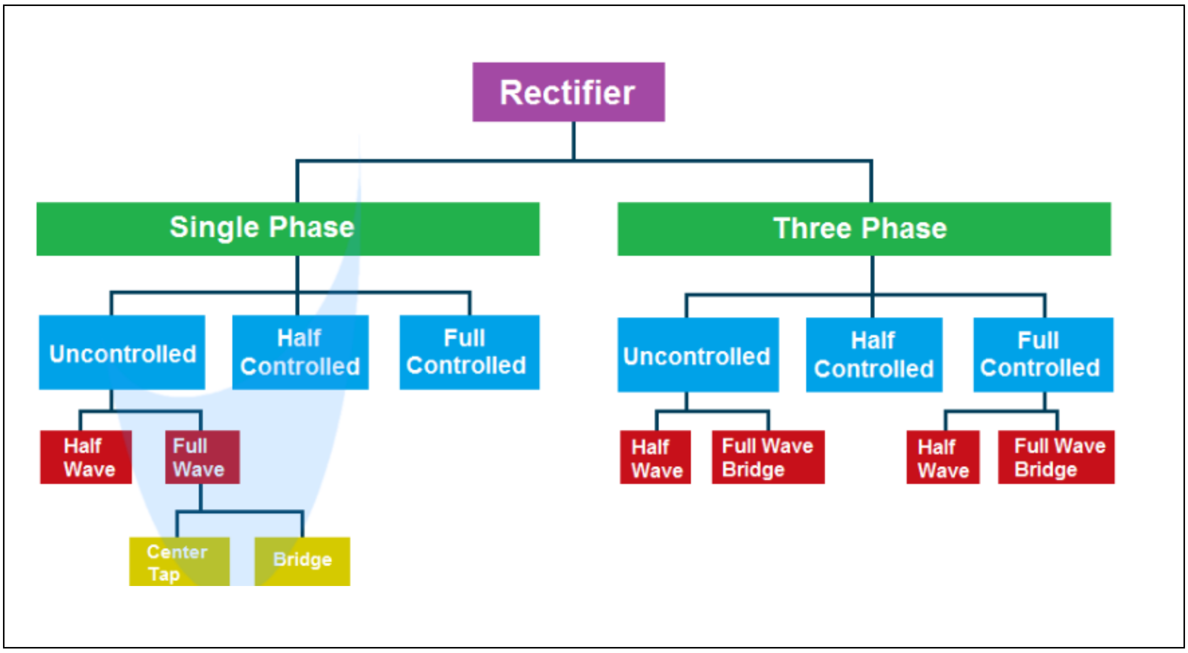
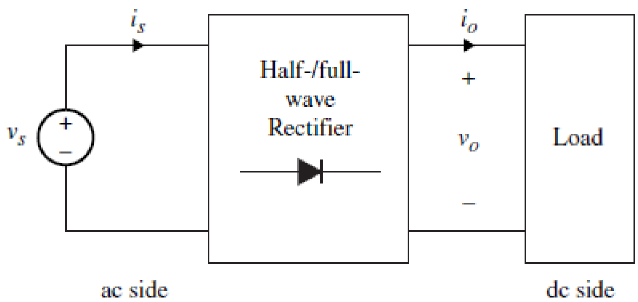
Lecture -2-

❖ Rectifiers



Single phase uncontrolled Rectifiers

There are two types of single-phase diode rectifier that convert a single-phase ac supply into a dc voltage, namely, single-phase half-wave rectifiers and single-phase full-wave rectifiers as shown in the block diagram below.



➤ *Half-Wave Rectifiers*

In practice, the half-wave rectifier is used most often in low-power applications because the average current in the supply will not be zero, and nonzero average current may cause problems in transformer performance, hence applications of this circuit are limited. it is very worthwhile to analyze the half-wave rectifier in detail. A thorough understanding of the half-wave rectifier circuit will enable the student to advance to the analysis of more complicated circuits with a minimum of effort.

