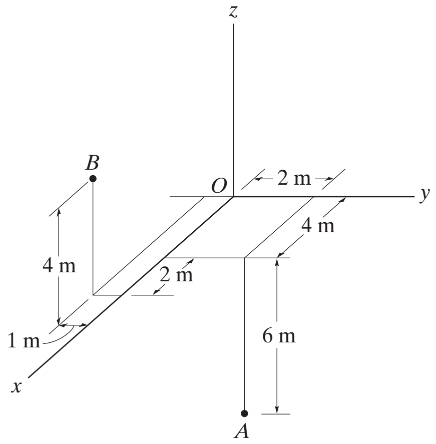


Fig. 2–32

2.7 Position Vectors

In this section we will introduce the concept of a position vector. It will be shown that this vector is of importance in formulating a Cartesian force vector directed between two points in space.

x, y, z Coordinates. Throughout the book we will use a *right-handed* coordinate system to reference the location of points in space. We will also use the convention followed in many technical books, which requires the positive z axis to be directed *upward* (the zenith direction) so that it measures the height of an object or the altitude of a point. The x, y axes then lie in the horizontal plane, Fig. 2–32. Points in space are located relative to the origin of coordinates, O, by successive measurements along the x, y, z axes. For example, the coordinates of point A are obtained by starting at O and measuring $x_A = +4$ m along the x axis, then $y_A = +2$ m along the y axis, and finally $z_A = -6$ m along the z axis, so that $A(4\text{ m}, 2\text{ m}, -6\text{ m})$. In a similar manner, measurements along the x, y, z axes from O to B yield the coordinates of B, that is, $B(6\text{ m}, -1\text{ m}, 4\text{ m})$.

Position Vector. A ***position vector*** $\mathbf{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example, if $\mathbf{r}$ extends from the origin of coordinates, O, to point $P(x, y, z)$, Fig. 2–33a, then $\mathbf{r}$ can be expressed in Cartesian vector form as

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

Note how the head-to-tail vector addition of the three components yields vector $\mathbf{r}$, Fig. 2–33b. Starting at the origin O, one "travels" x in the $+\mathbf{i}$ direction, then y in the $+\mathbf{j}$ direction, and finally z in the $+\mathbf{k}$ direction to arrive at point $P(x, y, z)$.

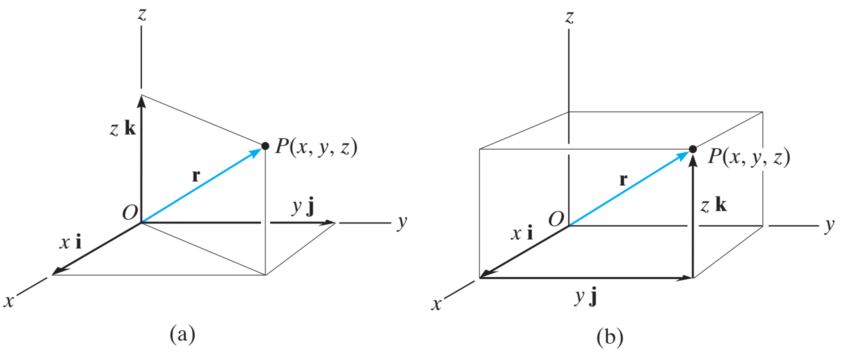
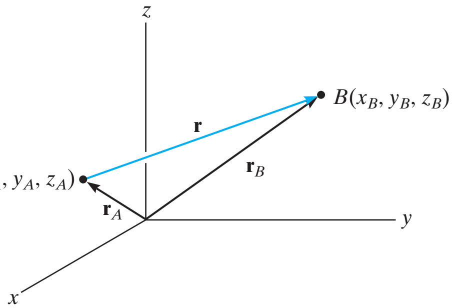


Fig. 2–33

In the more general case, the position vector may be directed from point A to point B in space, Fig. 2–34*a*. This vector is also designated by the symbol $\mathbf{r}$. As a matter of convention, we will *sometimes* refer to this vector with *two subscripts* to indicate from and to the point where it is directed. Thus, $\mathbf{r}$ can also be designated as $\mathbf{r}_{AB}$. Also, note that $\mathbf{r}_A$ and $\mathbf{r}_B$ in Fig. 2–34*a* are referenced with only one subscript since they extend from the origin of coordinates.

From Fig. 2–34*a*, by the head-to-tail vector addition, using the triangle rule, we require

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$



(a)

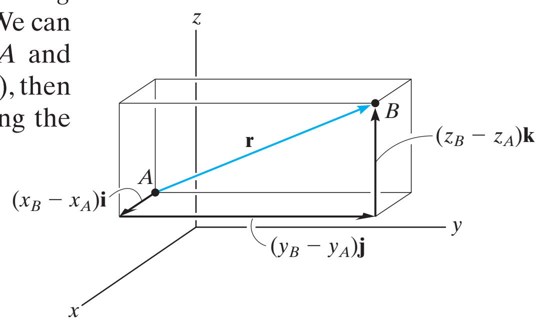
Solving for $\mathbf{r}$ and expressing $\mathbf{r}_A$ and $\mathbf{r}_B$ in Cartesian vector form yields

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$

or

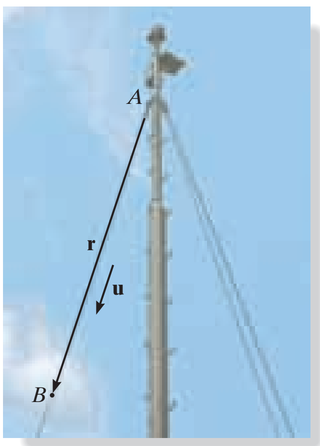
$$\boxed{\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}} \qquad (2\text{–}11)$$

Thus, the $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ *components of the position vector* $\mathbf{r}$ *may be formed by taking the coordinates of the tail of the vector* $A(x_A, y_A, z_A)$ *and subtracting them from the corresponding coordinates of the head* $B(x_B, y_B, z_B)$. We can also form these components *directly*, Fig. 2–34*b*, by starting at A and moving through a distance of $(x_B - x_A)$ along the positive x axis $(+\mathbf{i})$, then $(y_B - y_A)$ along the positive y axis $(+\mathbf{j})$, and finally $(z_B - z_A)$ along the positive z axis $(+\mathbf{k})$ to get to B.



(b)

Fig. 2–34



If an x, y, z coordinate system is established, then the coordinates of two points A and B on the cable can be determined. From this the position vector $\mathbf{r}$ acting along the cable can be formulated. Its magnitude represents the distance from A to B, and its unit vector, $\mathbf{u} = \mathbf{r}/r$, gives the direction defined by α, β, γ. (© Russell C. Hibbeler)

EXAMPLE 2.10

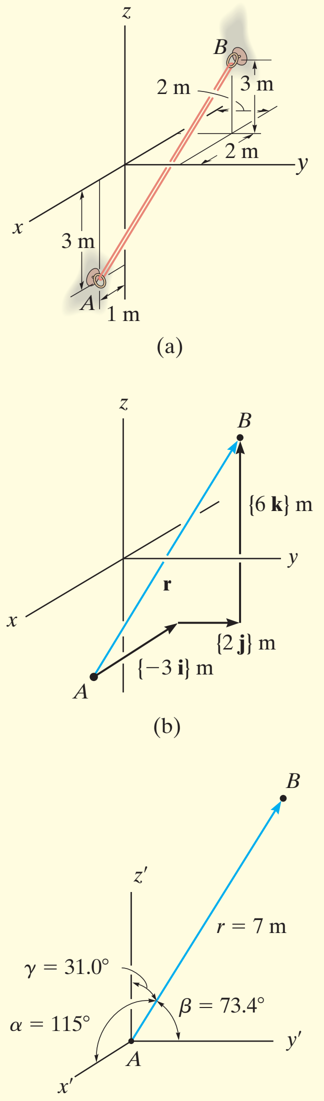


Fig. 2–35

An elastic rubber band is attached to points A and B as shown in Fig. 2–35*a*. Determine its length and its direction measured from A toward B.

SOLUTION

We first establish a position vector from A to B, Fig. 2–35*b*. In accordance with Eq. 2–11, the coordinates of the tail $A(1\text{ m}, 0, -3\text{ m})$ are subtracted from the coordinates of the head $B(-2\text{ m}, 2\text{ m}, 3\text{ m})$, which yields

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

These components of $\mathbf{r}$ can also be determined *directly* by realizing that they represent the direction and distance one must travel along each axis in order to move from A to B, i.e., along the x axis $\{-3\mathbf{i}\}$ m, along the y axis $\{2\mathbf{j}\}$ m, and finally along the z axis $\{6\mathbf{k}\}$ m.

The length of the rubber band is therefore

$$r = \sqrt{(-3\text{ m})^2 + (2\text{ m})^2 + (6\text{ m})^2} = 7\text{ m} \qquad \textit{Ans.}$$

Formulating a unit vector in the direction of $\mathbf{r}$, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

The components of this unit vector give the coordinate direction angles

$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115° \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4° \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0° \qquad \textit{Ans.}$$

NOTE: These angles are measured from the *positive axes* of a localized coordinate system placed at the tail of $\mathbf{r}$, as shown in Fig. 2–35*c*.

