4.7 Simplification of a Force and Couple System

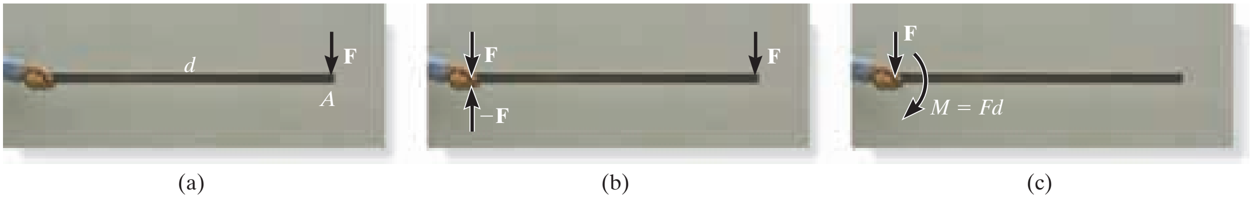
Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an ***equivalent system***, consisting of a single resultant force acting at a specific point and a resultant couple moment. A system is equivalent if the ***external effects*** it produces on a body are the same as those caused by the original force and couple moment system. In this context, the external effects of a system refer to the *translating and rotating motion* of the body if the body is free to move, or it refers to the *reactive forces* at the supports if the body is held fixed.

For example, consider holding the stick in Fig. 4–34*a*, which is subjected to the force **F** at point *A*. If we attach a pair of equal but opposite forces **F** and −**F** at point *B*, which is *on the line of action* of **F**, Fig. 4–34*b*, we observe that −**F** at *B* and **F** at *A* will cancel each other, leaving only **F** at *B*, Fig. 4–34*c*. Force **F** has now been moved from *A* to *B* without modifying its *external effects* on the stick; i.e., the reaction at the grip remains the same. This demonstrates the ***principle of transmissibility***, which states that a force acting on a body (stick) is a ***sliding vector*** since it can be applied at any point along its line of action.

We can also use the above procedure to move a force to a point that is *not* on the line of action of the force. If **F** is applied perpendicular to the stick, as in Fig. 4–35*a*, then we can attach a pair of equal but opposite forces **F** and −**F** to *B*, Fig. 4–35*b*. Force **F** is now applied at *B*, and the other two forces, **F** at *A* and −**F** at *B*, form a couple that produces the couple moment $M = Fd$, Fig. 4–35*c*. Therefore, the force **F** can be moved from *A* to *B* provided a couple moment **M** is added to maintain an equivalent system. This couple moment is determined by taking the moment of **F** about *B*. Since **M** is actually a *free vector*, it can act at any point on the stick. In both cases the systems are equivalent, which causes a downward force **F** and clockwise couple moment $M = Fd$ to be felt at the grip.

(a) (b) (c)

Fig. 4–34 (© Russell C. Hibbeler)



(a) (b) (c)

Fig. 4–35 (© Russell C. Hibbeler)

System of Forces and Couple Moments. Using the above method, a system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force acting at a point O and a resultant couple moment. For example, in Fig. 4–36*a*, O is not on the line of action of $\mathbf{F}_1$, and so this force can be moved to point O provided a couple moment $(\mathbf{M}_O)_1 = \mathbf{r}_1 \times \mathbf{F}$ is added to the body. Similarly, the couple moment $(\mathbf{M}_O)_2 = \mathbf{r}_2 \times \mathbf{F}_2$ should be added to the body when we move $\mathbf{F}_2$ to point O. Finally, since the couple moment $\mathbf{M}$ is a free vector, it can just be moved to point O. By doing this, we obtain the equivalent system shown in Fig. 4–36*b*, which produces the same external effects (support reactions) on the body as that of the force and couple system shown in Fig. 4–36*a*. If we sum the forces and couple moments, we obtain the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and the resultant couple moment $(\mathbf{M}_R)_O = \mathbf{M} + (\mathbf{M}_O)_1 + (\mathbf{M}_O)_2$, Fig. 4–36*c*.

Notice that $\mathbf{F}_R$ is independent of the location of point O since it is simply a summation of the forces. However, $(\mathbf{M}_R)_O$ depends upon this location since the moments $\mathbf{M}_1$ and $\mathbf{M}_2$ are determined using the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, which extend from O to each force. Also note that $(\mathbf{M}_R)_O$ is a free vector and can act at *any point* on the body, although point O is generally chosen as its point of application.

We can generalize the above method of reducing a force and couple system to an equivalent resultant force $\mathbf{F}_R$ acting at point O and a resultant couple moment $(\mathbf{M}_R)_O$ by using the following two equations.

$$\boxed{\begin{aligned}\mathbf{F}_R &= \Sigma\mathbf{F}\\ (\mathbf{M}_R)_O &= \Sigma\mathbf{M}_O + \Sigma\mathbf{M}\end{aligned}} \tag{4–17}$$

The first equation states that the resultant force of the system is equivalent to the sum of all the forces; and the second equation states that the resultant couple moment of the system is equivalent to the sum of all the couple moments $\Sigma\mathbf{M}$ plus the moments of all the forces $\Sigma\mathbf{M}_O$ about point O. If the force system lies in the x–y plane and any couple moments are perpendicular to this plane, then the above equations reduce to the following three scalar equations.

$$\boxed{\begin{aligned}(F_R)_x &= \Sigma F_x\\ (F_R)_y &= \Sigma F_y\\ (M_R)_O &= \Sigma M_O + \Sigma M\end{aligned}} \tag{4–18}$$

Here the resultant force is determined from the vector sum of its two components $(F_R)_x$ and $(F_R)_y$.

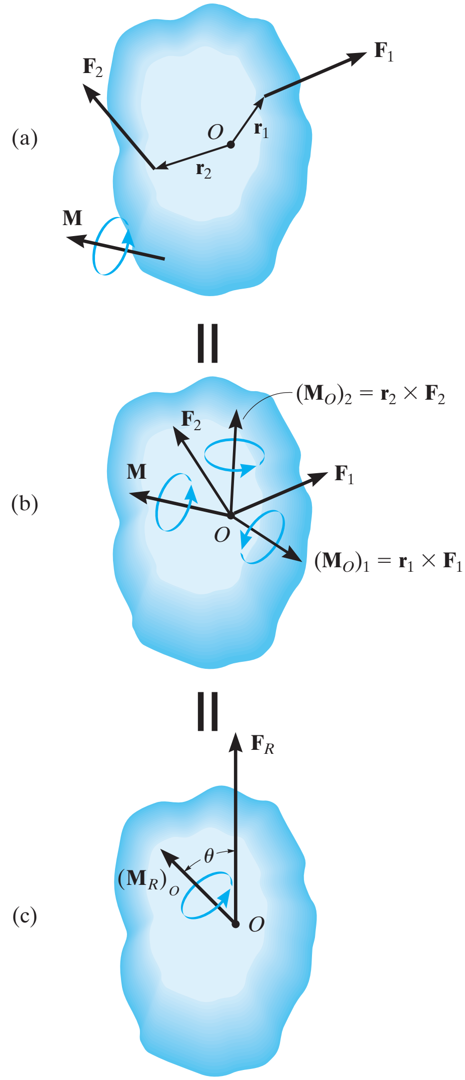
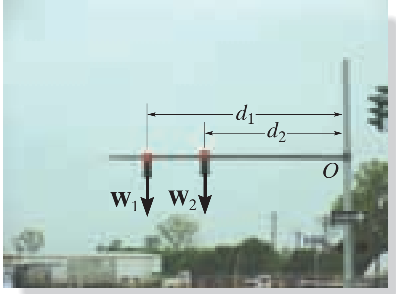
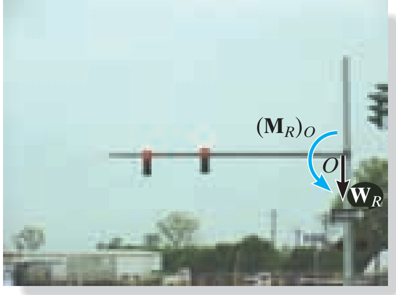


Fig. 4–36





The weights of these traffic lights can be replaced by their equivalent resultant force $W_R = W_1 + W_2$ and a couple moment $(M_R)_O = W_1 d_1 + W_2 d_2$ at the support, O. In both cases the support must provide the same resistance to translation and rotation in order to keep the member in the horizontal position. (© Russell C. Hibbeler)

Important Points

- Force is a sliding vector, since it will create the same external effects on a body when it is applied at any point P along its line of action. This is called the principle of transmissibility.

- A couple moment is a free vector since it will create the same external effects on a body when it is applied at any point P on the body.

- When a force is moved to another point P that is not on its line of action, it will create the same external effects on the body if a couple moment is also applied to the body. The couple moment is determined by taking the moment of the force about point P.

Procedure for Analysis

The following points should be kept in mind when simplifying a force and couple moment system to an equivalent resultant force and couple system.

- Establish the coordinate axes with the origin located at point O and the axes having a selected orientation.

Force Summation.

- If the force system is *coplanar*, resolve each force into its x and y components. If a component is directed along the positive x or y axis, it represents a positive scalar; whereas if it is directed along the negative x or y axis, it is a negative scalar.

- In three dimensions, represent each force as a Cartesian vector before summing the forces.

