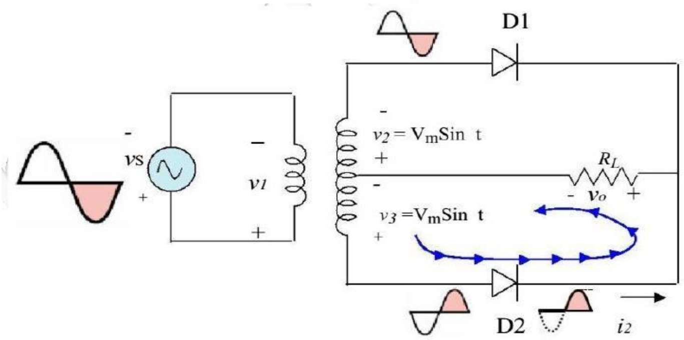
i_o
v_o


v_s
V_m
$-V_m$
v_o
v_{D_1}, v_{D_2}
v_{D_3}, v_{D_4}
i_o
i_{D_1}, i_{D_2}
i_{D_3}, i_{D_4}
i_s
t

➢ *full-wave rectifier with a center-tapped transformer:*

It is clear that each diode acts as a half-wave rectifier together with the associated half of the transformer. The outputs of the two half-wave rectifiers are combined to produce full-wave rectification in the load.





The Center-Tapped Transformer Rectifier

The voltage waveforms for a resistive load for the rectifier using the center tapped transformer are shown in Fig. 4-2. Some basic observations for this circuit are as follows:

1. Kirchhoff's voltage law shows that only one diode can conduct at a time. Load current can bepositive or zero but never negative.

2. The output voltage is + *vs*1 when *D*1 conducts and is -*vs*2 when *D*2 conducts. The transformersecondary voltages are related to the source voltage by *vs*1= *vs*2= *vs* (N2/2*N*1).

3. Kirchhoff's voltage law around the transformer secondary windings, *D*1, and *D*2 shows thatthe maximum voltage across a reverse-biased diode is *twice* the peak value of the load voltage.

4. Current in each half of the transformer secondary is reflected to the primary, resulting in anaverage source current of zero.

5. The transformer provides electrical isolation between the source and the load.

6. The fundamental frequency of the output voltage is 2ω since two periods of the output occurfor every period of the input.

The lower peak diode voltage in the bridge rectifier makes it more suitable for high-voltage applications. The center-tapped transformer rectifier, in addition to including electrical isolation, has only one diode voltage drop between the source and load, making it desirable for low-voltage, high-current applications.