2.8 Force Vector Directed Along a Line

Quite often in three-dimensional statics problems, the direction of a force is specified by two points through which its line of action passes. Such a situation is shown in Fig. 2–36, where the force **F** is directed along the cord AB. We can formulate **F** as a Cartesian vector by realizing that it has the *same direction* and *sense* as the position vector **r** directed from point A to point B on the cord. This common direction is specified by the ***unit vector*** $\mathbf{u} = \mathbf{r}/r$. Hence,

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

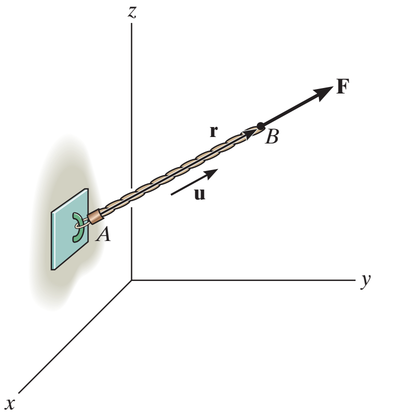
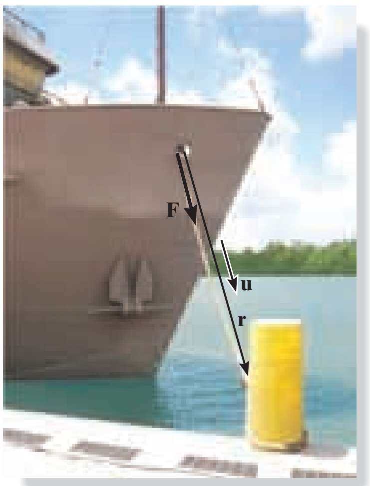


Fig. 2–36

Although we have represented **F** symbolically in Fig. 2–36, note that it has *units of force*, unlike **r**, which has units of length.



The force **F** acting along the rope can be represented as a Cartesian vector by establishing x, y, z axes and first forming a position vector **r** along the length of the rope. Then the corresponding unit vector $\mathbf{u} = \mathbf{r}/r$ that defines the direction of both the rope and the force can be determined. Finally, the magnitude of the force is combined with its direction, $\mathbf{F} = F\mathbf{u}$. (© Russell C. Hibbeler)

Important Points

- A position vector locates one point in space relative to another point.
- The easiest way to formulate the components of a position vector is to determine the distance and direction that must be traveled along the x, y, z directions—going from the tail to the head of the vector.
- A force **F** acting in the direction of a position vector **r** can be represented in Cartesian form if the unit vector **u** of the position vector is determined and it is multiplied by the magnitude of the force, i.e., $\mathbf{F} = F\mathbf{u} = F(\mathbf{r}/r)$.

EXAMPLE 2.11

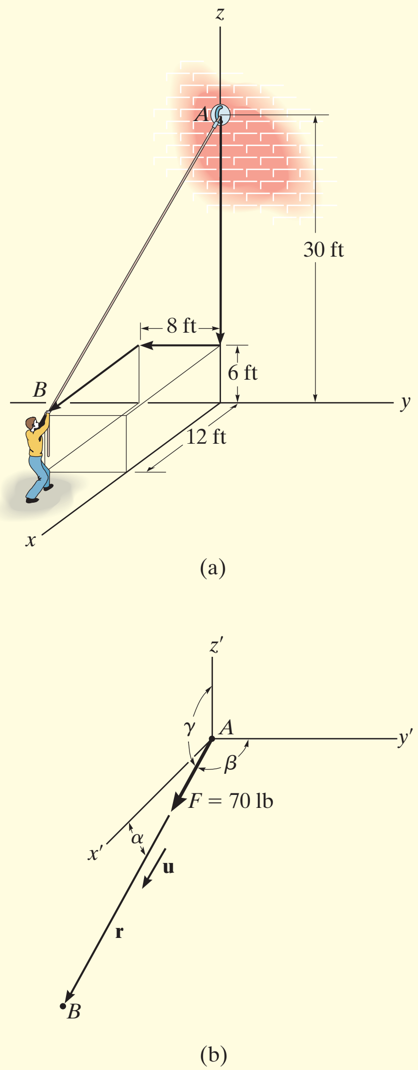


Fig. 2–37

The man shown in Fig. 2–37*a* pulls on the cord with a force of 70 lb. Represent this force acting on the support *A* as a Cartesian vector and determine its direction.

SOLUTION

Force **F** is shown in Fig. 2–37*b*. The *direction* of this vector, **u**, is determined from the position vector **r**, which extends from *A* to *B*. Rather than using the coordinates of the end points of the cord, **r** can be determined *directly* by noting in Fig. 2–37*a* that one must travel from *A* {−24**k**} ft, then {−8**j**} ft, and finally {12**i**} ft to get to *B*. Thus,

$$\mathbf{r} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ ft}$$

The magnitude of **r**, which represents the *length* of cord *AB*, is

$$r = \sqrt{(12\text{ ft})^2 + (-8\text{ ft})^2 + (-24\text{ ft})^2} = 28\text{ ft}$$

Forming the unit vector that defines the direction and sense of both **r** and **F**, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = \frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}$$

Since **F** has a *magnitude* of 70 lb and a *direction* specified by **u**, then

$$\mathbf{F} = F\mathbf{u} = 70\text{ lb}\left(\frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right)$$

$$= \{30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}\}\text{ lb} \qquad \textit{Ans.}$$

The coordinate direction angles are measured between **r** (or **F**) and the *positive axes* of a localized coordinate system with origin placed at *A*, Fig. 2–37*b*. From the components of the unit vector:

$$\alpha = \cos^{-1}\left(\frac{12}{28}\right) = 64.6^\circ \qquad \textit{Ans.}$$

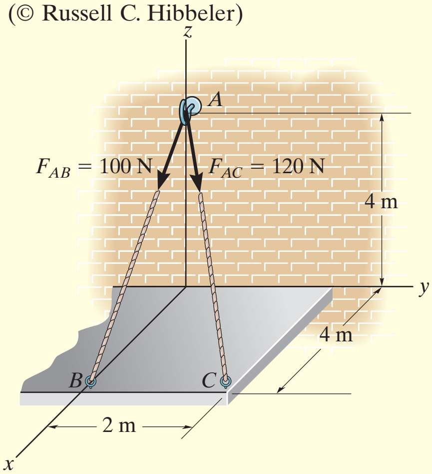
$$\beta = \cos^{-1}\left(\frac{-8}{28}\right) = 107^\circ \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-24}{28}\right) = 149^\circ \qquad \textit{Ans.}$$

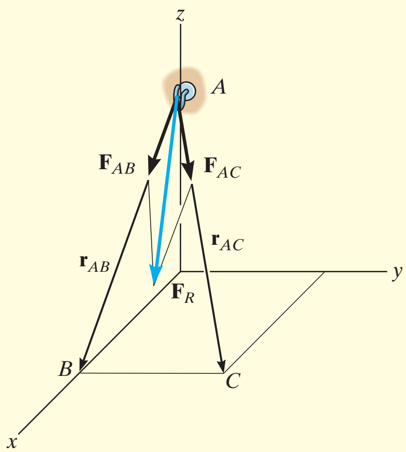
NOTE: These results make sense when compared with the angles identified in Fig. 2–37*b*.

EXAMPLE 2.12

(© Russell C. Hibbeler)



(a)



(b)

Fig. 2–38

The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the wall hook at A as shown in Fig. 2–38*a*, determine the resultant force acting at A. Express the result as a Cartesian vector.

SOLUTION

The resultant force $\mathbf{F}_R$ is shown graphically in Fig. 2–38*b*. We can express this force as a Cartesian vector by first formulating $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ as Cartesian vectors and then adding their components. The directions of $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ are specified by forming unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ along the cables. These unit vectors are obtained from the associated position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$. With reference to Fig. 2–38*a*, to go from A to B, we must travel $\{-4\mathbf{k}\}$ m, and then $\{4\mathbf{i}\}$ m. Thus,

$$\mathbf{r}_{AB} = \{4\mathbf{i} - 4\mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{(4 \text{ m})^2 + (-4 \text{ m})^2} = 5.66 \text{ m}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = (100 \text{ N})\left(\frac{4}{5.66}\mathbf{i} - \frac{4}{5.66}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{70.7\mathbf{i} - 70.7\mathbf{k}\} \text{ N}$$

To go from A to C, we must travel $\{-4\mathbf{k}\}$ m, then $\{2\mathbf{j}\}$ m, and finally $\{4\mathbf{i}\}$. Thus,

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(4 \text{ m})^2 + (2 \text{ m})^2 + (-4 \text{ m})^2} = 6 \text{ m}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = (120 \text{ N})\left(\frac{4}{6}\mathbf{i} + \frac{2}{6}\mathbf{j} - \frac{4}{6}\mathbf{k}\right)$$

$$= \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\} \text{ N}$$

The resultant force is therefore

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} = \{70.7\mathbf{i} - 70.7\mathbf{k}\} \text{ N} + \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\} \text{ N}$$

$$= \{151\mathbf{i} + 40\mathbf{j} - 151\mathbf{k}\} \text{ N} \qquad \textit{Ans.}$$

EXAMPLE 2.13

The force in Fig. 2–39*a* acts on the hook. Express it as a Cartesian vector.