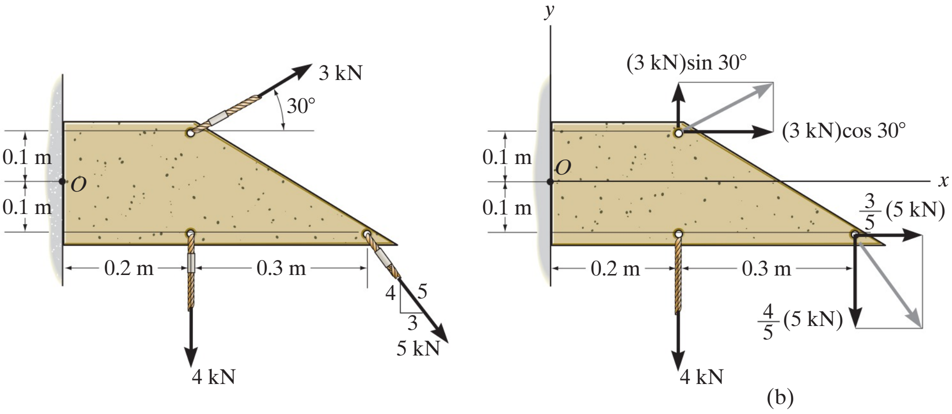
Moment Summation.

- When determining the moments of a *coplanar* force system about point O, it is generally advantageous to use the principle of moments, i.e., determine the moments of the components of each force, rather than the moment of the force itself.

- In three dimensions use the vector cross product to determine the moment of each force about point O. Here the position vectors extend from O to any point on the line of action of each force.

EXAMPLE 4.14

Replace the force and couple system shown in Fig. 4–37*a* by an equivalent resultant force and couple moment acting at point *O*.



SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their *x* and *y* components as shown in Fig. 4–37*b*. We have

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37*c*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN} \qquad \textit{Ans.}$$

Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3^\circ \qquad \textit{Ans.}$$

Moment Summation. The moments of 3 kN and 5 kN about point *O* will be determined using their *x* and *y* components. Referring to Fig. 4–37*b*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3\text{ kN})\sin 30^\circ(0.2\text{ m}) - (3\text{ kN})\cos 30^\circ(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})\,(0.1\text{ m})$$
$$- \left(\tfrac{4}{5}\right)(5\text{ kN})\,(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$
$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–37*c*.

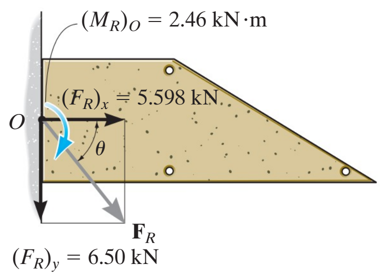


Fig. 4–37

NOTE: Realize that the resultant force and couple moment in Fig. 4–37*c* will produce the same external effects or reactions at the supports as those produced by the force system, Fig. 4–37*a*.

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point O.

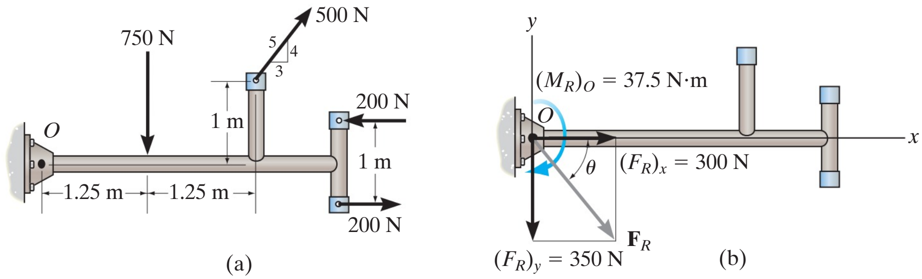


Fig. 4–38

SOLUTION

Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its x and y components, thus,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \;\; (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \;\; (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N} \qquad \textit{Ans.}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ \qquad \textit{Ans.}$$

Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 4–38*a*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

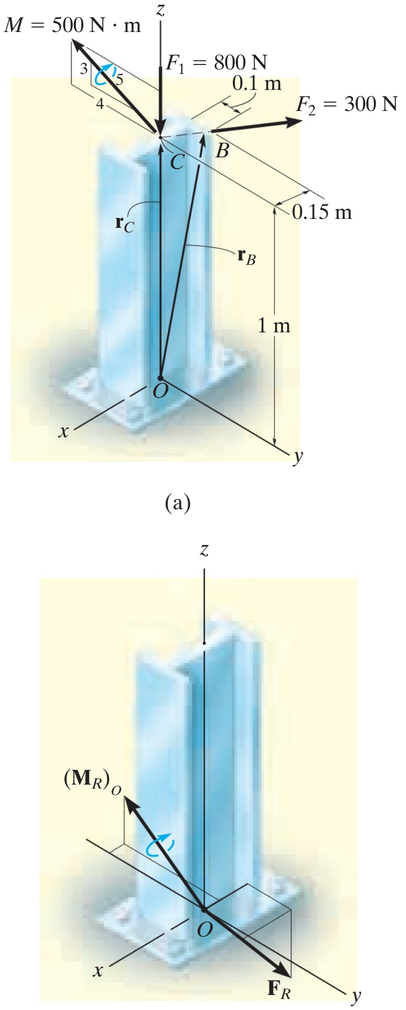
$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–38*b*.

EXAMPLE 4.16

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.



(a)

(b)

Fig. 4–39

SOLUTION *(VECTOR ANALYSIS)*

The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have

$$\mathbf{F}_1 = \{-800\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = (300\text{ N})\mathbf{u}_{CB}$$

$$= (300\text{ N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

$$= 300\text{ N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\}\text{ m}}{\sqrt{(-0.15\text{ m})^2 + (0.1\text{ m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\}\text{ N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\}\text{ N}\cdot\text{m}$$

Force Summation.

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = -800\mathbf{k} - 249.6\mathbf{i} + 166.4\mathbf{j}$$

$$= \{-250\mathbf{i} + 166\mathbf{j} - 800\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

Moment Summation.

$$(\mathbf{M}_R)_o = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

$$(\mathbf{M}_R)_o = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$(\mathbf{M}_R)_o = (-400\mathbf{j} + 300\mathbf{k}) + (1\mathbf{k}) \times (-800\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.1 & 1 \\ -249.6 & 166.4 & 0 \end{vmatrix}$$

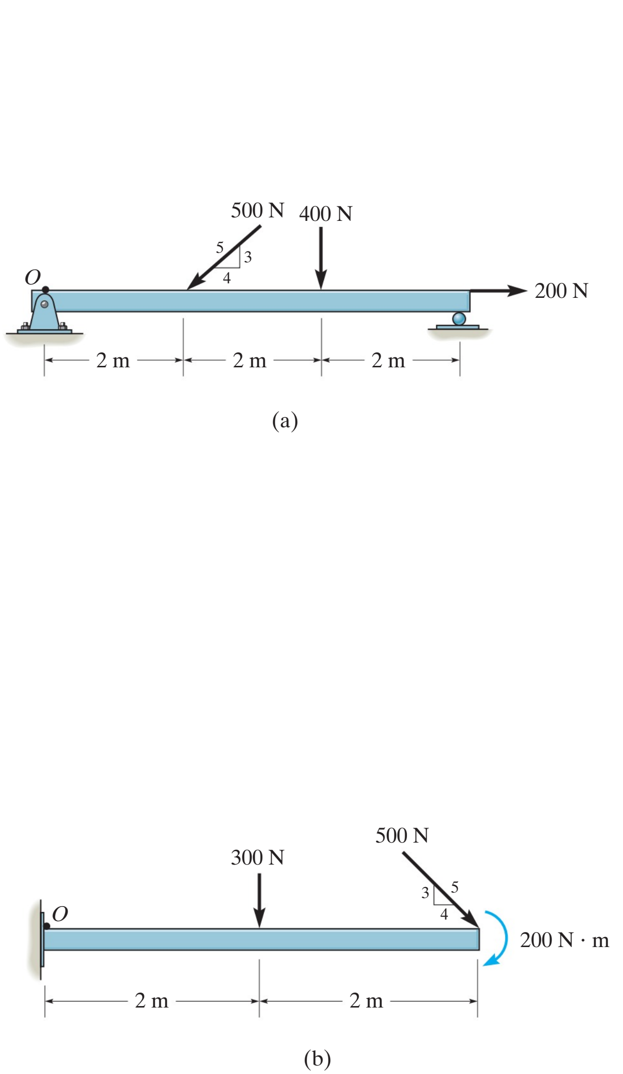
$$= (-400\mathbf{j} + 300\mathbf{k}) + (0) + (-166.4\mathbf{i} - 249.6\mathbf{j})$$

$$= \{-166\mathbf{i} - 650\mathbf{j} + 300\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The results are shown in Fig. 4–39*b*.

PRELIMINARY PROBLEM

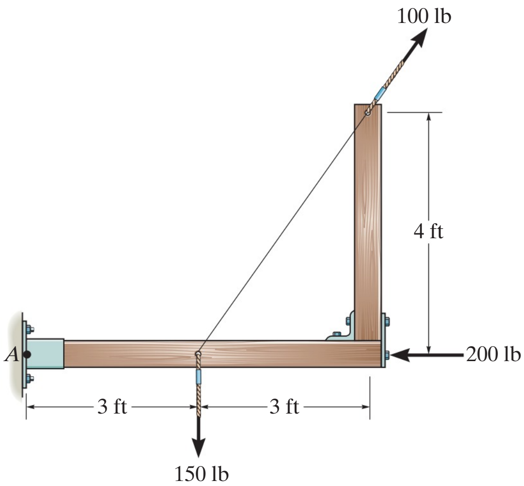
P4–5. In each case, determine the x and y components of the resultant force and the resultant couple moment at point O.