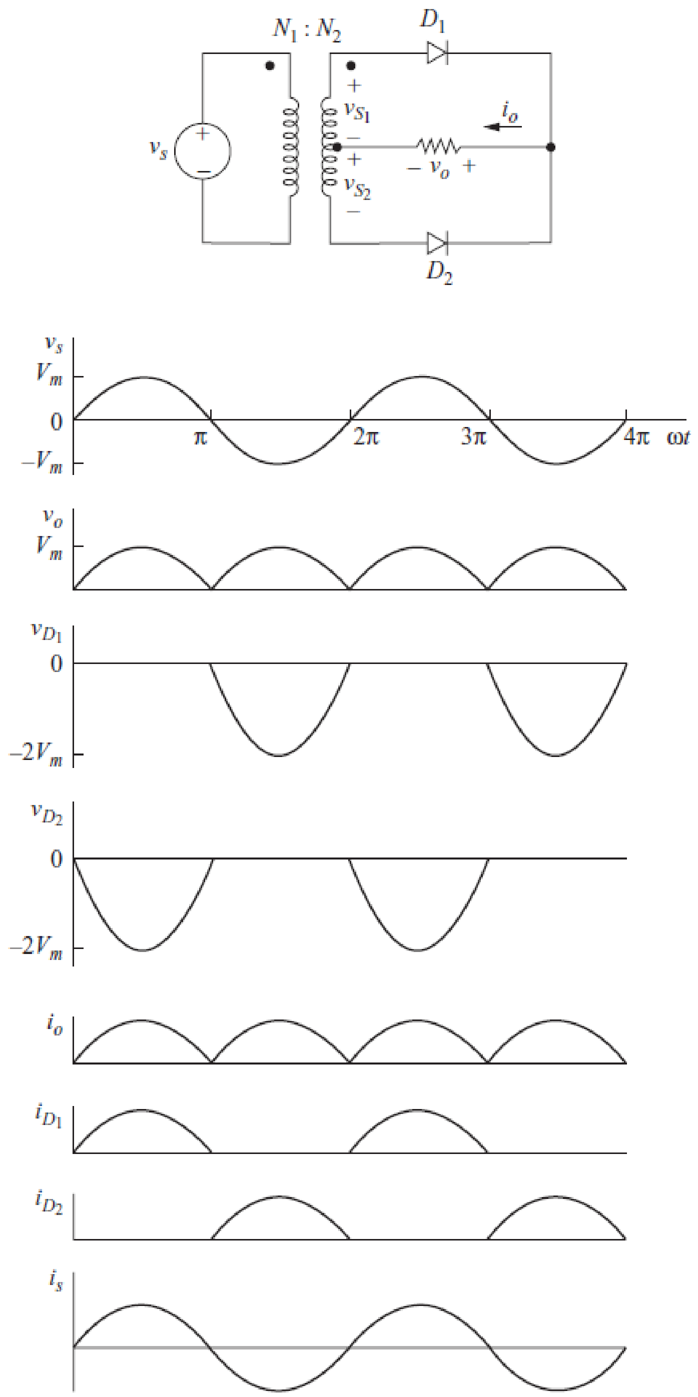


Figure 4-2 Full-wave center-tapped rectifier (circuit and voltages and currents).

The following discussion focuses on the full-wave bridge rectifier but generally applies to thecenter-tapped circuit as well.

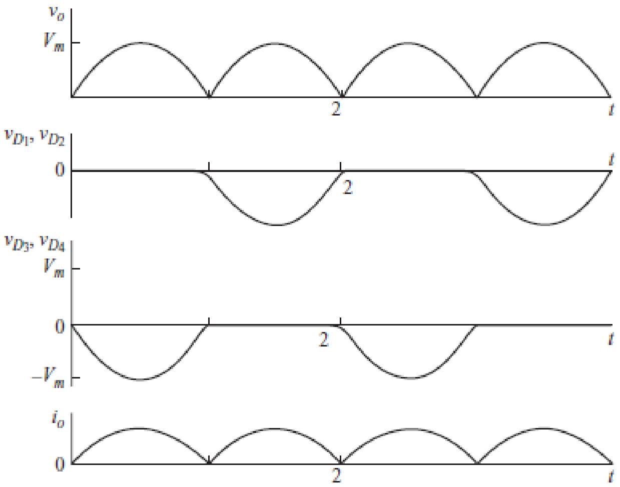
1) full-wave rectifier with Resistive Load

The voltage across a resistive load for the bridge rectifier of Fig. 4-1 is expressed as

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } 0 \leq \omega t \leq \pi \\ -V_m \sin \omega t & \text{for } \pi \leq \omega t \leq 2\pi \end{cases}$$

The dc component of the output voltage is the average value, and load current is simply the resistorvoltage divided by resistance.

$$\boxed{\begin{aligned} V_o &= \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t) = \frac{2V_m}{\pi} \\ I_o &= \frac{V_o}{R} = \frac{2V_m}{\pi R} \end{aligned}}$$



It's obvious That the output RMS value equal the source value

$$V_{o,rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\omega t)\ d(\omega t)} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t\ \ d(\omega t)} = \frac{\mathbf{V_m}}{\sqrt{\mathbf{2}}} = V_{s,rms}$$