1) *Half Wave Rectifier with Resistive Load (R- Load).*

basic half-wave rectifier with a resistive load is shown in Fig. 3-1a. The source is ac, and the objective is to create a load voltage that has a nonzero dc component. The diode is a basic electronic switch that allows current in one direction only.

For the positive half-cycle of the source in this circuit, the diode is on (forward-biased). Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive. For the negative half-cycle of the source, the diode is reverse-biased, making the current zero. The voltage across the reverse-biased diode is the source voltage, which has a negative value.

The voltage waveforms across the source, load, and diode are shown in Fig. 3-1b. Note that the units on the horizontal axis are in terms of angle (wt). This representation is useful because the values are independent of frequency.

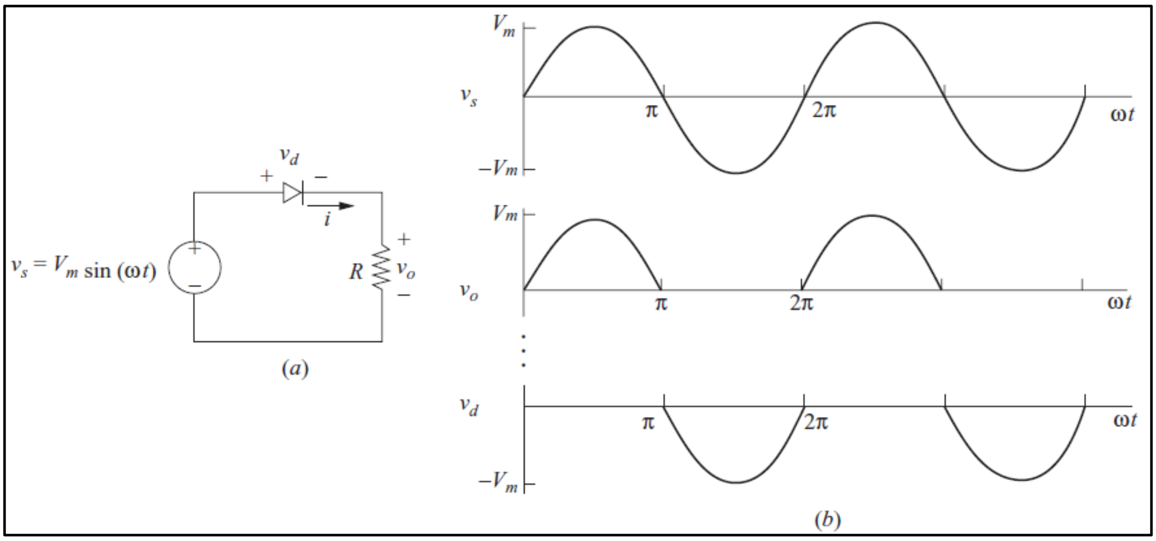


Figure 3-1 (*a*) Half-wave rectifier with resistive load; (*b*) Voltage waveforms.

The average value of output voltage or $V_{dc(load)}$ = $V_o = \frac{1}{T}\int_0^T v_o(t)\,dt$

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & for\ 0 \le \omega t \le \pi \\ 0 & otherwise \end{cases}$$

$$V_{DC(load)} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(wt)\,dwt$$

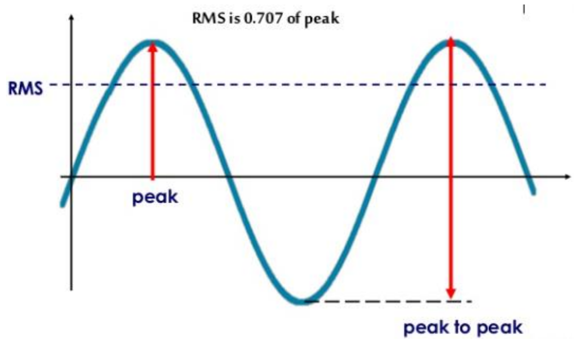
To make the soluation simpler, we assume wt = θ