Prob. P4–5

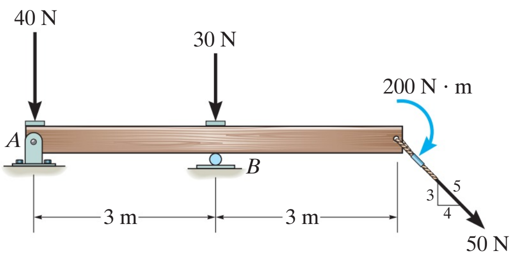
FUNDAMENTAL PROBLEMS

F4–25. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



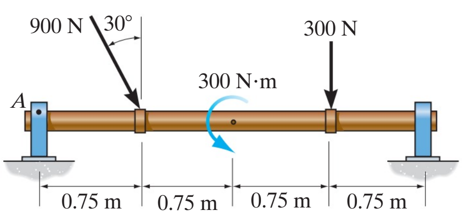
Prob. F4–25

F4–26. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



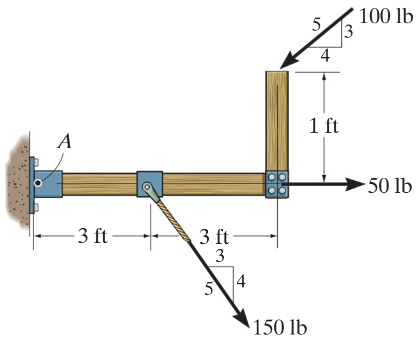
Prob. F4–26

F4–27. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



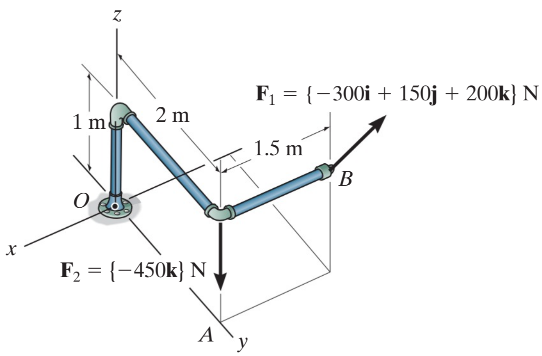
Prob. F4–27

F4–28. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



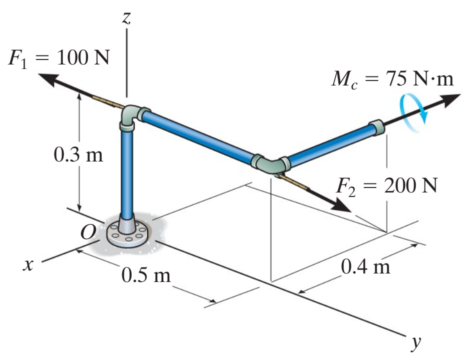
Prob. F4–28

F4–29. Replace the loading system by an equivalent resultant force and couple moment acting at point O.



Prob. F4–29

F4–30. Replace the loading system by an equivalent resultant force and couple moment acting at point O.

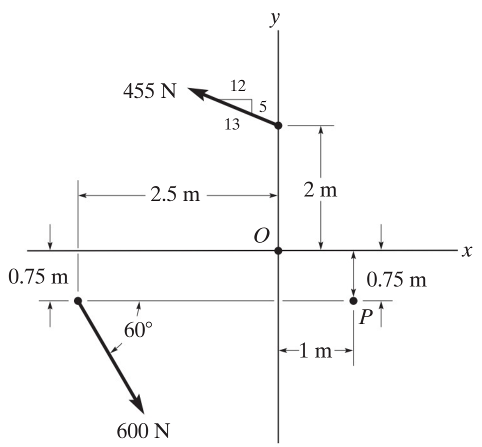


Prob. F4–30

FUNDAMENTAL PROBLEMS

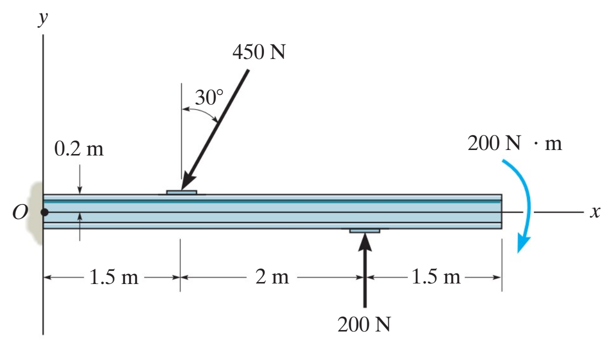
4–97. Replace the force system by an equivalent resultant force and couple moment at point O.

4–98. Replace the force system by an equivalent resultant force and couple moment at point P.



Probs. 4–97/98

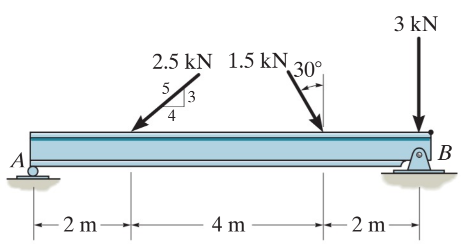
4–101. Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.



Prob. 4–101

4–99. Replace the force system acting on the beam by an equivalent force and couple moment at point A.

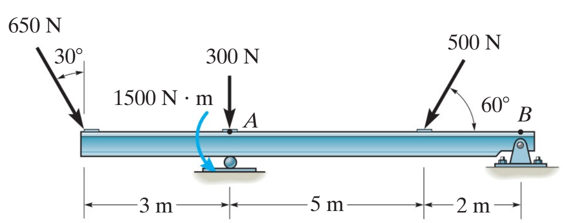
***4–100.** Replace the force system acting on the beam by an equivalent force and couple moment at point B.



Probs. 4–99/100

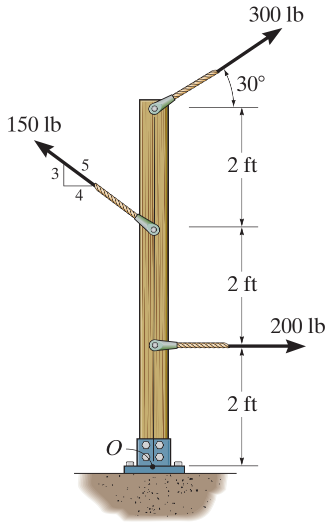
4–102. Replace the loading system acting on the post by an equivalent resultant force and couple moment at point A.

4–103. Replace the loading system acting on the post by an equivalent resultant force and couple moment at point B.



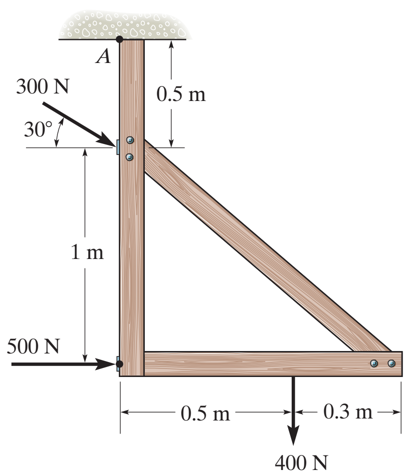
Probs. 4–102/103

***4–104.** Replace the force system acting on the post by a resultant force and couple moment at point O.



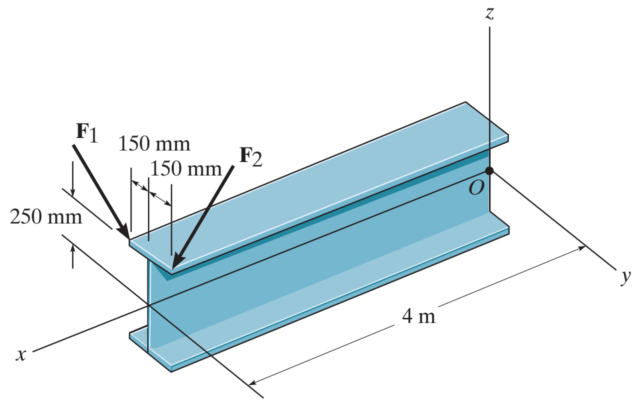
Prob. 4–104

4–105. Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.



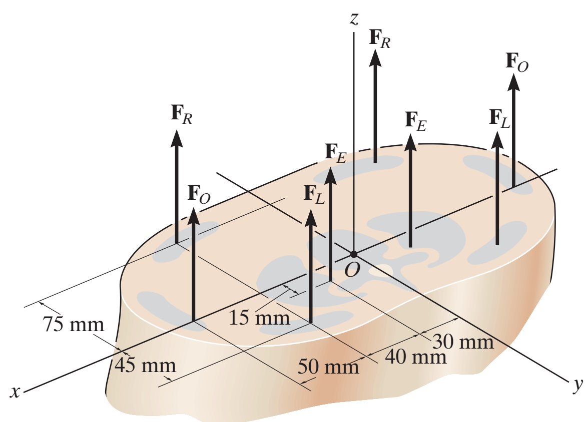
Prob. 4–105

4–106. The forces $\mathbf{F}_1 = \{-4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ kN and $\mathbf{F}_2 = \{3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}\}$ kN act on the end of the beam. Replace these forces by an equivalent force and couple moment acting at point O.



