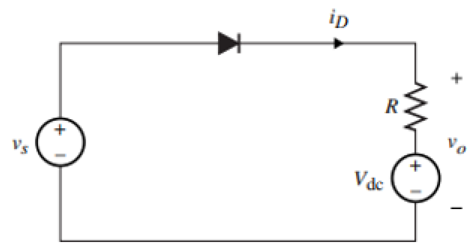
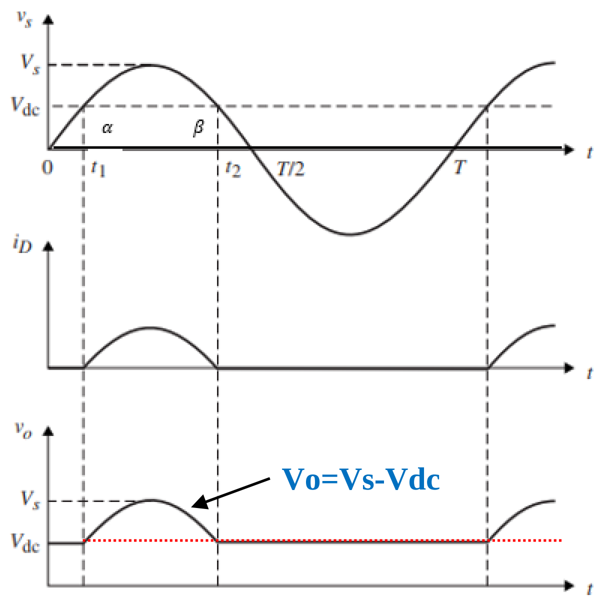
Lecture -3-

4) Half-wave Rectifier with (R load + DC Source)

- If the output is connected to a battery, the rectifier can be used as a battery charger. This is shown in Figure (a).
- diode D conducts (turn on) at Vs > E,
- diode D conducts (turn off) at Vs < E,



(a)



(b)

The angle α can be found from:

$$E = Vm \sin\alpha = Vdc \quad \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \quad \alpha = \sin^{-1}\left(\frac{E}{Vm}\right)$$

$$V_{o(load)} = \frac{1}{2\pi}\int_{\alpha}^{\beta}(Vm \sin wt - E)\, d(wt)$$

$$= \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} (Vm \sin wt . d(wt) - \int_{\alpha}^{\beta} E . d(wt)\right]$$

$$= \frac{1}{2\pi} \ ([-Vm \cos wt]_{\alpha}^{\beta} - \ [E\ wt]_{\alpha}^{\beta}$$

$$= \frac{1}{2\pi} \ [-Vm (\cos \beta - \cos \alpha) - E(\beta - \ \alpha)]$$

$$= \frac{1}{2\pi} \ [-Vm \cos \beta + Vm \cos \alpha - \ E\beta + \ E\alpha] \qquad at \quad \beta = \pi - \alpha$$

$$= \frac{1}{2\pi} \ [-Vm \cos (\pi - \alpha) + Vm \cos \alpha - \ E\ (\pi - \alpha) + \ E\alpha]$$

for $\cos(\pi - x) = -\cos x$

$$= \frac{1}{2\pi} \ [\ Vm \cos \alpha + Vm \cos \alpha - \ \pi E + \ E\alpha \ + \ E\alpha]$$

$$\boxed{V_{o(load)} = \frac{1}{2\pi} \ [\, 2\, Vm \cos \alpha + 2\, E\alpha - \ \pi E\,]}$$

$$\boxed{I_{o(load)} = \frac{1}{2\pi R} \ [\, 2\, Vm \cos \alpha + 2\, E\alpha - \ \pi E\,]}$$

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$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\beta=\pi-\alpha} (V_m \sin \omega t - E)^2 \ d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\left[\left(\frac{V_m^{\ 2}}{2} + E^2\right)(\pi - 2\alpha) + \frac{V_m^{\ 2}}{2} \sin 2\alpha - 4V_m E \cos \alpha\right]}$$

$$I_{rms} = \frac{V_{o,rms}}{R}$$

$$P_{R(resistor)} = I_{rms}^2 R; P_{battery} = P_{dc} = EI_{dc}$$

5) *Half-wave Rectifier with (RL- load + DC Source)*

Another variation of the half-wave rectifier is shown in Fig.a. The load consists of a resistance, an inductance, and a dc voltage. Starting the analysis at wt = 0 and assuming the initial current is zero, recognize that the diode will remain off as long as the voltage of the ac source is less than the dc voltage.