$$V_{DC(load)} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\theta)\,d\theta = \frac{V_m}{2\pi}[-\cos\theta\,]_0^{\pi} = -\frac{V_m}{2\pi}[\cos(\pi) - \cos(0)]$$

$$\boxed{\boldsymbol{V_{DC(load)} = \frac{V_m}{\pi}}}$$

$$\boxed{I_{DC(load)} = \frac{V_m}{\pi R}}$$

The RMS value of output voltage,



$$V_{RMS(load)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}[V_m \ \sin(\theta)]^2 \, d\theta}$$

$$V_{RMS(load)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(V_m)^2 \ (\sin(\theta))^2 \, d\theta} \ = \sqrt{\frac{(V_m)^2}{2\pi}\int_0^{\pi}\frac{1}{2}(\ 1 - \cos(2\theta)\ ) \ d\theta}$$

$$V_{RMS(load)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\int_0^{\pi} 1 \ d\theta - \int_0^{\pi} \cos(2\theta) \, d\theta} = \frac{V_m}{2\sqrt{\pi}}\sqrt{[\ \theta\ ]_0^{\pi} - [\sin(2\theta)]_0^{\pi}}$$

$$V_{RMS(load)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\pi}$$

$$\boxed{V_{RMS(load)} = \frac{V_m}{2}}$$

$$\boxed{I_{RMS(load)} = \frac{V_m}{2R}}$$

The power dissipated in the resistor is

$$\mathrm{P_R} = \frac{V_{o,rms}^2}{R} = I_{o,rms}^2\mathrm{R}$$

$$\mathrm{PF} = \frac{P}{S} = \frac{\mathrm{P_R}}{\mathrm{V_{s,rms}\ I_{s,rms}}}$$

Example:

For the half-wave rectifier, the source is a sinusoid of 120 V rms at a frequency of 60 Hz. The load resistor is 5 Ω. Determine (a) the average load current, (b) the average power absorbed by the load and (c) the power factor of the circuit.

■ **Solution**

(*a*) The voltage across the resistor is a half-wave rectified sine wave with peak value $V_m = 120\sqrt{2} = 169.7$ V. From Eq. (3-2), the average voltage is V_m/π, and average current is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}(120)}{5\pi} = 10.8 \text{ A}$$

(*b*) From Eq. (3-3), the rms voltage across the resistor for a half-wave rectified sinusoid is

$$V_{\text{rms}} = \frac{V_m}{2} = \frac{\sqrt{2}(120)}{2} = 84.9 \text{ V}$$

The power absorbed by the resistor is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{84.9^2}{4} = 1440 \text{ W}$$

The rms current in the resistor is $V_m/(2R) = 17.0$ A, and the power could also be calculated from $I_{\text{rms}}^2 R = (17.0)^2(5) = 1440$ W.

(*c*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{1440}{(120)(17)} = 0.707$$

2) *Half Wave Rectifier with (RL- Load)*

The half-wave rectifier with an **inductive load** (RL) is shown in Fig. 3-2*a*. Industrial loads typically contain inductance as well as resistance. As the source voltage goes through zero, becoming positive in the circuit of Fig. 3-2*a*, the diode becomes forward-biased.