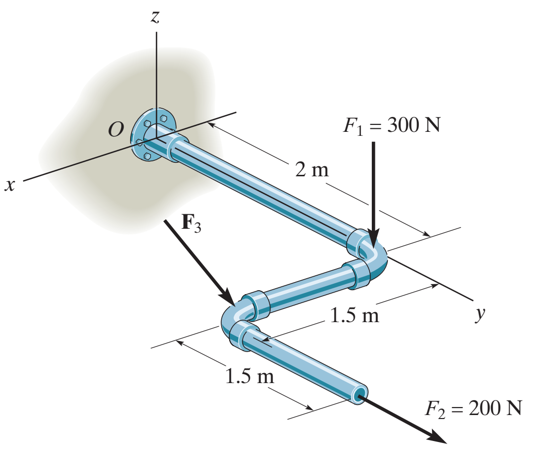
Prob. 4–106

4–107. A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y–z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.



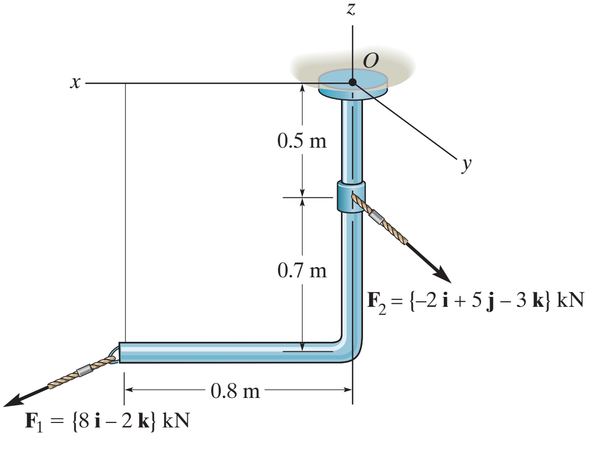
Prob. 4–107

***4–108.** Replace the force system by an equivalent resultant force and couple moment at point O. Take $\mathbf{F}_3 = \{-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k}\}$ N.



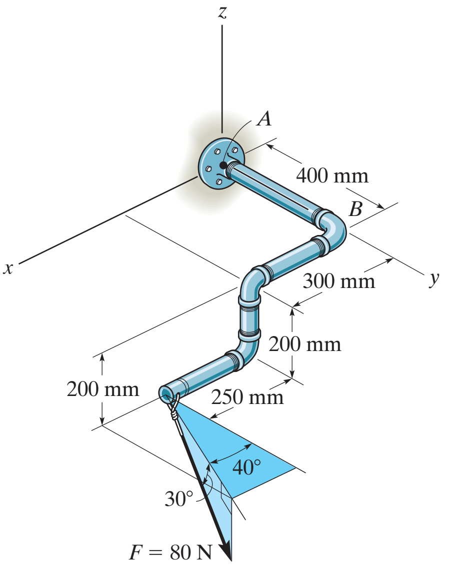
Prob. 4–108

4–109. Replace the loading by an equivalent resultant force and couple moment at point O.



Prob. 4–109

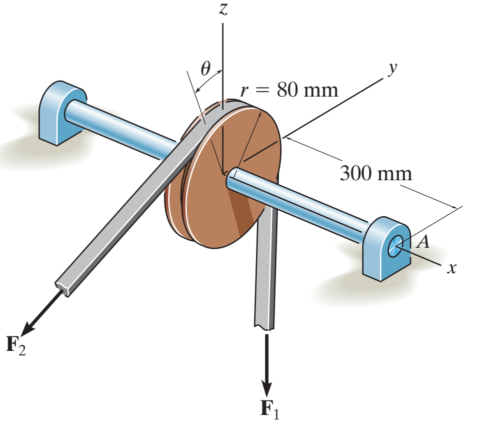
4–110. Replace the force of $F = 80$ N acting on the pipe assembly by an equivalent resultant force and couple moment at point A.



Prob. 4–110

4–111. The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.

***4–112.** The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



Probs. 4–111/112

4.8 Further Simplification of a Force and Couple System

In the preceding section, we developed a way to reduce a force and couple moment system acting on a rigid body into an equivalent resultant force $\mathbf{F}_R$ acting at a specific point O and a resultant couple moment $(\mathbf{M}_R)_O$. The force system can be further reduced to an equivalent single resultant force provided the lines of action of $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are *perpendicular* to each other. Because of this condition, concurrent, coplanar, and parallel force systems can be further simplified.

Concurrent Force System. Since a ***concurrent force system*** is one in which the lines of action of all the forces intersect at a common point O, Fig. 4–40*a*, then the force system produces no moment about this point. As a result, the equivalent system can be represented by a single resultant force $\mathbf{F}_R = \Sigma\mathbf{F}$ acting at O, Fig. 4–40*b*.

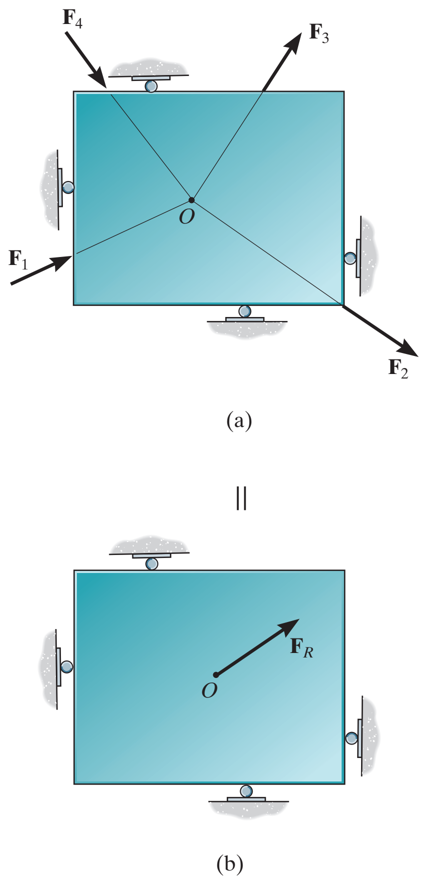


Fig. 4–40

Coplanar Force System. In the case of a ***coplanar force system***, the lines of action of all the forces lie in the same plane, Fig. 4–41*a*, and so the resultant force $\mathbf{F}_R = \Sigma\mathbf{F}$ of this system also lies in this plane. Furthermore, the moment of each of the forces about any point O is directed perpendicular to this plane. Thus, the resultant moment $(\mathbf{M}_R)_O$ and resultant force $\mathbf{F}_R$ will be *mutually perpendicular*, Fig. 4–41*b*. The resultant moment can be replaced by moving the resultant force $\mathbf{F}_R$ a perpendicular or moment arm distance d away from point O such that $\mathbf{F}_R$ produces the *same moment* $(\mathbf{M}_R)_O$ about point O, Fig. 4–41*c*. This distance d can be determined from the scalar equation $(M_R)_O = F_R d = \Sigma M_O$ or $d = (M_R)_O/F_R$.

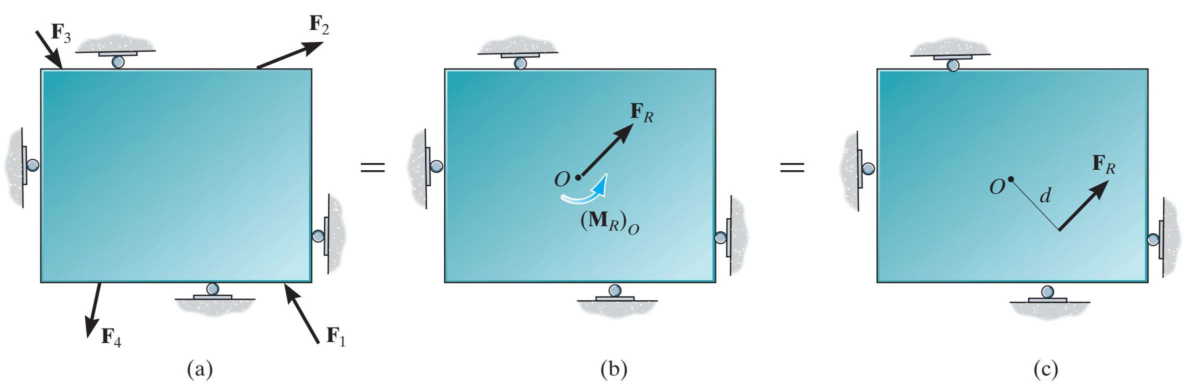


Fig. 4–41

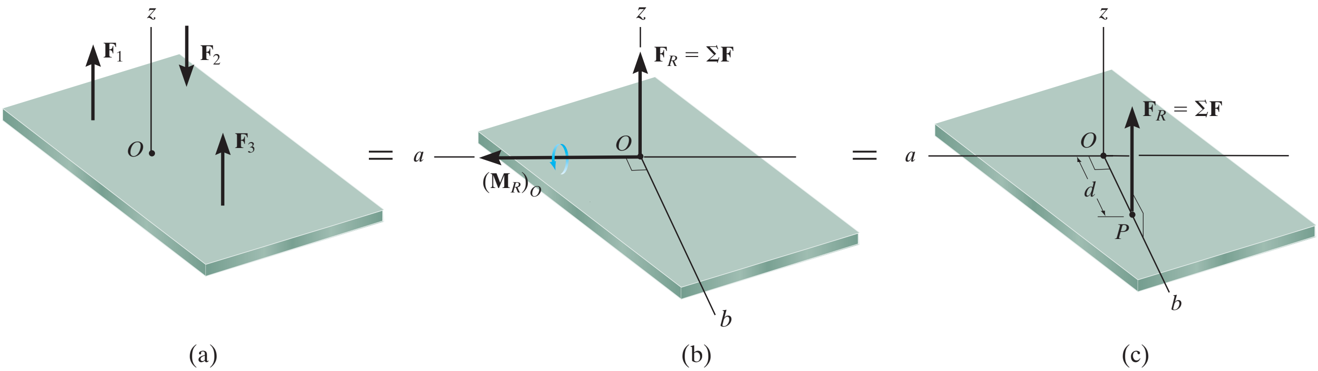


Fig. 4–42

Parallel Force System. The ***parallel force system*** shown in Fig. 4–42*a* consists of forces that are all parallel to the z axis. Thus, the resultant force $\mathbf{F}_R = \Sigma\mathbf{F}$ at point O must also be parallel to this axis, Fig. 4–42*b*. The moment produced by each force lies in the plane of the plate, and so the resultant couple moment, $(\mathbf{M}_R)_O$, will also lie in this plane, along the moment axis a since $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are mutually perpendicular. As a result, the force system can be further reduced to an equivalent single resultant force $\mathbf{F}_R$, acting through point P located on the perpendicular b axis, Fig. 4–42*c*. The distance d along this axis from point O requires $(M_R)_O = F_R d = \Sigma M_O$ or $d = \Sigma M_O / F_R$.