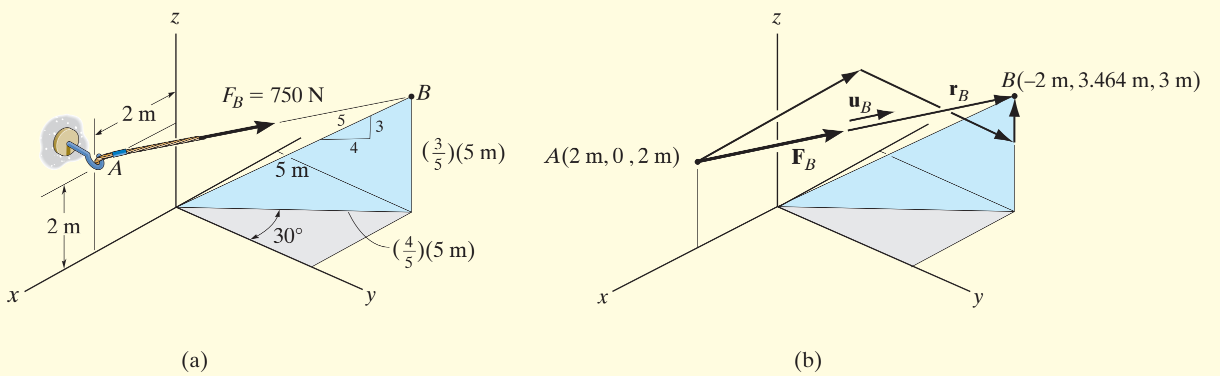


Fig. 2–39

SOLUTION

As shown in Fig. 2–39*b*, the coordinates for points *A* and *B* are

$$A\,(2\text{ m}, 0, 2\text{ m})$$

and

$$B\left[-\left(\frac{4}{5}\right)5\sin 30^\circ\text{ m}, \left(\frac{4}{5}\right)5\cos 30^\circ\text{ m}, \left(\frac{3}{5}\right)5\text{ m}\right]$$

or

$$B(-2\text{ m}, 3.464\text{ m}, 3\text{ m})$$

Therefore, to go from *A* to *B*, one must travel $\{-4\mathbf{i}\}$ m, then $\{3.464\mathbf{j}\}$ m, and finally $\{1\mathbf{k}\}$ m. Thus,

$$\mathbf{u}_B = \left(\frac{\mathbf{r}_B}{r_B}\right) = \frac{\{-4\mathbf{i} + 3.464\mathbf{j} + 1\mathbf{k}\}\text{ m}}{\sqrt{(-4\text{ m})^2 + (3.464\text{ m})^2 + (1\text{ m})^2}}$$

$$= -0.7428\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k}$$

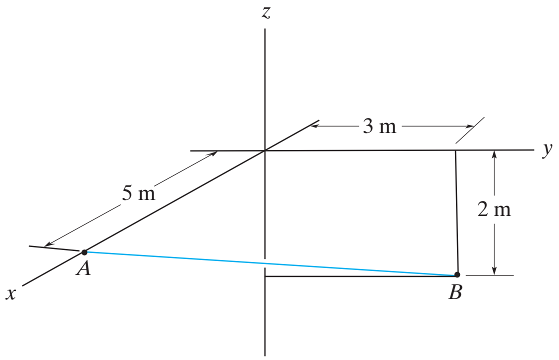
Force $\mathbf{F}_B$ expressed as a Cartesian vector becomes

$$\mathbf{F}_B = F_B\,\mathbf{u}_B = (750\text{ N})(-0.74281\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k})$$

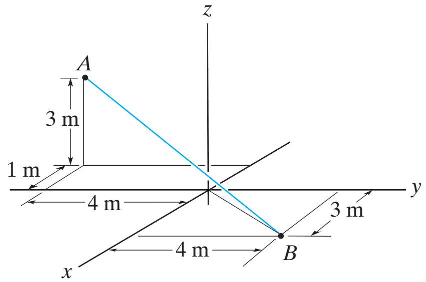
$$= \{-557\mathbf{i} + 482\mathbf{j} + 139\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

PRELIMINARY PROBLEMS

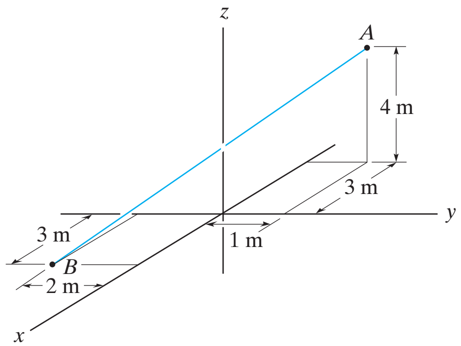
P2–6. In each case, establish a position vector from point A to point B.



(a)



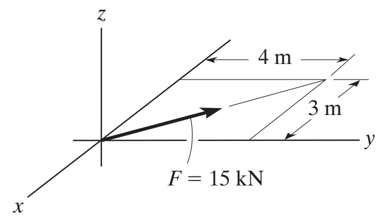
(b)



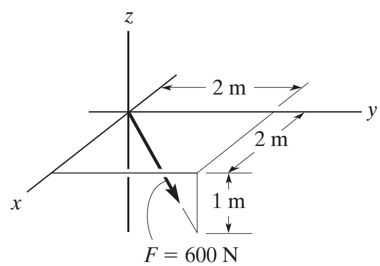
(c)

Prob. P2–6

P2–7. In each case, express **F** as a Cartesian vector.



(a)



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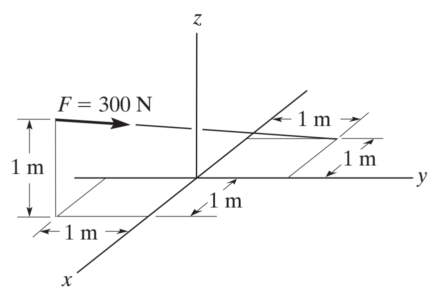
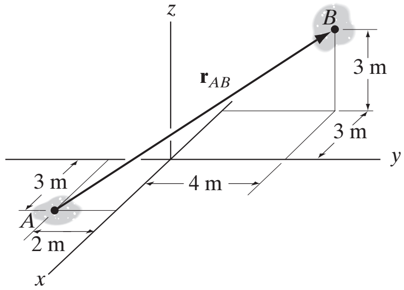


(c)

Prob. P2–7

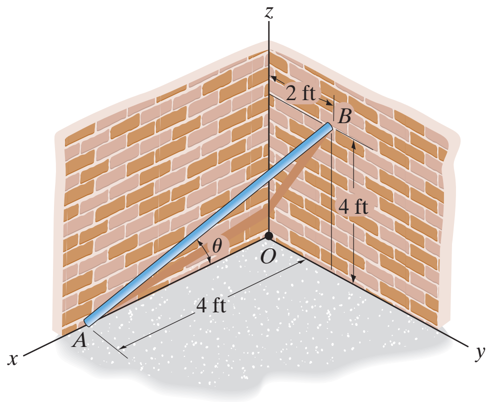
FUNDAMENTAL PROBLEMS

F2–19. Express the position vector $\mathbf{r}_{AB}$ in Cartesian vector form, then determine its magnitude and coordinate direction angles.



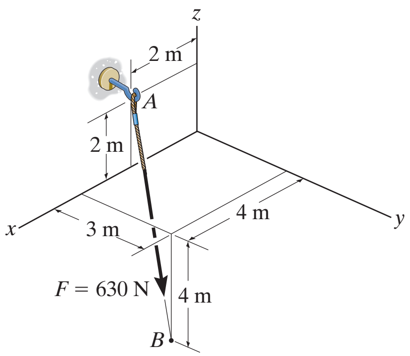
Prob. F2–19

F2–20. Determine the length of the rod and the position vector directed from A to B. What is the angle θ?



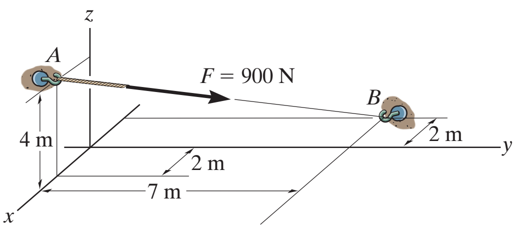
Prob. F2–20

F2–21. Express the force as a Cartesian vector.



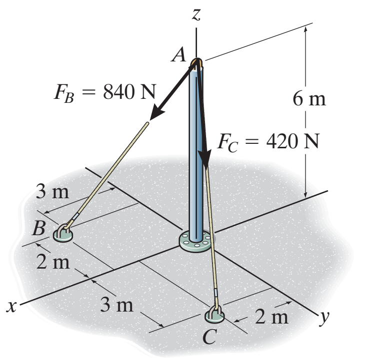
Prob. F2–21

F2–22. Express the force as a Cartesian vector.



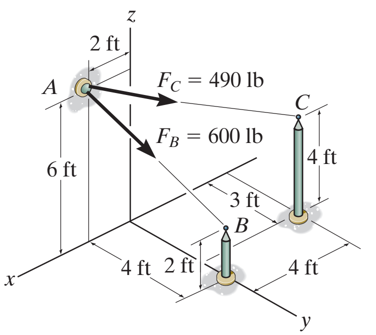
Prob. F2–22

F2–23. Determine the magnitude of the resultant force at A.