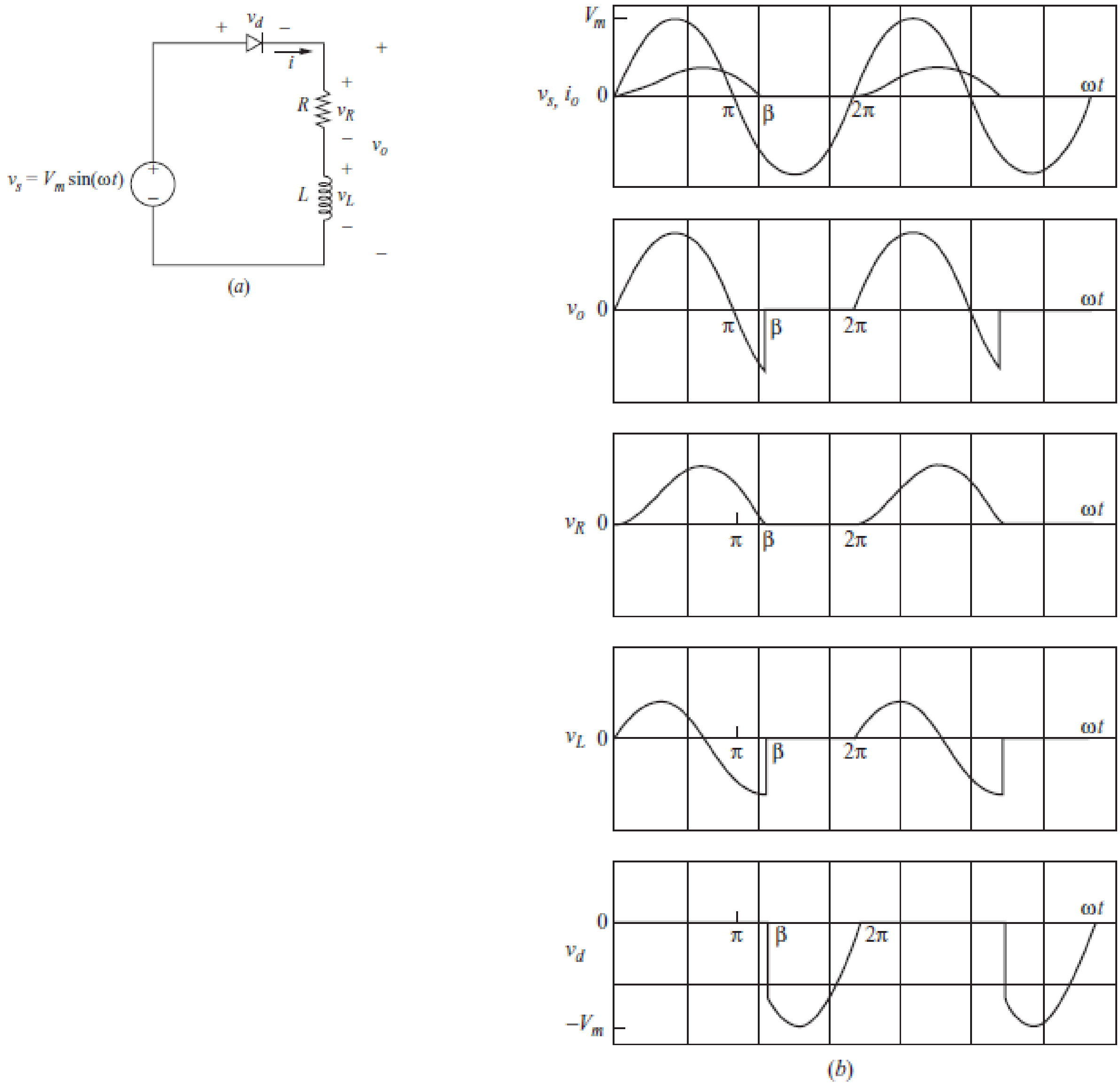


Figure 3-2 (*a*) Half-wave rectifier with an *RL* load; (*b*) Waveforms

The Kirchhoff voltage law equation that describes the current in the circuit for the forward-biased ideal diode is:

$$v_s = v_R + v_L$$

$$V_m \sin(wt) = Ri(t) + L\,\frac{di(t)}{dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The solution of equation (1) can be obtained by expressing the current as the sum of the forced response and the natural response:

$$i(t) = \, i_f(t) + \, i_n(t)$$

The forced response for this circuit is the current that exists after the natural response has decayed to zero. In this case, the forced response is the steady-state sinusoidal current that would exist in the circuit if the diode were not present. This steady-state current can be found from phasor analysis, resulting in