Procedure for Analysis

The technique used to reduce a coplanar or parallel force system to a single resultant force follows a similar procedure outlined in the previous section.

- Establish the x, y, z, axes and locate the resultant force $\mathbf{F}_R$ an arbitrary distance away from the origin of the coordinates.

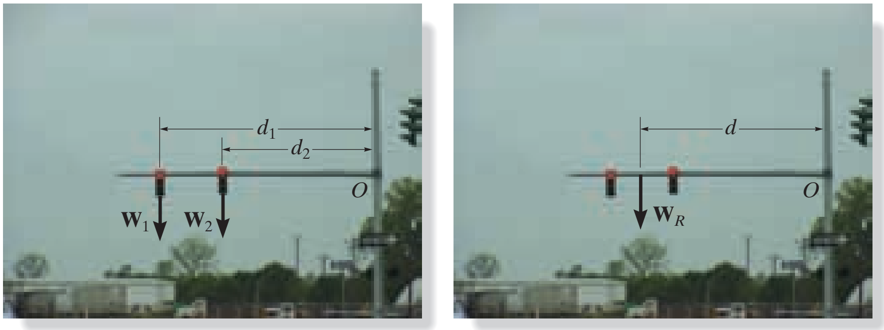
Force Summation.

- The resultant force is equal to the sum of all the forces in the system.
- For a coplanar force system, resolve each force into its x and y components. Positive components are directed along the positive x and y axes, and negative components are directed along the negative x and y axes.

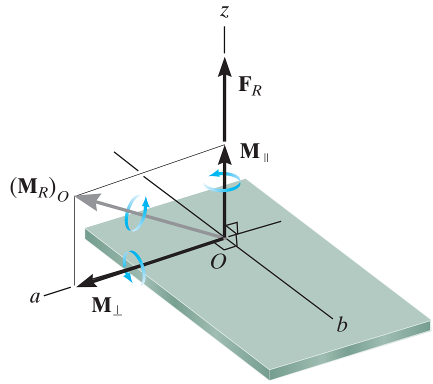
Moment Summation.

- The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O.
- This moment condition is used to find the location of the resultant force from point O.

The four cable forces are all concurrent at point O on this bridge tower. Consequently they produce no resultant moment there, only a resultant force $\mathbf{F}_R$. Note that the designers have positioned the cables so that $\mathbf{F}_R$ is directed *along* the bridge tower directly to the support, so that it does not cause any bending of the tower. (© Russell C. Hibbeler)

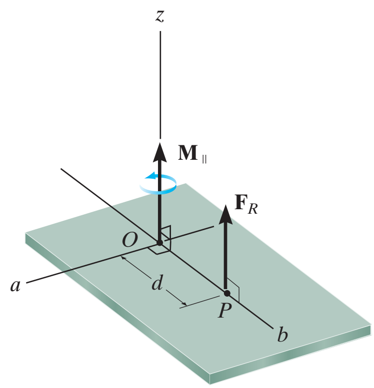


Here the weights of the traffic lights are replaced by their resultant force $W_R = W_1 + W_2$ which acts at a distance $d = (W_1 d_1 + W_2 d_2)/W_R$ from O. Both systems are equivalent. (© Russell C. Hibbeler)



(a)

||



(b)

||

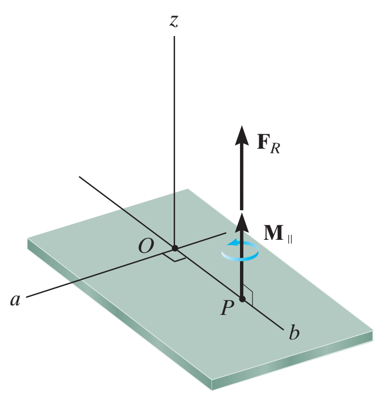


Fig. 4–43

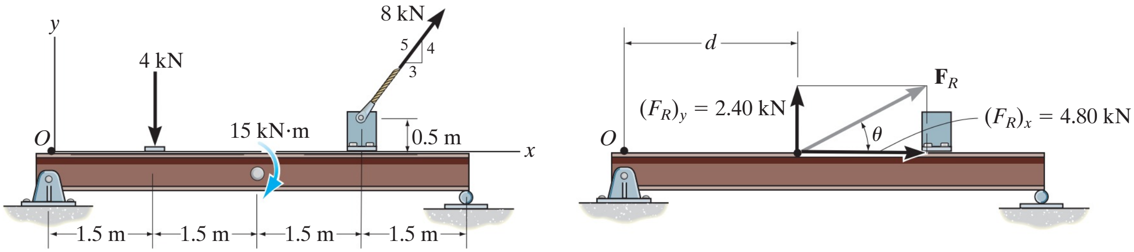
Reduction to a Wrench. In general, a three-dimensional force and couple moment system will have an equivalent resultant force $\mathbf{F}_R$ acting at point O and a resultant couple moment $(\mathbf{M}_R)_O$ that are *not perpendicular* to one another, as shown in Fig. 4–43*a*. Although a force system such as this cannot be further reduced to an equivalent single resultant force, the resultant couple moment $(\mathbf{M}_R)_O$ can be resolved into components parallel and perpendicular to the line of action of $\mathbf{F}_R$, Fig. 4–43*a*. If this appears difficult to do in three dimensions, use the dot product to get $\mathbf{M}_{\parallel} = (\mathbf{M}_R) \cdot \mathbf{u}_{F_R}$ and then $\mathbf{M}_{\perp} = \mathbf{M}_R - \mathbf{M}_{\parallel}$. The perpendicular component $\mathbf{M}_{\perp}$ can be replaced if we move $\mathbf{F}_R$ to point P, a distance d from point O along the b axis, Fig. 4–43*b*. As shown, this axis is perpendicular to both the a axis and the line of action of $\mathbf{F}_R$. The location of P can be determined from $d = M_{\perp}/F_R$. Finally, because $\mathbf{M}_{\parallel}$ is a free vector, it can be moved to point P, Fig. 4–43*c*. This combination of a resultant force $\mathbf{F}_R$ and collinear couple moment $\mathbf{M}_{\parallel}$ will tend to translate and rotate the body about its axis and is referred to as a ***wrench*** or ***screw.*** A wrench is the simplest system that can represent any general force and couple moment system acting on a body.

Important Point

- A concurrent, coplanar, or parallel force system can always be reduced to a single resultant force acting at a specific point P. For any other type of force system, the simplest reduction is a wrench, which consists of resultant force and collinear couple moment acting at a specific point P.

EXAMPLE 4.17

Replace the force and couple moment system acting on the beam in Fig. 4–44*a* by an equivalent resultant force, and find where its line of action intersects the beam, measured from point *O*.



(a) (b)

Fig. 4–44

SOLUTION

Force Summation. Summing the force components,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN} \qquad \textit{Ans.}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ \qquad \textit{Ans.}$$

Moment Summation. We must equate the moment of $\mathbf{F}_R$ about point *O* in Fig. 4–44*b* to the sum of the moments of the force and couple moment system about point *O* in Fig. 4–44*a*. Since the line of action of $(\mathbf{F}_R)_x$ acts through point *O*, *only* $(\mathbf{F}_R)_y$ *produces a moment about this point*. Thus,

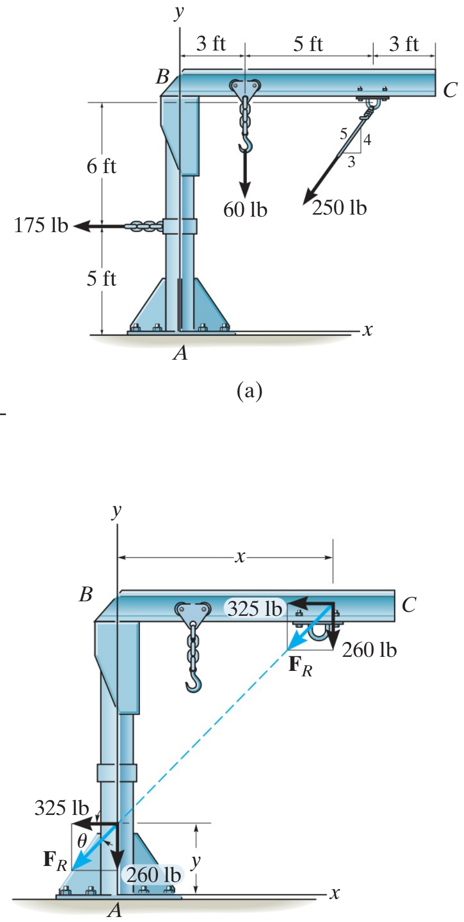
$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m} \qquad \textit{Ans.}$$

EXAMPLE 4.18

The jib crane shown in Fig. 4–45*a* is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column *AB* and boom *BC*.