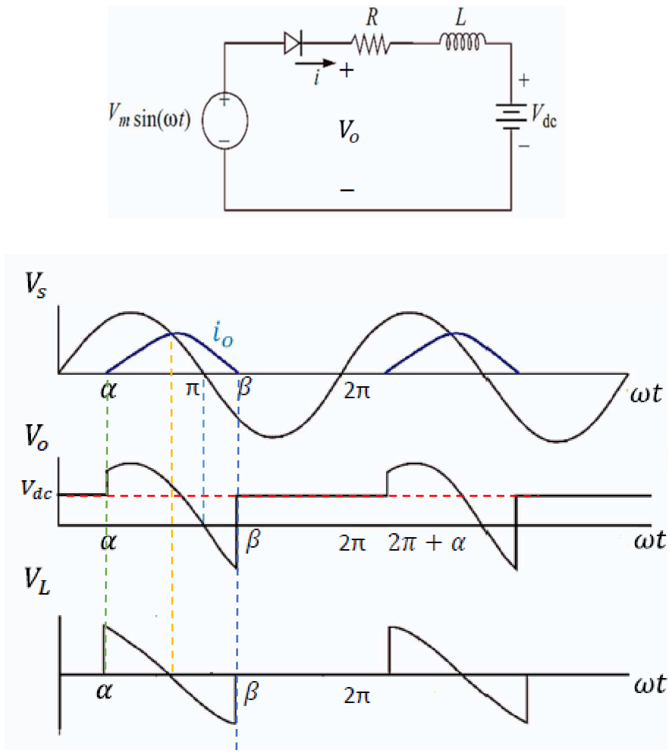

Figure (*a*) Half-wave rectifier with *RL* source load

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} + V_{dc}$$

$$i(t) = i_f(t) + i_n(t)$$

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) - \frac{V_{dc}}{R}$$

$$i_n(t) = Ae^{-t/\tau}$$

$$i_o(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \frac{V_{dc}}{R} + Ae^{\left(\frac{-\omega t}{\omega \tau}\right)}\right] & for\ \alpha < \omega t < \beta \\ 0 & otherwise \end{cases}$$

$$where \quad A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta) + \frac{V_{dc}}{R}\right]e^{\left(\frac{\alpha}{\omega \tau}\right)}$$

The constant A is evaluated by using the initial condition for current $i(\alpha) = 0$

where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ $\tau = \frac{L}{R}$

$$V_{o(load)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(wt) + \int_{\alpha}^{\beta} (Vm \sin wt.d(wt) + \int_{\beta}^{2\pi} E.d(wt)\right]$$

$$= \frac{1}{2\pi}\,[\,E\alpha + \,Vm\,(\cos\alpha - \cos\beta) + E(\,2\pi - \beta)]$$

$$= \frac{1}{2\pi}\,[\;Vm\,(\cos\alpha - \cos\beta) + E(\alpha + \;2\pi - \beta)]$$

$$I_{o(load)} = \frac{V_{o(load)} - E}{R}$$

$$I_{o,avg} = \frac{1}{2\pi}\int_{\alpha}^{\beta} i_o(\omega t)\,d\omega t$$

$$I_{o,rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i_o^2(\omega t)\,d\omega t}$$

$$P_R = I_{o,rms}^2 R\;, \qquad P_{dc} = I_{o,avg} V_{dc}$$

$$P_{ac} = P_{dc} + P_R = I_{o,avg}V_{dc} + I_{o,rms}^2 R$$

$$\text{PF} = \frac{P}{S} = \frac{P_{ac}}{S}$$

Example:

For the half-wave rectifier with RL-Source Load, R =2 Ω, L = 20 mH, and Vdc = 100 V. The ac source is 120 V rms at 60 Hz. Determine (a) an expression for the current in the circuit, (b) the power absorbed by the dc source, ***β* = 3.37 rad (193°).**

■ Solution

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = 7.80\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 1.31 \text{ rad}$$
$$\alpha = \sin^{-1}(100/169.7) = 36.1° = 0.630 \text{ rad}$$
$$\omega\tau = 377(0.02/2) = 3.77 \text{ rad}$$

(a) Using Eq.

$$i_o(\omega t) = \begin{cases} \frac{V_m}{Z}\left[sin(\omega t-\theta) - \frac{V_{dc}}{R} + Ae^{\left(\frac{-\omega t}{\omega\tau}\right)}\right] & for\ \alpha < \omega t < \beta \\ 0 & otherwise \end{cases}$$

$$where \qquad A = \left[-\frac{V_m}{Z}sin(\alpha-\theta) + \frac{V_{dc}}{R}\right]e^{\left(\frac{\alpha}{\omega\tau}\right)}$$

$$A = \left[\left(\frac{120\sqrt{2}}{7.8}\right)\sin(0.63-1.31) + \frac{100}{2}\right]e^{\left(\frac{0.63}{3.77}\right)}$$

$$A \ = [-21.75\ (-0.628) + 50]e^{0.167} = (63.659\ )e^{0.167} = 75.23$$

OR//

$$\theta = 1.32 * \frac{180}{\pi} = 75.05°$$

$$A = \left[\left(\frac{120\sqrt{2}}{7.8}\right)\sin(36.1-75.05) + \frac{100}{2}\right]e^{\left(\frac{0.63}{3.77}\right)} \ = 75.3$$

$$i(wt) = 21.75\sin(wt-1.31) - 50 + 75.23\ e^{\left(\frac{wt}{3.77}\right)} \qquad A$$

b) The power absorbed by the dc source is IoVdc.

$$V_{o(load)} = \frac{1}{2\pi}\ [\ Vm\,(\cos\alpha - \cos\beta) + E(\alpha +\ 2\pi - \beta)]$$

$$= \frac{1}{2\pi}\ [\ 120\ \sqrt{2}\,(\cos 0.63 - \cos 3.37) + E(0.63 +\ 2\pi - 3.37)]$$

$$= \frac{1}{2\pi}\ [169.7(0.8+0.97) + 100(3.54)] = 104.146 \qquad v$$

$$I_{o(load)} = \frac{V_{o(load)} - E}{R} = \frac{104.146-100}{2} = 2.07 \quad A$$

$$P_{dc} = I_{o(load)} * V_{dc} \ = 2.07 * 100 = 207\ w$$