Prob. F2–23

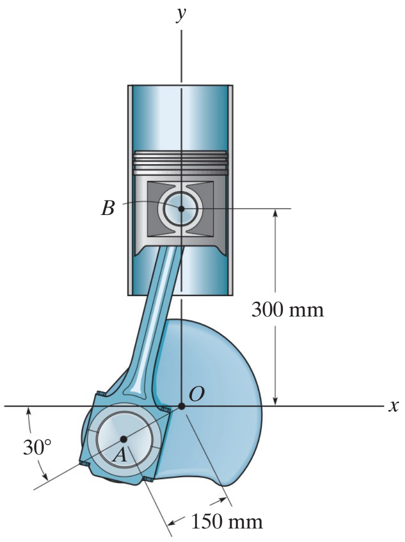
F2–24. Determine the resultant force at A.



Prob. F2–24

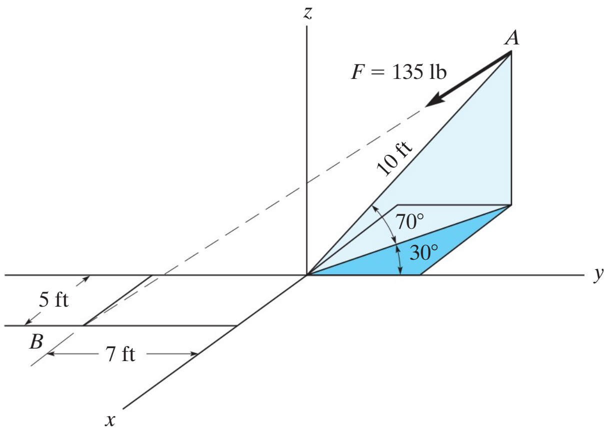
PROBLEMS

2–86. Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



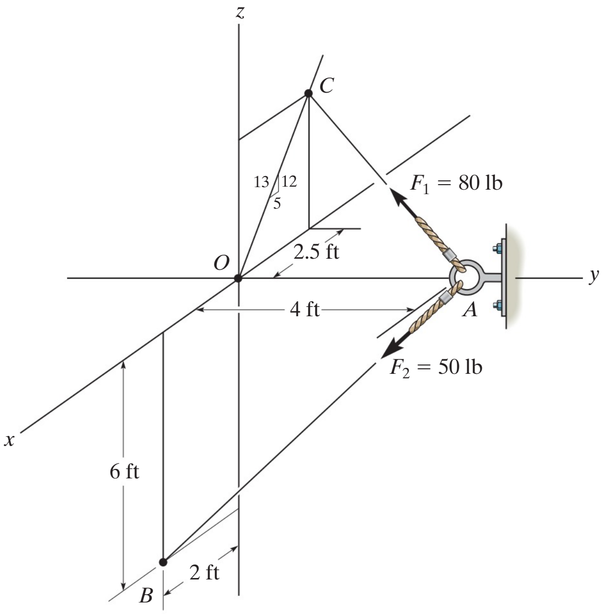
Prob. 2–86

2–87. Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



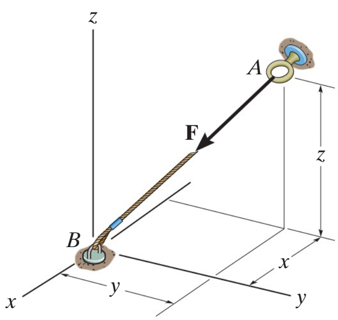
Prob. 2–87

***2–88.** Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–88

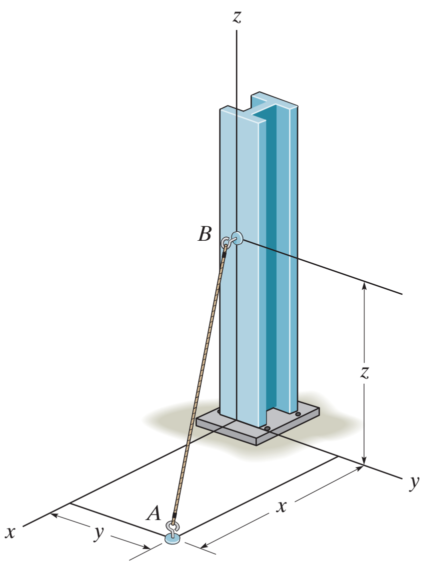
2–89. If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



Prob. 2–89

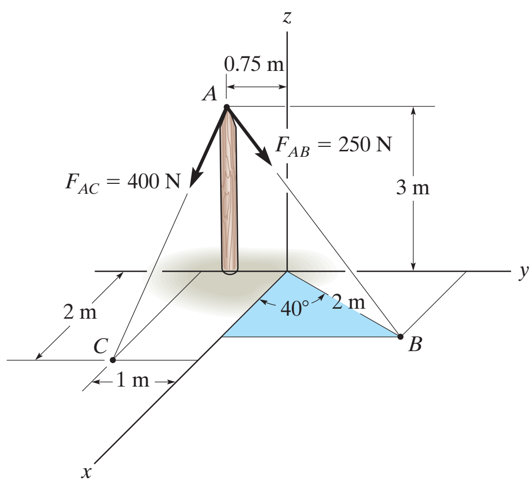
2–90. The 8-m-long cable is anchored to the ground at A. If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.

2–91. The 8-m-long cable is anchored to the ground at A. If $z = 5$ m, determine the location $+x, +y$ of point A. Choose a value such that $x = y$.



Probs. 2–90/91

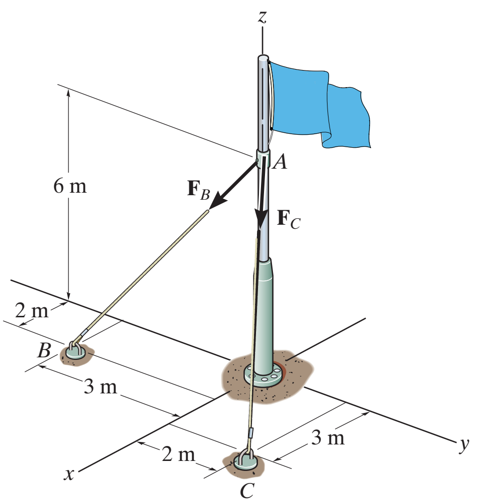
***2–92.** Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–92

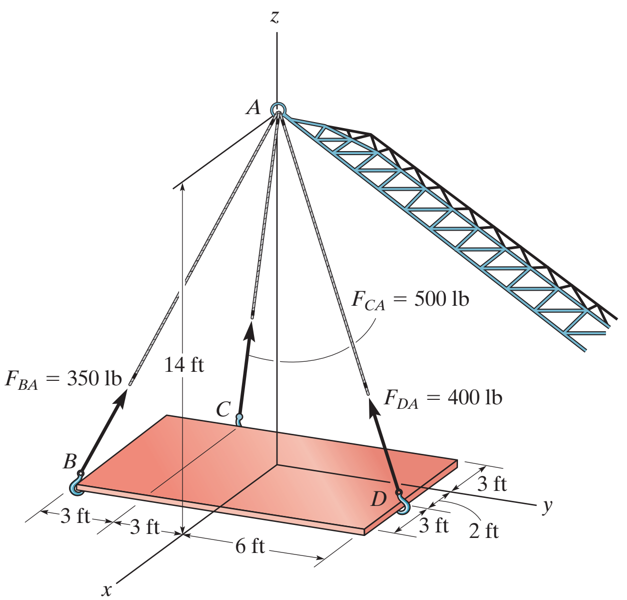
2–93. If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

2–94. If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



Probs. 2–93/94

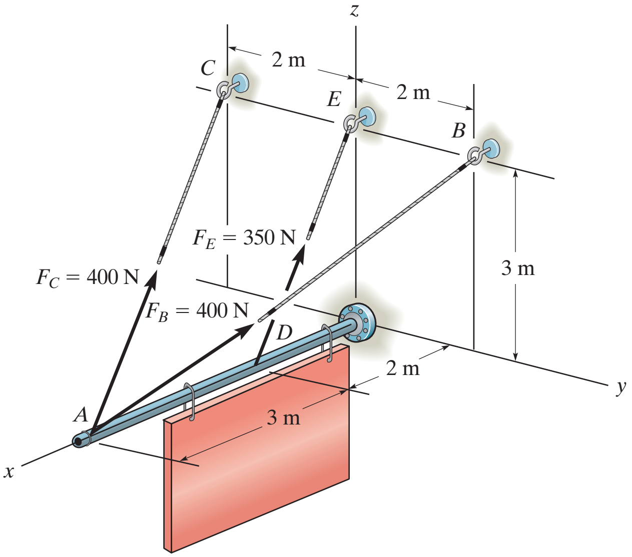
2–95. The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



Prob. 2–95

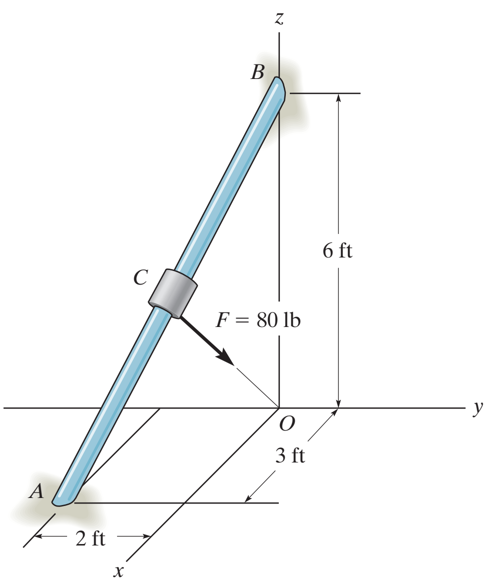
*2–96. The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.