(a)

(b)

Fig. 4–45

SOLUTION

Force Summation. Resolving the 250-lb force into x and y components and summing the force components yields

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = -250\text{ lb}\left(\tfrac{3}{5}\right) - 175\text{ lb} = -325\text{ lb} = 325\text{ lb} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -250\text{ lb}\left(\tfrac{4}{5}\right) - 60\text{ lb} = -260\text{ lb} = 260\text{ lb}\downarrow$$

As shown by the vector addition in Fig. 4–45*b*,

$$F_R = \sqrt{(325\text{ lb})^2 + (260\text{ lb})^2} = 416\text{ lb} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{260\text{ lb}}{325\text{ lb}}\right) = 38.7° \qquad \textit{Ans.}$$

Moment Summation. Moments will be summed about point A. Assuming the line of action of $\mathbf{F}_R$ *intersects AB* at a distance y from A, Fig. 4–45*b*, we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(y) + 260\text{ lb }(0)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$y = 2.29\text{ ft} \qquad \textit{Ans.}$$

By the principle of transmissibility, $\mathbf{F}_R$ can be placed at a distance x where it intersects BC, Fig. 4–45*b*. In this case we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(11\text{ ft}) - 260\text{ lb }(x)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$x = 10.9\text{ ft} \qquad \textit{Ans.}$$

EXAMPLE 4.19

The slab in Fig. 4–46*a* is subjected to four parallel forces. Determine the magnitude and direction of a resultant force equivalent to the given force system, and locate its point of application on the slab.

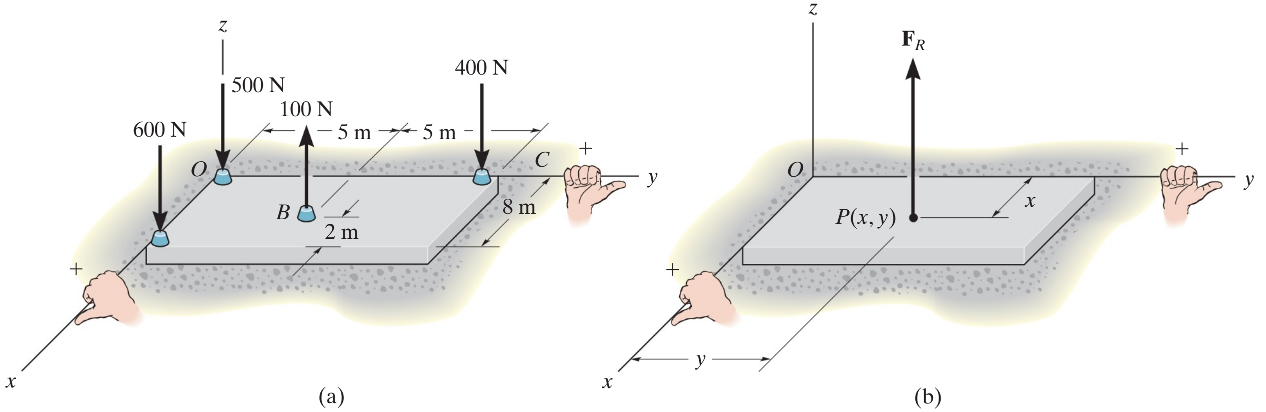


Fig. 4–46

SOLUTION (*SCALAR ANALYSIS*)

Force Summation. From Fig. 4–46*a*, the resultant force is

$$+\uparrow F_R = \Sigma F; \qquad F_R = -600\text{ N} + 100\text{ N} - 400\text{ N} - 500\text{ N}$$
$$= -1400\text{ N} = 1400\text{ N}\downarrow \qquad \textit{Ans.}$$

Moment Summation. We require the moment about the x axis of the resultant force, Fig. 4–46*b*, to be equal to the sum of the moments about the x axis of all the forces in the system, Fig. 4–46*a*. The moment arms are determined from the y coordinates, since these coordinates represent the *perpendicular distances* from the x axis to the lines of action of the forces. Using the right-hand rule, we have

$$(M_R)_x = \Sigma M_x;$$
$$-(1400\text{ N})y = 600\text{ N}(0) + 100\text{ N}(5\text{ m}) - 400\text{ N}(10\text{ m}) + 500\text{ N}(0)$$
$$-1400y = -3500 \qquad y = 2.50\text{ m} \qquad \textit{Ans.}$$

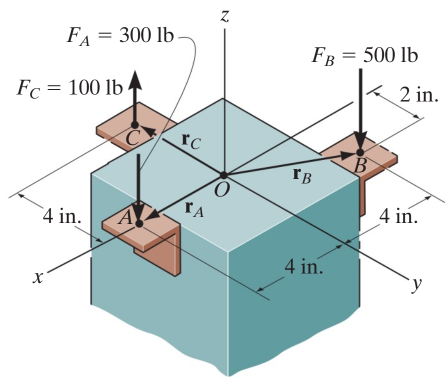
In a similar manner, a moment equation can be written about the y axis using moment arms defined by the x coordinates of each force.

$$(M_R)_y = \Sigma M_y;$$
$$(1400\text{ N})x = 600\text{ N}(8\text{ m}) - 100\text{ N}(6\text{ m}) + 400\text{ N}(0) + 500\text{ N}(0)$$
$$1400x = 4200$$
$$x = 3\text{ m} \qquad \textit{Ans.}$$

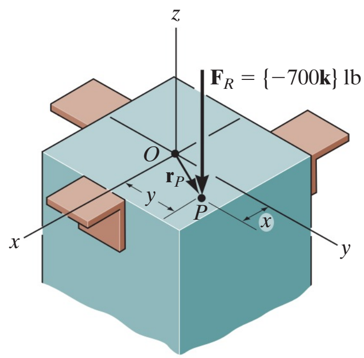
NOTE: A force of $F_R = 1400$ N placed at point $P(3.00\text{ m}, 2.50\text{ m})$ on the slab, Fig. 4–46*b*, is therefore equivalent to the parallel force system acting on the slab in Fig. 4–46*a*.

EXAMPLE 4.20

Replace the force system in Fig. 4–47*a* by an equivalent resultant force and specify its point of application on the pedestal.



(a)



(b)

Fig. 4–47

SOLUTION

Force Summation. Here we will demonstrate a vector analysis. Summing forces,

$$\mathbf{F}_R = \Sigma\mathbf{F};\ \mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$
$$= \{-300\mathbf{k}\}\text{ lb} + \{-500\mathbf{k}\}\text{ lb} + \{100\mathbf{k}\}\text{ lb}$$
$$= \{-700\mathbf{k}\}\text{ lb} \qquad \textit{Ans.}$$

Location. Moments will be summed about point O. The resultant force $\mathbf{F}_R$ is assumed to act through point $P\,(x, y, 0)$, Fig. 4–47*b*. Thus

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;$$
$$\mathbf{r}_P \times \mathbf{F}_R = (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) + (\mathbf{r}_C \times \mathbf{F}_C)$$
$$(x\mathbf{i} + y\mathbf{j}) \times (-700\mathbf{k}) = [(4\mathbf{i}) \times (-300\mathbf{k})]$$
$$+ [(-4\mathbf{i} + 2\mathbf{j}) \times (-500\mathbf{k})] + [(-4\mathbf{j}) \times (100\mathbf{k})]$$
$$-700x(\mathbf{i} \times \mathbf{k}) - 700y(\mathbf{j} \times \mathbf{k}) = -1200(\mathbf{i} \times \mathbf{k}) + 2000(\mathbf{i} \times \mathbf{k})$$
$$- 1000(\mathbf{j} \times \mathbf{k}) - 400(\mathbf{j} \times \mathbf{k})$$
$$700x\mathbf{j} - 700y\mathbf{i} = 1200\mathbf{j} - 2000\mathbf{j} - 1000\mathbf{i} - 400\mathbf{i}$$

Equating the **i** and **j** components,

$$-700y = -1400 \qquad (1)$$
$$y = 2\text{ in.} \qquad \textit{Ans.}$$
$$700x = -800 \qquad (2)$$
$$x = -1.14\text{ in.} \qquad \textit{Ans.}$$

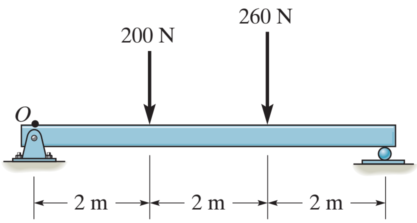
The negative sign indicates that the x coordinate of point P is negative.