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta)$$

The natural response is the transient that occurs when the load is energized. It is the solution to the homogeneous differential equation for the circuit without the source or diode.

$$i_n(t) = A\,e^{-t/\tau}$$

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

The constant **A** is evaluated by using the initial condition for current: **t = 0, i(*ω*t) = 0**.

$$i(0) = \frac{V_m}{Z}\sin(0 - \theta) + Ae^{0} = 0$$

$$A = -\frac{V_m}{Z}\sin(-\theta) = \frac{V_m}{Z}\sin\theta$$

Substituting for ***A*** in Eq.

$$i(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + \frac{V_m}{Z}\sin(\theta)e^{-t/\tau}$$

$$= \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-t/\tau}\right]$$

It is often to write the function in terms of the angle ***ω*t** rather than time **t**.

$$\boxed{i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}}$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$, $\tau = \frac{L}{R}$

The voltage waveforms for each element are shown in Fig. 3-2b. Note that the diode remains forward-biased longer than (π) and that the source is negative for the last part of the conduction interval. This is due to the energy stored in the inductor where it continues to discharge for intervals π to β. Also note that the inductor voltage is negative when the current is decreasing (VL =Ldi/dt).

***extinction angle β*:** The first positive value of ωt that results in zero current.

The average dc component of the output is:

$$V_{DC(load)} = \frac{1}{2\pi}\int_0^{\beta} V_m \sin(wt)\,dwt \longrightarrow \boxed{V_{DC(load)} = \frac{V_m}{2\pi}[1 - \cos(\beta)]}$$

$$\boxed{I_{DC(load)} = \frac{V_m}{2\pi R}[1 - \cos(\beta)]}$$

The *rms* value of I_o can be written as

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\, d(\omega t)}$$

Or it can be written as

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} (V_m \sin\omega t)^2\, d\omega t} = \sqrt{\frac{V_m{}^2}{4\pi}\left(\beta - \frac{1}{2}\sin 2\beta\right)}$$

$$I_{rm} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{V_m{}^2}{4\pi}\left(\beta - \frac{1}{2}\sin 2\beta\right)}$$

Example: For the RL half-wave rectifier, R=100Ω, L=0.1 H, ω=377 rad/s, and Vm=100 V. Determine (a) an expression for the current in this circuit, (b) the average current, (c) the rms current, (d) the power absorbed by the RL load, and (e) the power factor. ***β* is found to be 3.50 rad, or 201°.**

Solution:

a) $$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)\, e^{-\omega t/\omega\tau}\right]$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ $Z = \sqrt{R^2 + (\omega L)^2}$

$$Z = \sqrt{(100)^2 + (377 * 0.1)^2} = 106.9\ \Omega$$

$$\theta = tan^{-1}\left(377 * \frac{0.1}{100}\right) = 0.36\, rad = 20.6\,°$$

$$w\tau = \frac{wL}{R} = 377 * \frac{0.1}{100} = 0.377\, rad$$

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)\, e^{-\omega t/\omega\tau}\right]$$

$$i(wt) = \frac{100}{106.9}\left[sin(wt - 0.36) + sin(0.36)\, e^{-wt/0.377}\right]$$

$$i(wt) = 0.935\left[sin(wt - 0.36) + sin(0.36)\, e^{-wt/0.377}\right]$$

$$i(wt) = 0.935\ sin(wt - 0.36) + 0.35\ e^{-wt/0.377} \quad \text{A} \qquad \text{for } 0 \le wt \le \beta$$

(b) the average current

$$I_o = I_{dc} = \frac{Vm}{2\pi R}(1-\cos\beta) = \frac{100}{2\pi * 100}(1-\cos 201) = 0.3077\ \ A$$ degree

$$I_o = I_{dc} = \frac{Vm}{2\pi R}(1-\cos\beta) = \frac{100}{2\pi * 100}(1-\cos 3.5) = 0.308\ \ \ A$$ rad