2–97. Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting on the sign at point A.



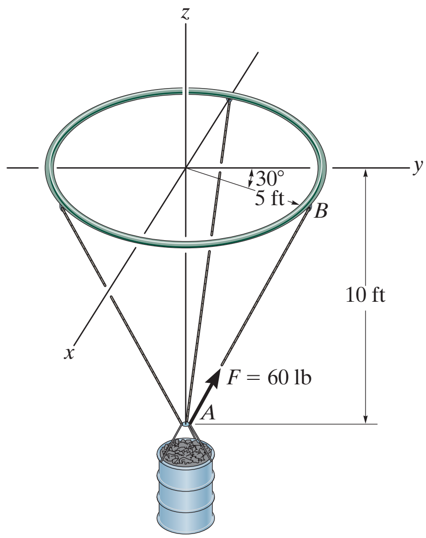
Probs. 2–96/97

2–98. The force **F** has a magnitude of 80 lb and acts at the midpoint C of the thin rod. Express the force as a Cartesian vector.



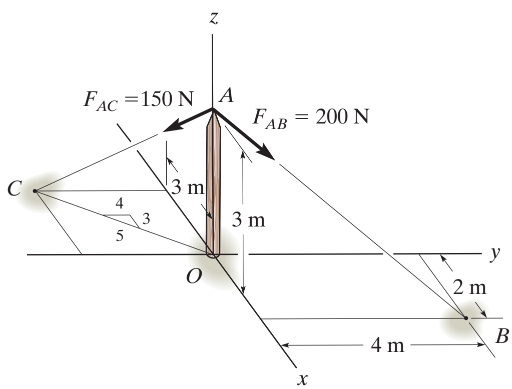
Prob. 2–98

2–99. The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



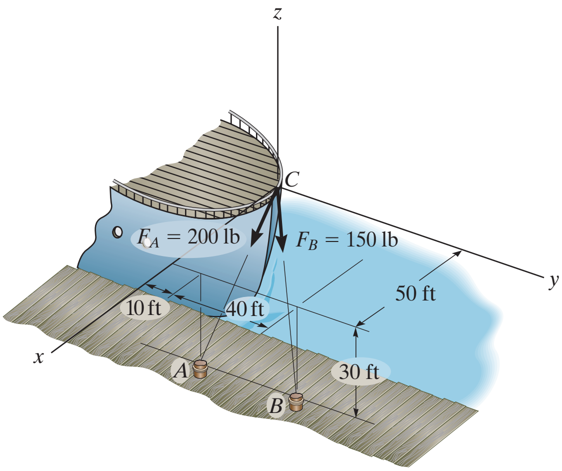
Prob. 2–99

*2–100. Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



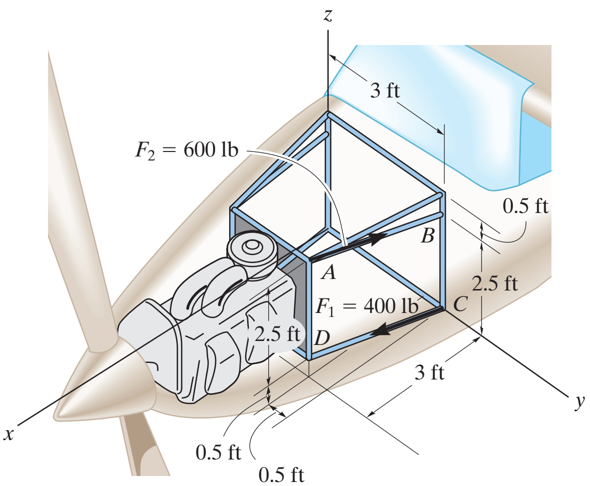
Prob. 2–100

2–101. The two mooring cables exert forces on the stern of a ship as shown. Represent each force as as Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.



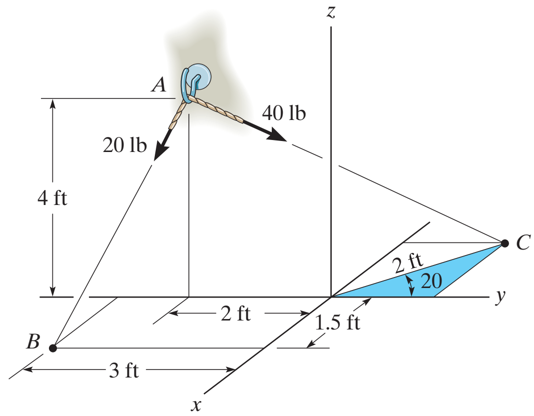
Prob. 2–101

2–102. The engine of the lightweight plane is supported by struts that are connected to the space truss that makes up the structure of the plane. The anticipated loading in two of the struts is shown. Express each of those forces as Cartesian vector.



Prob. 2–102

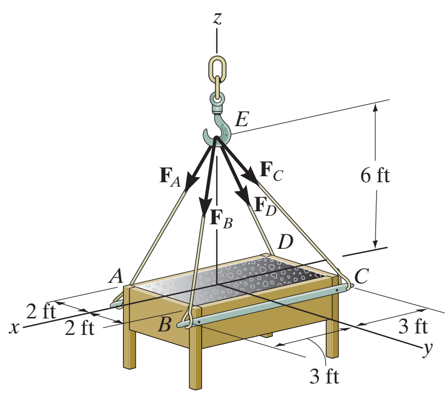
2–103. Determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–103

***2–104.** If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.

2–105. If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



Probs. 2–104/105

2.9 Dot Product

Occasionally in statics one has to find the angle between two lines or the components of a force parallel and perpendicular to a line. In two dimensions, these problems can readily be solved by trigonometry since the geometry is easy to visualize. In three dimensions, however, this is often difficult, and consequently vector methods should be employed for the solution. The dot product, which defines a particular method for "multiplying" two vectors, can be used to solve the above-mentioned problems.

The ***dot product*** of vectors **A** and **B**, written $\mathbf{A} \cdot \mathbf{B}$ and read "**A** dot **B**," is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle θ between their tails, Fig. 2–40. Expressed in equation form,

$$\boxed{\mathbf{A} \cdot \mathbf{B} = AB\cos\theta} \qquad (2\text{–}12)$$

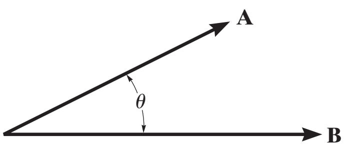


Fig. 2–40

where $0° \leq \theta \leq 180°$. The dot product is often referred to as the *scalar product* of vectors since the result is a *scalar* and not a vector.

Laws of Operation.

1. Commutative law: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
2. Multiplication by a scalar: $a(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B})$
3. Distributive law: $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$

It is easy to prove the first and second laws by using Eq. 2–12. The proof of the distributive law is left as an exercise (see Prob. 2–112).

Cartesian Vector Formulation.

Equation 2–12 must be used to find the dot product for any two Cartesian unit vectors. For example, $\mathbf{i} \cdot \mathbf{i} = (1)(1)\cos 0° = 1$ and $\mathbf{i} \cdot \mathbf{j} = (1)(1)\cos 90° = 0$. If we want to find the dot product of two general vectors **A** and **B** that are expressed in Cartesian vector form, then we have

$$\begin{aligned}
\mathbf{A} \cdot \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\
&= A_xB_x(\mathbf{i} \cdot \mathbf{i}) + A_xB_y(\mathbf{i} \cdot \mathbf{j}) + A_xB_z(\mathbf{i} \cdot \mathbf{k}) \\
&\quad + A_yB_x(\mathbf{j} \cdot \mathbf{i}) + A_yB_y(\mathbf{j} \cdot \mathbf{j}) + A_yB_z(\mathbf{j} \cdot \mathbf{k}) \\
&\quad + A_zB_x(\mathbf{k} \cdot \mathbf{i}) + A_zB_y(\mathbf{k} \cdot \mathbf{j}) + A_zB_z(\mathbf{k} \cdot \mathbf{k})
\end{aligned}$$

Carrying out the dot-product operations, the final result becomes

$$\boxed{\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z} \qquad (2\text{–}13)$$

Thus, to determine the dot product of two Cartesian vectors, multiply their corresponding x, y, z components and sum these products algebraically. Note that the result will be either a positive or negative *scalar*, or it could be zero.