NOTE: It is also possible to establish Eq. 1 and 2 directly by summing moments about the x and y axes. Using the right-hand rule, we have

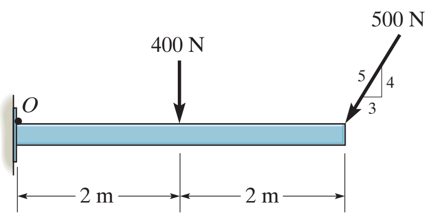
$$(M_R)_x = \Sigma M_x; \qquad -700y = -100\text{ lb}(4\text{ in.}) - 500\text{ lb}(2\text{ in.})$$
$$(M_R)_y = \Sigma M_y; \qquad 700x = 300\text{ lb}(4\text{ in.}) - 500\text{ lb}(4\text{ in.})$$

PRELIMINARY PROBLEMS

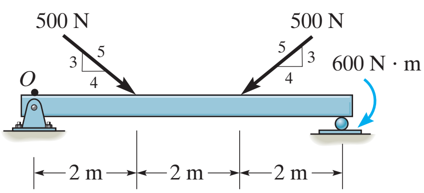
P4–6. In each case, determine the x and y components of the resultant force and specify the distance where this force acts from point O.



(a)



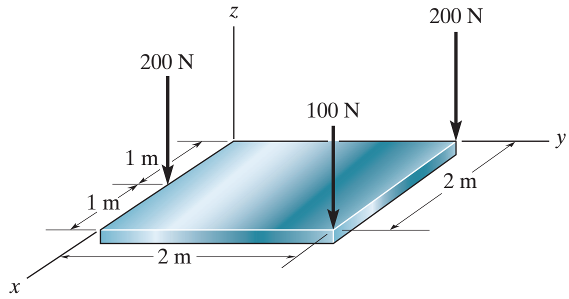
(b)



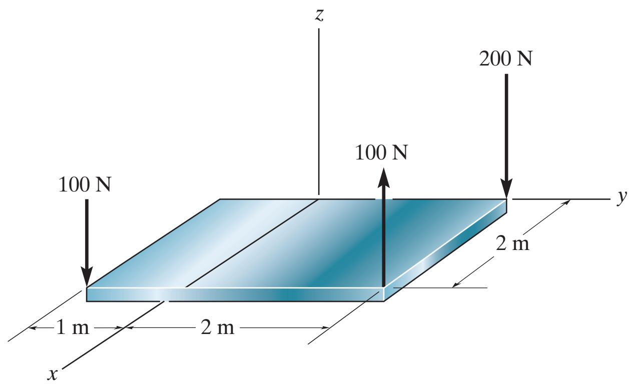
(c)

Prob. P4–6

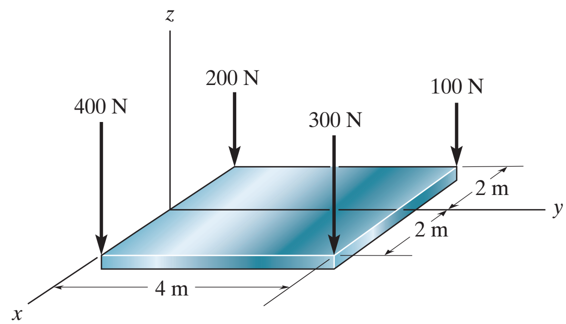
P4–7. In each case, determine the resultant force and specify its coordinates x and y where it acts on the x–y plane.



(a)



(b)

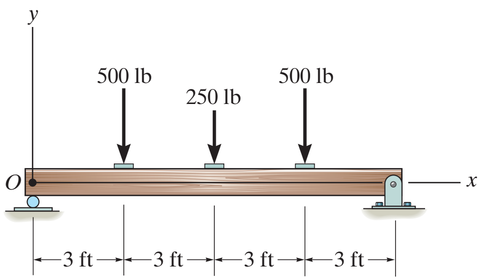


(c)

Prob. P4–7

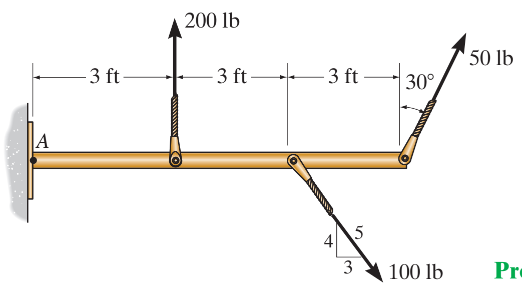
FUNDAMENTAL PROBLEMS

F4–31. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O.



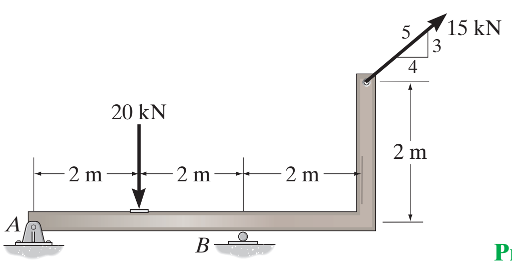
Prob. F4–31

F4–32. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from A.



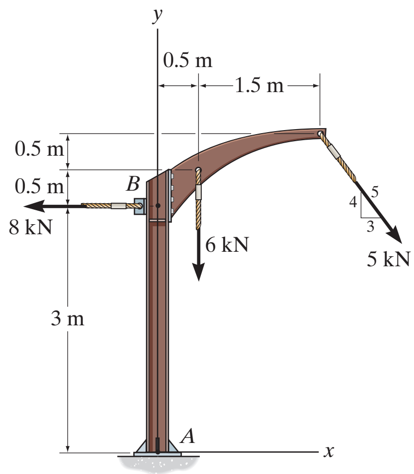
Prob. F4–32

F4–33. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the horizontal segment of the member measured from A.



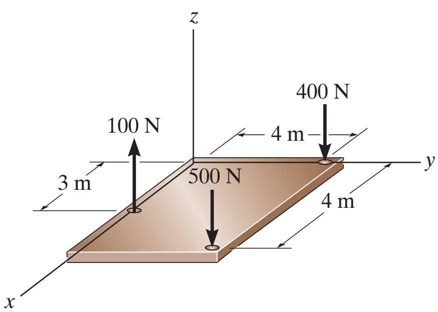
Prob. F4–33

F4–34. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member AB measured from A.



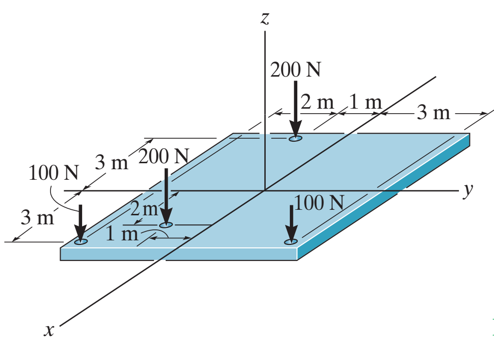
Prob. F4–34

F4–35. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



Prob. F4–35

F4–36. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.

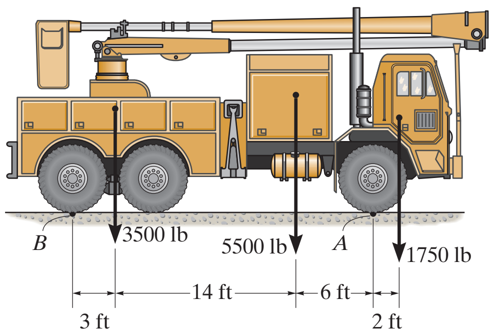


Prob. F4–36

PROBLEMS

4–113. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from B.

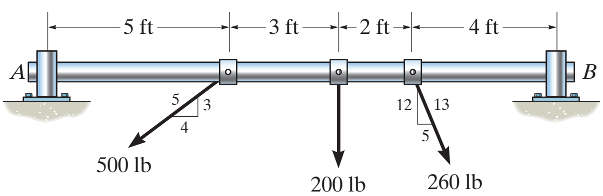
4–114. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.



Probs. 4–113/114

4–115. Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.

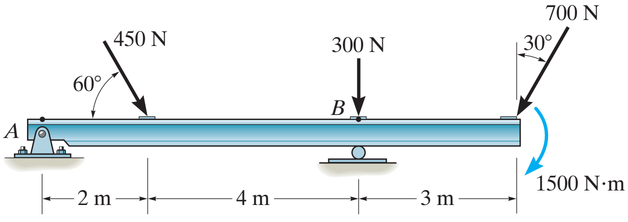
***4–116.** Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.



Probs. 4–115/116

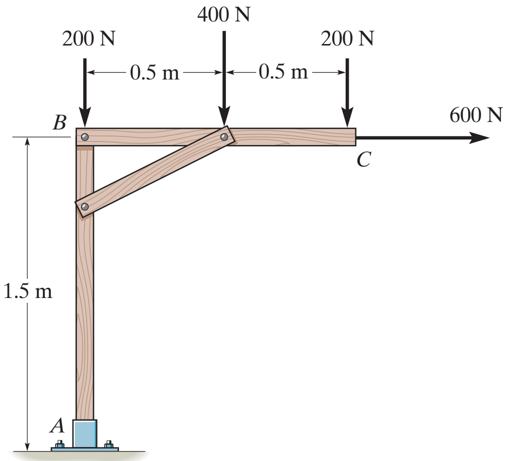
4–117. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.

4–118. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from B.