(c) **the rms current**

$$V_{rms(load)} = \sqrt{\frac{(Vm)^2}{4\pi}\ [\beta - \frac{1}{2}\sin(2\beta)]}$$

$$V_{rms(load)} = \sqrt{\frac{(100)^2}{4\pi}\left[3.5 - \frac{1}{2}\sin(2 * 3.5)\right]} = 50.23\ \ v$$

$$\boldsymbol{I_{rms(load)}} = \frac{\boldsymbol{V_{rms(load)}}}{\boldsymbol{Z}} = \frac{50.23}{106.9} = 0.47\ A$$

(d) the power absorbed by the RL load

$$P = (I_{rms})^2 * R = (0.47)^2 * 100 = 22.09\ \ w$$

(e) the power factor

$$\boldsymbol{PF} = \frac{\boldsymbol{P}}{\boldsymbol{S}} = \frac{\boldsymbol{P}}{\boldsymbol{V_{s,rms}} * \boldsymbol{I_{rms}}} = \frac{22.09}{\left(\frac{100}{\sqrt{2}}\right) * 0.47} = 0.66$$

3) *R-L Load with Freewheeling Diode (D_{FW}).*

A freewheeling diode can be connected across an RL load as shown in Figure 3-3. The behavior of this circuit is somewhat different from that of the half wave rectifier of Fig. 3-2. The key to the analysis of this circuit is to determine when each diode conducts. First, it is observed that both diodes cannot be forward-biased at the same time. Kirchhoff's voltage law around the path containing the source and the two diodes shows that one diode must be reverse biased. The freewheeling diode also known as a flyback diode, snubber diode.

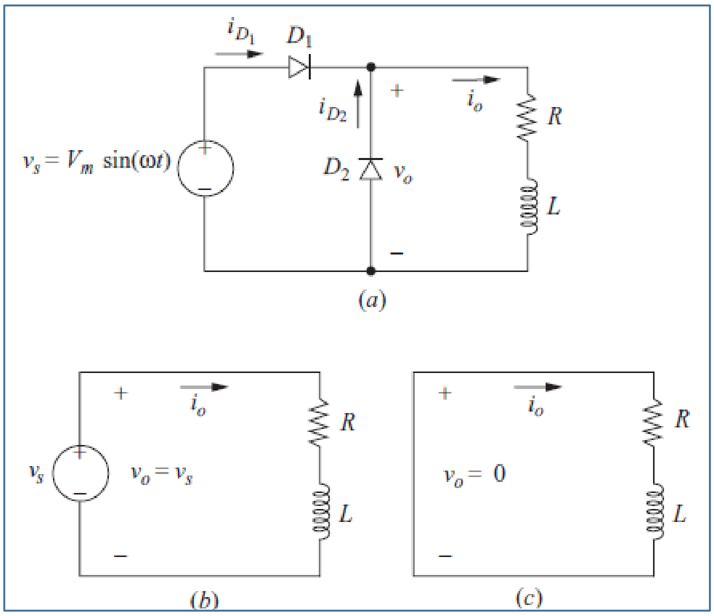


Figure 3-3 *a*) Half-wave rectifier with freewheeling diode; (*b*) Equivalent circuit for *vs* > 0; (*c*) Equivalent circuit for *vs* < 0.

For a positive source voltage,

- *D*1 is on.
- *D*2 is off.
- The equivalent circuit is the same as that of Fig. 3-3 *b*.
- The voltage across the *RL* load is the same as the source.

For a negative source voltage,

- *D*1 is off.
- *D*2 is on.
- The equivalent circuit is the same at that of Fig. 3-3 *c*.
- The voltage across the *RL* load is zero.

The purpose of the freewheeling diode is

1) to **prevent** the output voltage from becoming negative.
2) the energy stored in inductance is transferred to resistance load through the freewheeling diode, which means the system efficiency is improved.
3) to prevent voltage spikes and protect other components in the circuit.