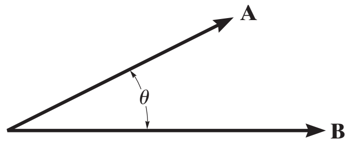
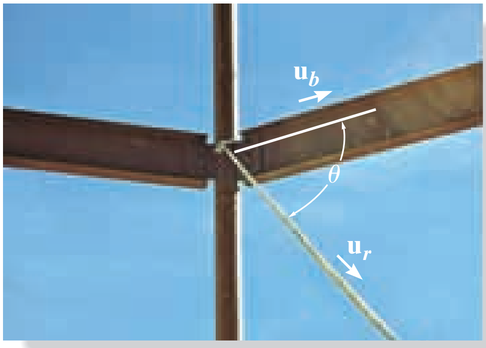
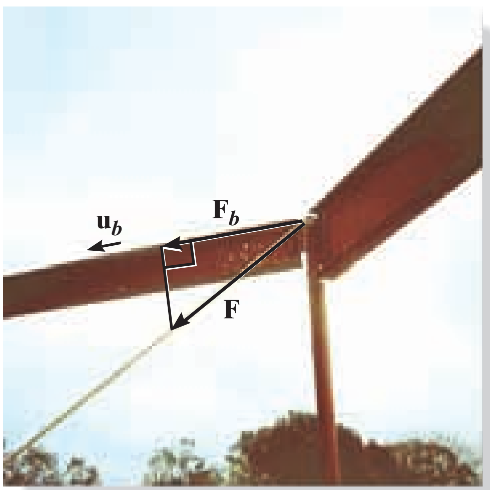


Fig. 2–40 (Repeated)



The angle θ between the rope and the beam can be determined by formulating unit vectors along the beam and rope and then using the dot product $\mathbf{u}_b \cdot \mathbf{u}_r = (1)(1)\cos\theta$. (© Russell C. Hibbeler)



The projection of the cable force $\mathbf{F}$ along the beam can be determined by first finding the unit vector $\mathbf{u}_b$ that defines this direction. Then apply the dot product, $F_b = \mathbf{F} \cdot \mathbf{u}_b$. (© Russell C. Hibbeler)

Applications. The dot product has two important applications in mechanics.

- ***The angle formed between two vectors or intersecting lines.*** The angle θ between the tails of vectors $\mathbf{A}$ and $\mathbf{B}$ in Fig. 2–40 can be determined from Eq. 2–12 and written as

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right) \qquad 0^\circ \le \theta \le 180^\circ$$

Here $\mathbf{A} \cdot \mathbf{B}$ is found from Eq. 2–13. In particular, notice that if $\mathbf{A} \cdot \mathbf{B} = 0, \theta = \cos^{-1} 0 = 90^\circ$ so that $\mathbf{A}$ will be *perpendicular* to $\mathbf{B}$.

- ***The components of a vector parallel and perpendicular to a line.*** The component of vector $\mathbf{A}$ parallel to or collinear with the line *aa* in Fig. 2–40 is defined by A_a where $A_a = A \cos\theta$. This component is sometimes referred to as the ***projection*** of $\mathbf{A}$ onto the line, since a *right angle* is formed in the construction. If the *direction* of the line is specified by the unit vector $\mathbf{u}_a$, then since $u_a = 1$, we can determine the magnitude of A_a directly from the dot product (Eq. 2–12); i.e.,

$$A_a = A\cos\theta = \mathbf{A} \cdot \mathbf{u}_a$$

Hence, the scalar projection of $\mathbf{A}$ *along a line is determined from the dot product of* $\mathbf{A}$ *and the unit vector* $\mathbf{u}_a$ *which defines the direction of the line*. Notice that if this result is positive, then $\mathbf{A}_a$ has a directional sense which is the same as $\mathbf{u}_a$, whereas if A_a is a negative scalar, then $\mathbf{A}_a$ has the opposite sense of direction to $\mathbf{u}_a$.

The component $\mathbf{A}_a$ represented as a *vector* is therefore

$$\mathbf{A}_a = A_a \mathbf{u}_a$$

The component of $\mathbf{A}$ that is *perpendicular* to line *aa* can also be obtained, Fig. 2–41. Since $\mathbf{A} = \mathbf{A}_a + \mathbf{A}_\perp$, then $\mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_a$. There are two possible ways of obtaining $A_\perp$. One way would be to determine θ from the dot product, $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{u}_A / A)$, then $A_\perp = A\sin\theta$. Alternatively, if A_a is known, then by Pythagorean's theorem we can also write $A_\perp = \sqrt{A^2 - A_a{}^2}$.

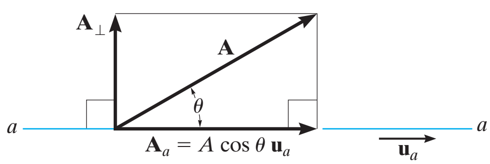


Fig. 2–41

Important Points

- The dot product is used to determine the angle between two vectors or the projection of a vector in a specified direction.
- If vectors **A** and **B** are expressed in Cartesian vector form, the dot product is determined by multiplying the respective x, y, z scalar components and algebraically adding the results, i.e., $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.
- From the definition of the dot product, the angle formed between the tails of vectors **A** and **B** is $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{B}/AB)$.
- The magnitude of the projection of vector **A** along a line aa whose direction is specified by $\mathbf{u}_a$ is determined from the dot product $A_a = \mathbf{A} \cdot \mathbf{u}_a$.

EXAMPLE 2.14

Determine the magnitudes of the projection of the force **F** in Fig. 2–42 onto the u and v axes.

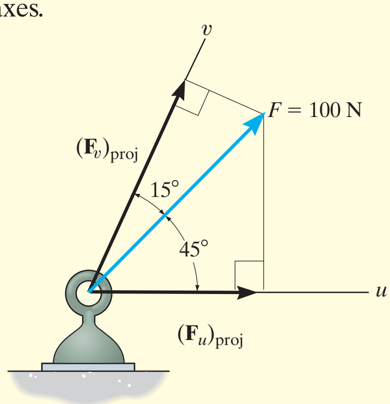


Fig. 2–42

SOLUTION

Projections of Force. The graphical representation of the *projections* is shown in Fig. 2–42. From this figure, the magnitudes of the projections of **F** onto the u and v axes can be obtained by trigonometry:

$$(F_u)_{\text{proj}} = (100 \text{ N})\cos 45^\circ = 70.7 \text{ N} \qquad \textit{Ans.}$$

$$(F_v)_{\text{proj}} = (100 \text{ N})\cos 15^\circ = 96.6 \text{ N} \qquad \textit{Ans.}$$

NOTE: These projections are not equal to the magnitudes of the components of force **F** along the u and v axes found from the parallelogram law. They will only be equal if the u and v axes are *perpendicular* to one another.

EXAMPLE 2.15

The frame shown in Fig. 2–43*a* is subjected to a horizontal force $\mathbf{F} = \{300\mathbf{j}\}$ N. Determine the magnitudes of the components of this force parallel and perpendicular to member AB.

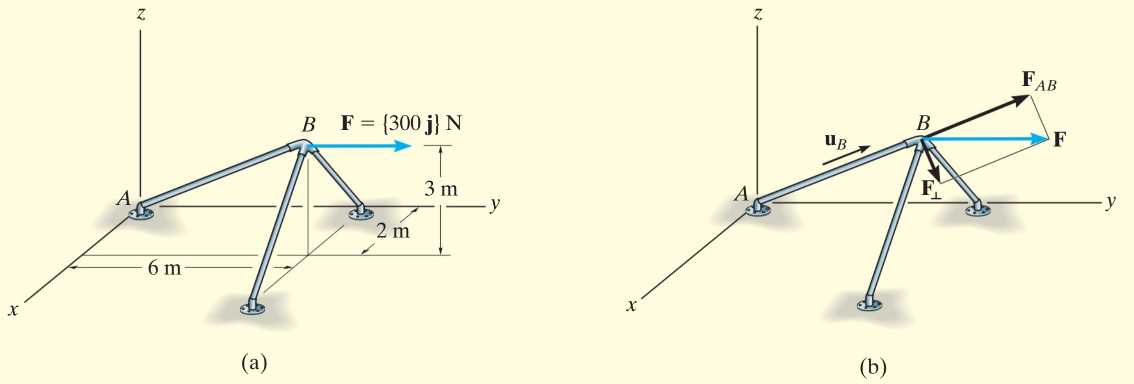


Fig. 2–43

SOLUTION

The magnitude of the component of $\mathbf{F}$ along AB is equal to the dot product of $\mathbf{F}$ and the unit vector $\mathbf{u}_B$, which defines the direction of AB, Fig. 2–43*b*. Since

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.286\,\mathbf{i} + 0.857\,\mathbf{j} + 0.429\,\mathbf{k}$$

then

$$\begin{aligned} F_{AB} &= F\cos\theta = \mathbf{F}\cdot\mathbf{u}_B = (300\mathbf{j})\cdot(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}) \\ &= (0)(0.286) + (300)(0.857) + (0)(0.429) \\ &= 257.1 \text{ N} \end{aligned}$$ *Ans.*

Since the result is a positive scalar, $\mathbf{F}_{AB}$ has the same sense of direction as $\mathbf{u}_B$, Fig. 2–43*b*.

Expressing $\mathbf{F}_{AB}$ in Cartesian vector form, we have

$$\begin{aligned} \mathbf{F}_{AB} &= F_{AB}\mathbf{u}_B = (257.1 \text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}) \\ &= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\} \text{N} \end{aligned}$$ *Ans.*

The perpendicular component, Fig. 2–43*b*, is therefore

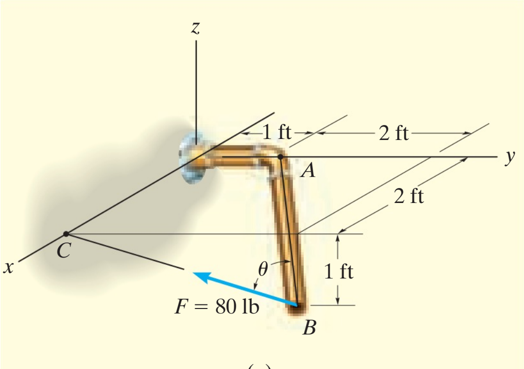
$$\begin{aligned} \mathbf{F}_\perp &= \mathbf{F} - \mathbf{F}_{AB} = 300\mathbf{j} - (73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}) \\ &= \{-73.5\mathbf{i} + 79.6\mathbf{j} - 110\mathbf{k}\} \text{N} \end{aligned}$$

Its magnitude can be determined either from this vector or by using the Pythagorean theorem, Fig. 2–43*b*:

$$\begin{aligned} F_\perp &= \sqrt{F^2 - F_{AB}^2} = \sqrt{(300 \text{ N})^2 - (257.1 \text{ N})^2} \\ &= 155 \text{ N} \end{aligned}$$ *Ans.*

EXAMPLE 2.16

The pipe in Fig. 2–44*a* is subjected to the force of $F = 80$ lb. Determine the angle θ between **F** and the pipe segment BA and the projection of **F** along this segment.