The same for RMS current :

$$\mathrm{I}_{o,rms} = \frac{V_{rms}}{R} = \frac{\mathbf{V_m}}{\sqrt{\mathbf{2}}\mathbf{R}} = I_{s,rms}$$

$$\mathrm{PF} = \frac{\mathrm{P_S} = \mathrm{P_R}}{\mathrm{V_{s,rms}}\,\mathrm{I_{s,rms}}} = \frac{\mathrm{I}_{o,rms}\mathrm{V}_{o,rms}}{\mathrm{V_{s,rms}}\,\mathrm{I_{s,rms}}} = 1$$

The average diode current is:

$$I_{D,avg} = \frac{I_o}{2}$$

$$I_{D,rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2(\omega t)\ d(\omega t)} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\pi}\int_0^{\pi} i^2(\omega t)\ d(\omega t)} = \frac{\mathbf{I_{o,rms}}}{\sqrt{\mathbf{2}}}$$

Example//

A single-phase rectifier has a resistive load of 18 Ω. Determine (a) the average load current, (b) the rms load current, (c) the average and rms current in each diode (d) power factor. For a bridge rectifier with an AC source of 120 V rms and 60 Hz.

a) $V_m = 120 \text{ x } \sqrt{2} = \mathbf{169.7}\text{ V}$

$$V_{o,avg} = \frac{2\,V_m}{\pi} \rightarrow I_{o,avg} = \frac{2V_m}{\pi R} = \frac{2\,x\,169.7}{\pi \text{ x } 18} = \mathbf{6}\text{ A}$$

b) $V_{o,rms} = V_{s,rms} = \mathbf{120}\text{ V} \rightarrow I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{120}{18} = \mathbf{6.67}\text{ A}$

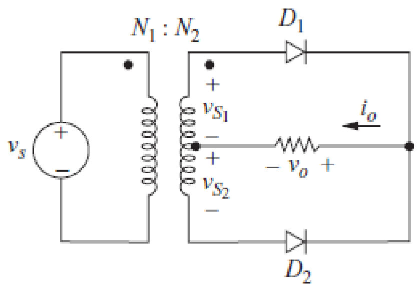
c) $I_{D,avg} = \frac{I_{o,avg}}{2} = \frac{6}{2} = \mathbf{3}\text{ A}$ ..... $I_{D,rms} = \frac{I_{o,rms}}{\sqrt{2}} = \frac{6.67}{\sqrt{2}} = \mathbf{4.7}\text{ A}$

d) $I_{s,rms} = I_{o,rms} = \mathbf{6.67}\text{ A}$

$PF = \mathbf{1}$ (note $\rightarrow$ $I_{s,rms} = I_{o,rms}$ ...., $V_{R,rms} = V_{o,rms} = V_{s,rms}$)

Example//

A single-phase center-tapped transformer rectifier has an AC source of 240 V rms and 60 Hz. The overall transformer turns ratio is 3:1 (80 V rms between the extreme ends of the secondary and 40 V rms on each tap). The load is a resistance of 4 Ω. Determine (a) the average load current, (b) the rms load current, (c) the average source current, and (d) the rms source current.



a) $V_{sec,rms} = \frac{240}{3} = 80$ v rms

$$V_m = \frac{V_{sec,rms}}{2} \text{ x } \sqrt{2} = 40 \text{ x } \sqrt{2} = 56.57 \text{ v}$$

$$V_{o,avg} = \frac{2\,V_m}{\pi} \rightarrow I_{o,avg} = \frac{2V_m}{\pi R} = \frac{2\,x\,56.57}{\pi \text{ x } 4} = \mathbf{9}\text{ A}$$

b) $V_{o,rms} = \frac{V_{se\ ,rms}}{2} = \mathbf{40}\text{ V} \rightarrow I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{40}{4} = \mathbf{10}\text{ A}$

c) $I_{s,avg} = \mathbf{0}\text{ A}$ ...... $I_{s,rms} = \frac{I_{o,rms}}{6} = \mathbf{1.67}\text{ A}$