Since the voltage across the load is the same as the source voltage when the source is positive and is zero when the source is negative, the load voltage is a half-wave rectified sine wave.The steady-state current is usually of greater interest than the transient that occurs when the circuit is first energized. Steady-state load and diode currents are shown in Fig. 3-4.

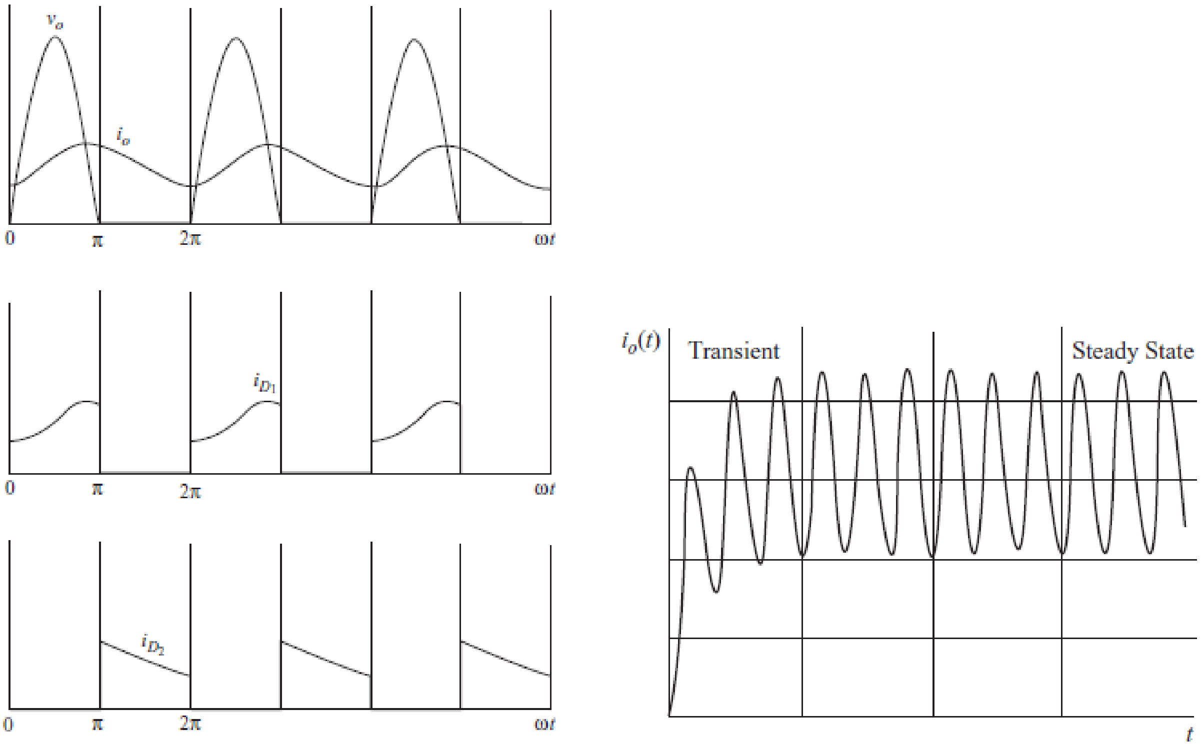


Figure 3-4 Steady-state load voltage and current waveforms with freewheeling diode.