(a)

SOLUTION

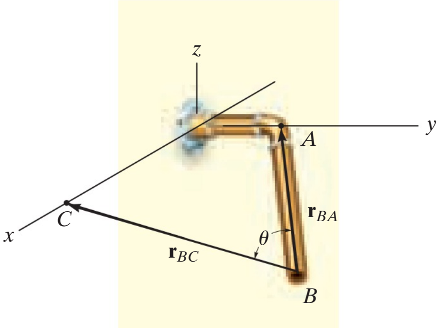
Angle θ. First we will establish position vectors from B to A and B to C; Fig. 2–44*b*. Then we will determine the angle θ between the tails of these two vectors.

$$\mathbf{r}_{BA} = \{-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}\text{ ft},\ r_{BA} = 3\text{ ft}$$
$$\mathbf{r}_{BC} = \{-3\mathbf{j} + 1\mathbf{k}\}\text{ ft},\ r_{BC} = \sqrt{10}\,ft$$

Thus,

$$\cos\theta = \frac{\mathbf{r}_{BA}\cdot\mathbf{r}_{BC}}{r_{BA}r_{BC}} = \frac{(-2)(0) + (-2)(-3) + (1)(1)}{3\sqrt{10}} = 0.7379$$

$$\theta = 42.5^\circ \qquad \textit{Ans.}$$



(b)

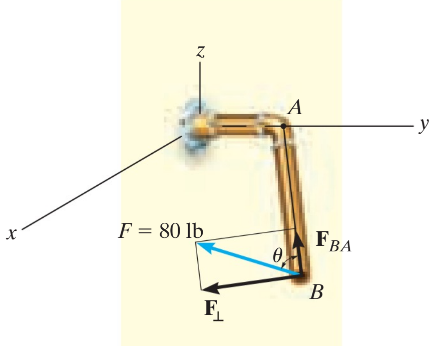
Components of F. The component of **F** along BA is shown in Fig. 2–44*c*. We must first formulate the unit vector along BA and force **F** as Cartesian vectors.

$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{(-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k})}{3} = -\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$$

$$\mathbf{F} = 80\text{ lb}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = 80\left(\frac{-3\mathbf{j} + 1\mathbf{k}}{\sqrt{10}}\right) = -75.89\mathbf{j} + 25.30\mathbf{k}$$

Thus,

$$F_{BA} = \mathbf{F}\cdot\mathbf{u}_{BA} = (-75.89\mathbf{j} + 25.30\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right)$$
$$= 0\left(-\frac{2}{3}\right) + (-75.89)\left(-\frac{2}{3}\right) + (25.30)\left(\frac{1}{3}\right)$$
$$= 59.0\text{ lb} \qquad \textit{Ans.}$$



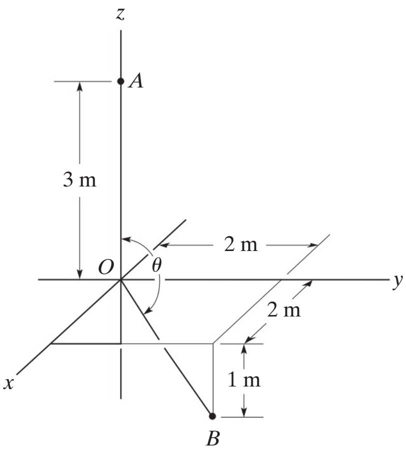
(c)

Fig. 2–44

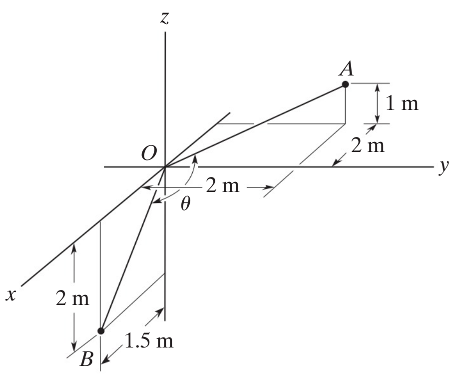
NOTE: Since θ has been calculated, then also, $F_{BA} = F\cos\theta = 80\text{ lb}\cos 42.5^\circ = 59.0$ lb.

PRELIMINARY PROBLEMS

P2–8. In each case, set up the dot product to find the angle θ. Do not calculate the result.



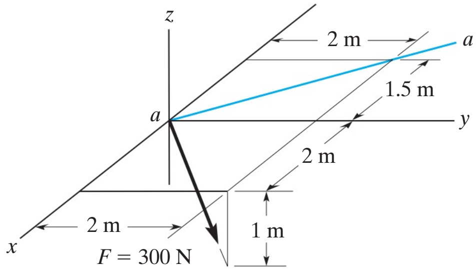
(a)



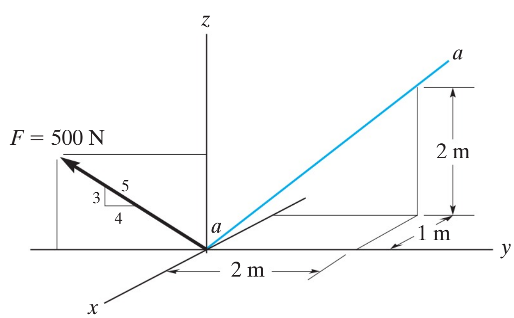
(b)

Prob. P2–8

P2–9. In each case, set up the dot product to find the magnitude of the projection of the force **F** along *a-a* axes. Do not calculate the result.



(a)

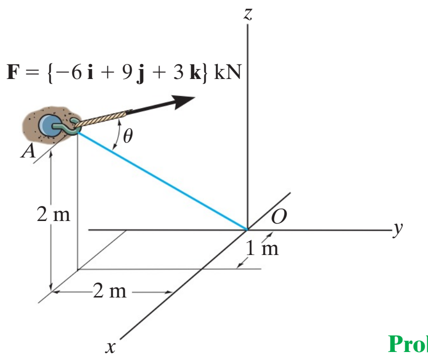


(b)

Prob. P2–9

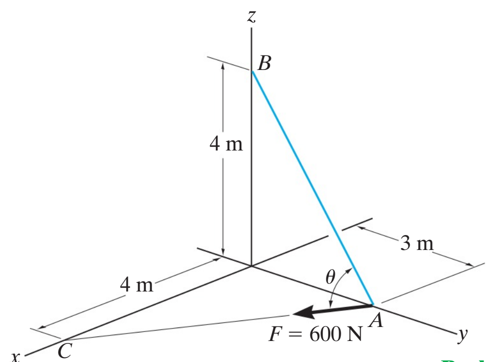
FUNDAMENTAL PROBLEMS

F2–25. Determine the angle θ between the force and the line AO.



Prob. F2–25

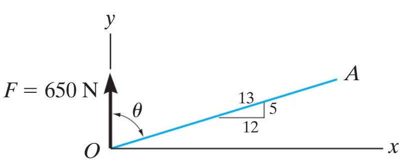
F2–26. Determine the angle θ between the force and the line AB.



Prob. F2–26

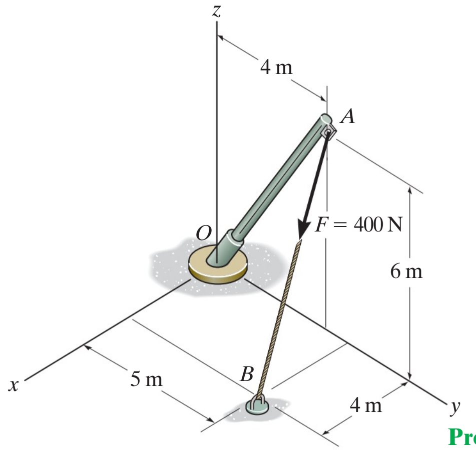
F2–27. Determine the angle θ between the force and the line OA.

F2–28. Determine the projected component of the force along the line OA.



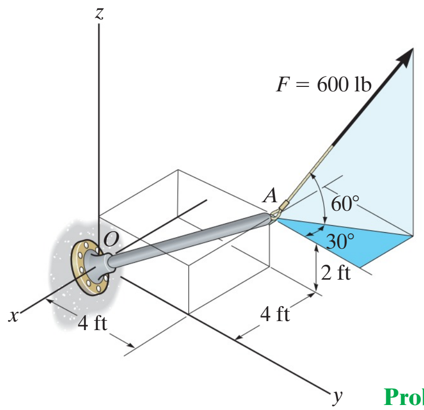
Probs. F2–27/28

F2–29. Find the magnitude of the projected component of the force along the pipe AO.