2) ***full-wave rectifier with RL Load***

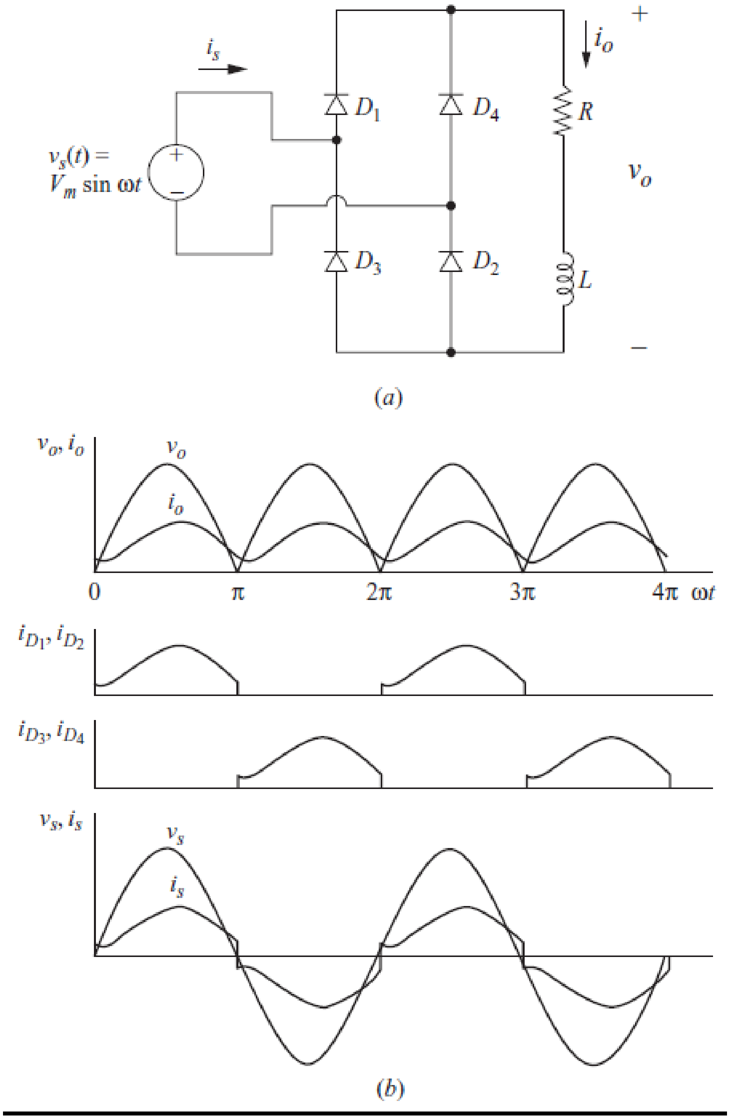


Figure 4-3 (a) Bridge rectifier with an RL load; (b) Voltages and currents;

For the bridge circuit, current is transferred from one pair of diodes to the other pair when the source changes polarity. The current reaches periodic steady state after a few periods (depending on the L/R time constant), which means that the current at the end of a period is the same as the current at the beginning of the period as shown in Fig. 4-3b. The voltage across the *RL* load is a full-wave rectified sinusoid, as it was for the resistive load.
The full-wave rectified sinusoidal voltage across the load can be expressed as a Fourier series consisting of a dc term and the even harmonics.

$$v_o(t) = V_o + \sum_{n=2,4\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

where (4-4)

$$V_o = \frac{2V_m}{\pi} \qquad \text{and} \qquad V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

$$\boxed{\begin{aligned} I_0 &= \frac{V_0}{R} \\ I_n &= \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega L|} \end{aligned}} \qquad (4\text{-}5)$$

$$Z_n = |R + jn\omega_0 L| = \sqrt{R^2 + (n\omega_0 L)^2}$$

$$I_{o,rms} = \sqrt{\sum_n I_{n,rms}^2} = \sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \sum_{n=2,4,6,..} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

Note that as the harmonic number n increases in Eq. (4-4), the voltage amplitude decreases. For an RL load, the impedance *Zn* increases as n increases. The combination of decreasing *Vn* and increasing *Zn* makes *In* decrease rapidly for increasing harmonic number. Therefore, the dc term and only a few, if any, of the ac terms are usually necessary to describe current in an RL load.