The current in the load can be expressed as a Fourier series by using superposition, taking each frequency separately. The Fourier series for the half-wave rectified sine wave for the voltage across the load is

$$i_o(t) \approx I_o = \frac{V_0}{R} = \frac{V_m}{\pi R}$$

$$v_o(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi} cos(n\omega_0 t)$$

$$where \;\; \mathrm{I}_n = \frac{\mathrm{V}_n}{\mathrm{Z}_n}\,, \qquad \mathrm{Z}_n = |\mathrm{R} + \mathrm{j}n\omega_0\mathrm{L}| = \sqrt{\mathrm{R}^2 + (\mathrm{n}\omega_0 L)^2} \quad , \omega_0 = 2\pi f_0 \;\; rad/sec$$

$$\mathrm{I}_{o,rm} = \sqrt{\sum_n \mathrm{I}_{n,rms}^2} = \sqrt{\mathrm{I}_0^2 + \left(\frac{\mathrm{I}_1}{\sqrt{2}}\right)^2 + \sum_{n=2,4,6,..} \left(\frac{\mathrm{I}_n}{\sqrt{2}}\right)^2}$$

$$\mathrm{P_L} = \mathrm{I}_{o,rms}^2\,\mathrm{R}$$

$$\mathrm{PF} = \frac{P}{S} = \frac{\mathrm{P_L}}{\mathrm{V_{s,rms}\,I_{s,rms}}} = \frac{\mathrm{I}_{o,rms}^2\,\mathrm{R}}{\mathrm{V_{s,rms}\,I_{s,rms}}}$$

$$Vn = \frac{2Vm}{(n^2-1)\pi}$$

Example: For half-wave rectifier with a freewheeling diode, determine the average load voltage and current, and determine the power absorbed by the resistor, where R = 2 Ω and L = 25 mH, Vm is 100V, and the frequency is 60 Hz.

Solution:

$$V_o = \frac{V_m}{\pi} = \frac{100}{\pi} = 31.8 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{31.8}{2} = 15.9 \text{ A}$$

Load power can be determined from $I_{rms}^2 R$, and rms current is determined from the Fourier components of current. The amplitudes of the ac current components are determined from phasor analysis:

$$I_n = \frac{V_n}{Z_n}$$

where $Z_n = |R + jn\omega_0 L| = |2 + jn377(0.025)|$

n=2,4,6,….

$$Z_n = |R + jn\omega_0 L| = \sqrt{R^2 + (n\omega_0 L)^2} \quad , \omega_0 = 2\pi f_0 \ rad/sec$$

$$Z_1 = \sqrt{4 + (1 * 377 * 0.025)^2} = 9.63\,\Omega$$

$$Z_2 = \sqrt{4 + (2 * 377 * 0.025)^2} = 18.96\,\Omega$$

$$Vn = \frac{2Vm}{(n^2 - 1)\pi}$$

$$V_1 = \frac{V_m}{2} = \frac{100}{2} = 50 \text{ V}$$

$$V_2 = \frac{2V_m}{(2^2 - 1)\pi} = 21.2 \text{ V}$$

$$V_4 = \frac{2V_m}{(4^2 - 1)\pi} = 4.24 \text{ V}$$

$$V_6 = \frac{2V_m}{(6^2 - 1)\pi} = 1.82 \text{ V}$$

$$I_1 = \frac{V_1}{Z_1} = \frac{50}{9.63} = 5.19 \quad A$$

$$I_2 = \frac{V_2}{Z_2} = \frac{21.2}{18.96} = 1.12 \quad A$$

The resulting Fourier terms are as follows:

n	V_n (V)	Z_n (Ω)	I_n (A)
0	31.8	2.00	15.9
1	50.0	9.63	5.19
2	21.2	18.96	1.12
4	4.24	37.75	0.11
6	1.82	56.58	0.03

$$I_{o,rm} = \sqrt{\sum_{n} I_{n,rms}^2} = \sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \sum_{n=2,4,6,..} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

$$I_{\text{rms}} = \sqrt{\sum_{k=0}^{\infty} I_{k,\text{rms}}} \approx \sqrt{15.9^2 + \left(\frac{5.19}{\sqrt{2}}\right)^2 + \left(\frac{1.12}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2} = 16.34 \text{ A}$$

Notice that the contribution to rms current from the harmonics decreases as n increases, and higher-order terms are not significant. Power in the resistor is $I_{\text{rms}}^2 R = (16.34)^2 2 = 534$ W.