Probs. 4–117/118

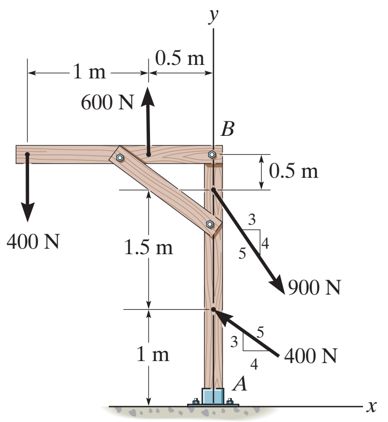
4–119. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



Prob. 4–119

***4–120.** Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member *AB*, measured from *A*.

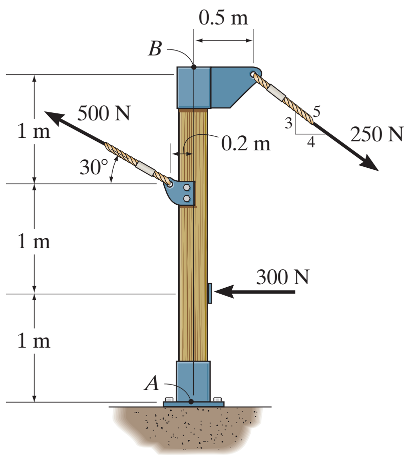
4–121. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member *CB*, measured from end *C*.



Probs. 4–120/121

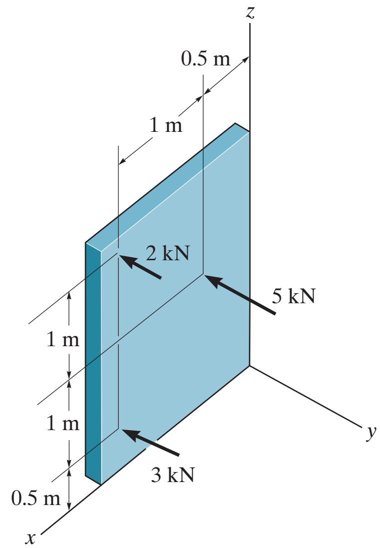
4–122. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *A*.

4–123. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *B*.



Probs. 4–122/123

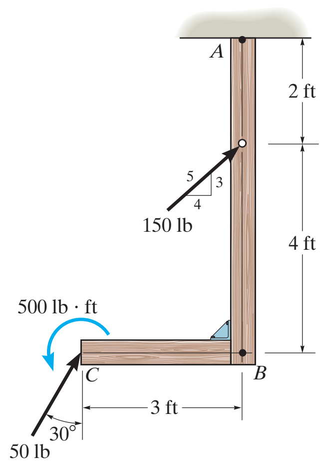
***4–124.** Replace the parallel force system acting on the plate by a resultant force and specify its location on the *x*-*z* plane.



Prob. 4–124

4–125. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *AB*, measured from *A*.

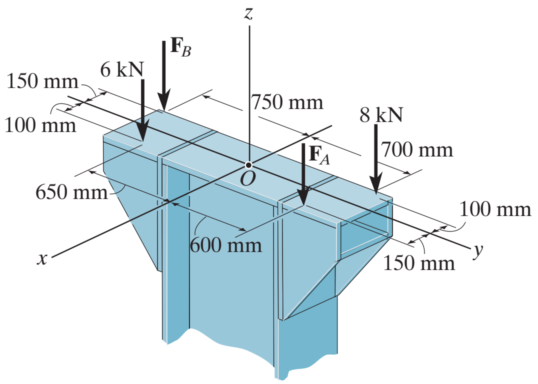
4–126. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from *B*.



Probs. 4–125/126

4–127. If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system acting on the corbels by a resultant force, and specify its location on the x–y plane.

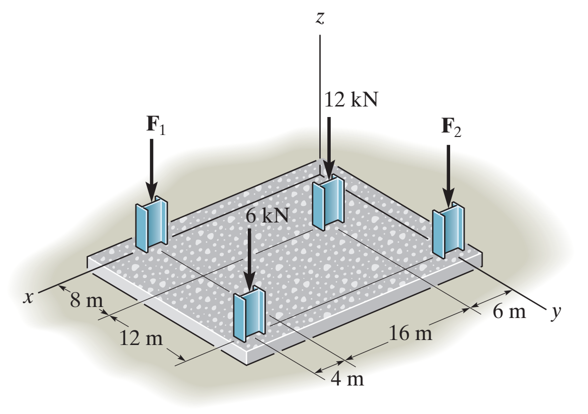
***4–128.** Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O of the column.



Probs. 4–127/128

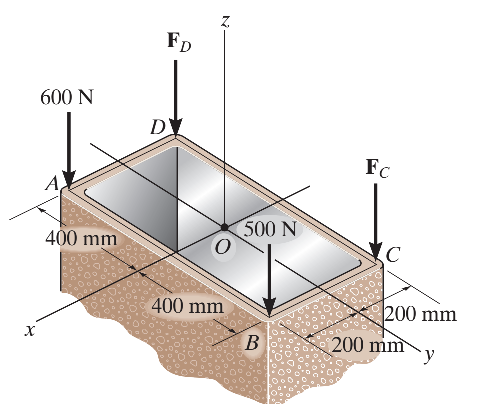
4–130. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 8$ kN and $F_2 = 9$ kN.

4–131. The building slab is subjected to four parallel column loadings. Determine $\mathbf{F}_1$ and $\mathbf{F}_2$ if the resultant force acts through point (12 m, 10 m).



Probs. 4–130/131

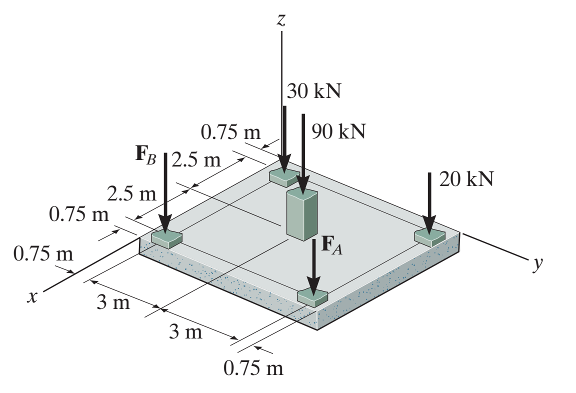
4–129. The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



Prob. 4–129

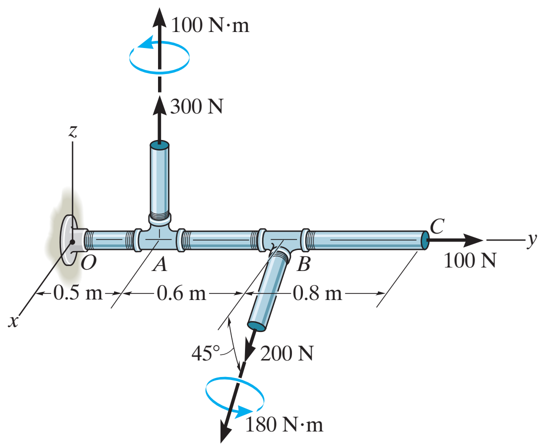
***4–132.** If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.

4–133. If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



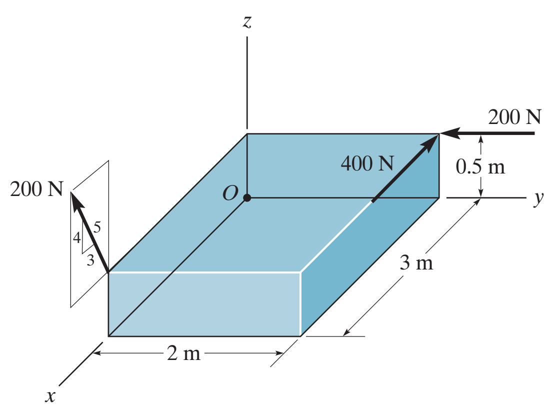
Probs. 4–132/133

4–134. Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point *O*.



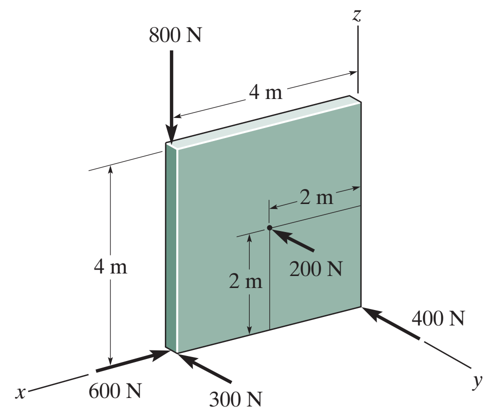
Prob. 4–134

4–135. Replace the force system by a wrench and specify the magnitude of the force and couple moment of the wrench and the point where the wrench intersects the *x–z* plane.



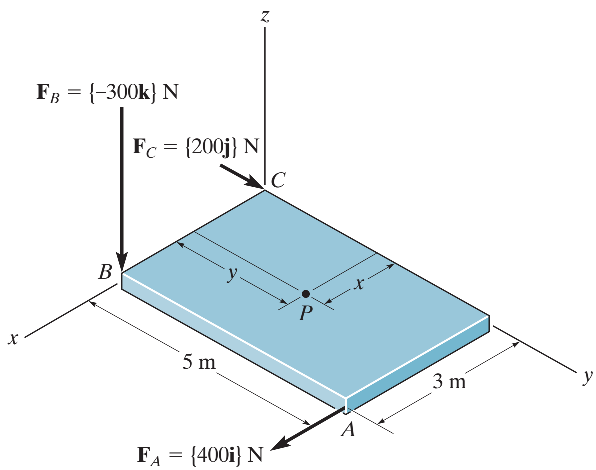
Prob. 4–135

***4–136.** Replace the five forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point *P*(*x*, *z*) where the wrench intersects the *x–z* plane.



Prob. 4–136

4–137. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point *P*(*x*, *y*) where the wrench intersects the plate.



Prob. 4–137