Example//

Uncontrolled Full -Wave bridge rectifier circuit has an ac source with *Vm* = 100 V at 60 Hz and a series *RL* load with *R* = 10 Ω and *L* =10 mH. (*a*) Determine the average current in the load. (*b*) Estimate the peak-to-peak variation in load current based on the first ac term in the Fourier series. (*c*) Determine the power absorbed by the load and the power factor of the circuit. (*d*) Determine the average and rms currents in the diodes.

■ ***Solution***

(*a*) The average load current is determined from the Average output voltage is

$$V_0 = \frac{2V_m}{\pi} = \frac{2(200)}{\pi} = 63.7 \text{ V}$$

and average load current is

$$I_0 = \frac{V_0}{R} = \frac{63.7 \text{ V}}{10 \ \Omega} = 6.37 \text{ A}$$

(*b*) Amplitudes of the ac voltage terms are determined from

$$V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

For *n* = 2 and 4,

$$V_2 = \frac{2(100)}{\pi}\left(\frac{1}{2-1} - \frac{1}{2+1}\right) = 42.4 \ V$$
$$V_4 = \frac{2(100)}{\pi}\left(\frac{1}{4-1} - \frac{1}{4+1}\right) = 8.49 \ V$$

The amplitudes of first two ac current terms in the current Fourier series are computed from Eq

$$I_n = \frac{V_n}{Z_n}\ ,\ Z_n = |R + jn\omega L| = \sqrt{R^2 + (n\omega L)^2} \qquad \text{where } \omega = 2\pi f = 377\ rad/sec$$

$$I_2 = \frac{V_2}{Z_2} = \frac{42.4}{\sqrt{10^2 + (2x377x0.01)^2}} = 3.39\ A$$
$$I_4 = \frac{V_4}{Z_4} = \frac{8.49}{\sqrt{10^2 + (4x377x0.01)^2}} = 0.47\ A$$

The current I_2 is much larger than I_4 and higher-order harmonics, so I_2 can be used to estimate the peak-to-peak variation in load current $\Delta i_o \approx 2(3.39) = 6.78$ A. Actual variation in i_o will be larger because of the higher-order terms.

(*c*) The power absorbed by the load is determined from *Irms*. The rms current is then determined from Eq

$$I_{o,rms} = \sqrt{\sum_n I_{n,rms}^2} = \sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \sum_{n=2,4,6,..} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

$$= \sqrt{(6.37)^2 + \left(\frac{3.39}{\sqrt{2}}\right)^2 + \left(\frac{0.47}{\sqrt{2}}\right)^2 + \cdots} \approx 6.81\ \text{A}$$

Adding more terms in the series would not be useful because they are small and have little effect on the result. Power in the load is

$$P = I_{\text{rms}}^2 R = (6.81)^2(10) = 464\ \text{W}$$

The rms source current is the same as the rms load current. Power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{464}{\left(\frac{100}{\sqrt{2}}\right)(6.81)} = 0.964$$

(*d*) Each diode conducts for one-half of the time, so

$$I_{D,\text{avg}} = \frac{I_o}{2} = \frac{6.37}{2} = 3.19\ \text{A}$$

and

$$I_{D,\text{rms}} = \frac{I_{\text{rms}}}{\sqrt{2}} = \frac{6.81}{\sqrt{2}} = 4.82\ \text{A}$$

If $(wL >> R)$

In some applications, the load inductance may be relatively large or made large by adding external inductance. If the inductive impedance for the ac terms in the Fourier series effectively eliminates the ac current terms in the load, the load current is essentially dc.

$$i(\omega t) \approx I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R} \qquad \text{for } \omega L \gg R$$

$$I_{\text{rms}} \approx I_o$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m sin\omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

$$I_{rms} \approx I_o = I_{dc}$$