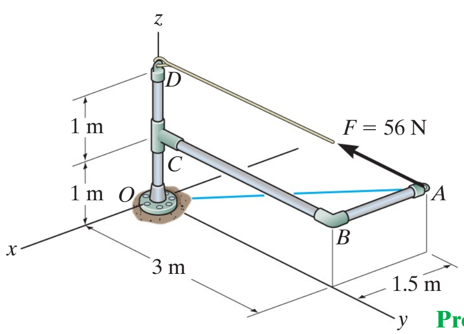
Prob. F2–29

F2–30. Determine the components of the force acting parallel and perpendicular to the axis of the pole.



Prob. F2–30

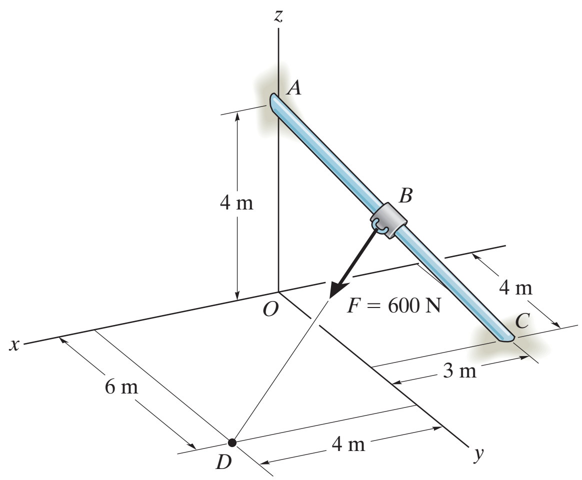
F2–31. Determine the magnitudes of the components of the force $F = 56$ N acting along and perpendicular to line AO.



Prob. F2–31

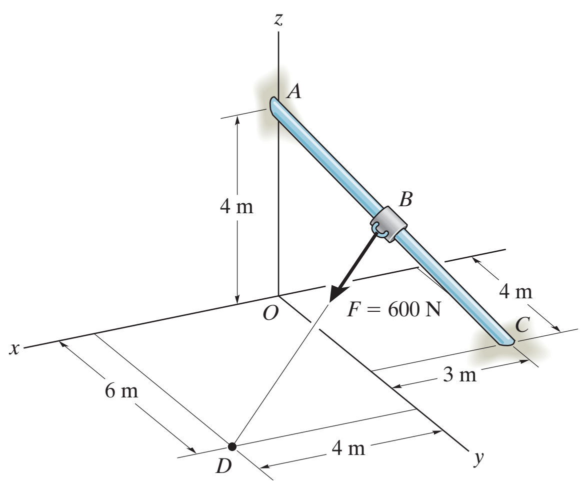
PROBLEMS

2–106. Express the force **F** in Cartesian vector form if it acts at the midpoint B of the rod.



Prob. 2–106

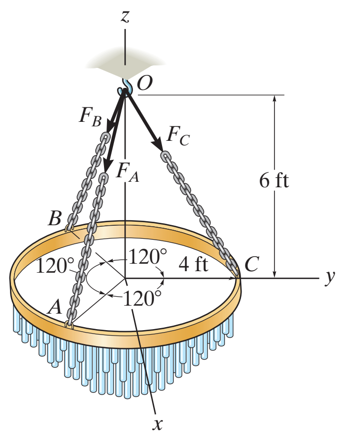
2–107. Express force **F** in Cartesian vector form if point B is located 3 m along the rod from end C.



Prob. 2–107

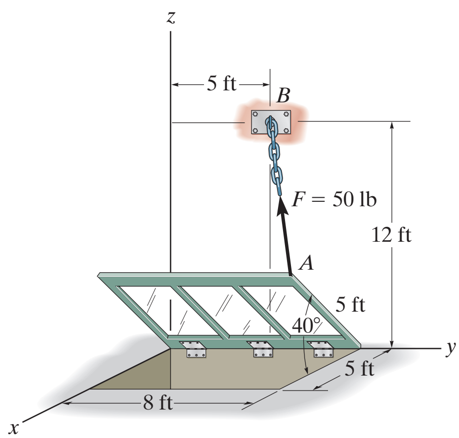
***2–108.** The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.

2–109. The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



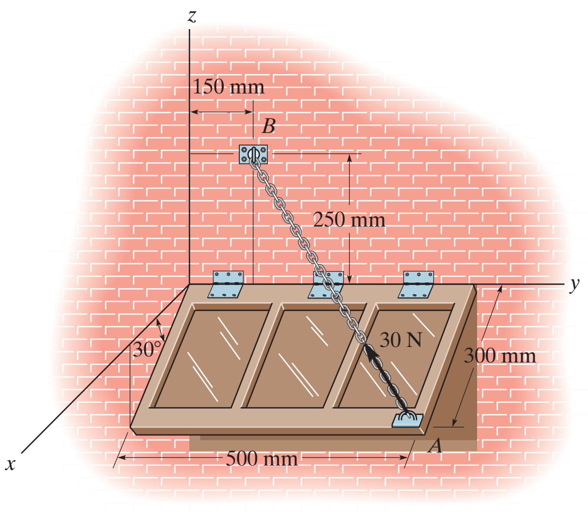
Probs. 2–108/109

2–110. The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



Prob. 2–110

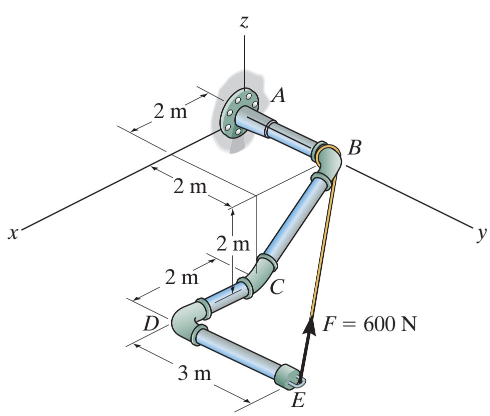
2–111. The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



Prob. 2–111

***2–112.** Given the three vectors **A, B,** and **D,** show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.

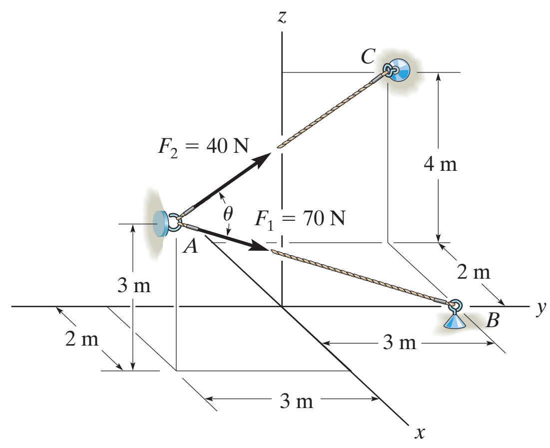
2–113. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



Probs. 2–112/113

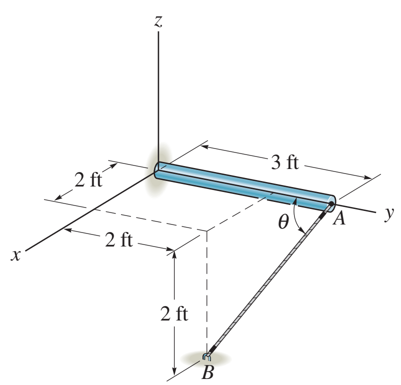
2–114. Determine the angle θ between the two cables.

2–115. Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



Probs. 2–114/115

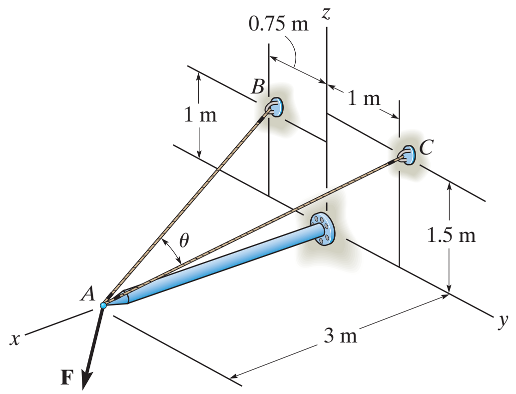
***2–116.** Determine the angle θ between the y axis of the pole and the wire AB.



Prob. 2–116

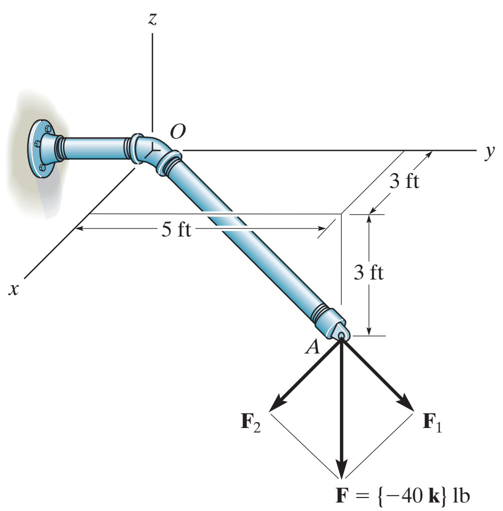
2–117. Determine the magnitudes of the projected components of the force $\mathbf{F} = [60\mathbf{i} + 12\mathbf{j} - 40\mathbf{k}]$ N along the cables AB and AC.

2–118. Determine the angle θ between cables AB and AC.



Probs. 2–117/118

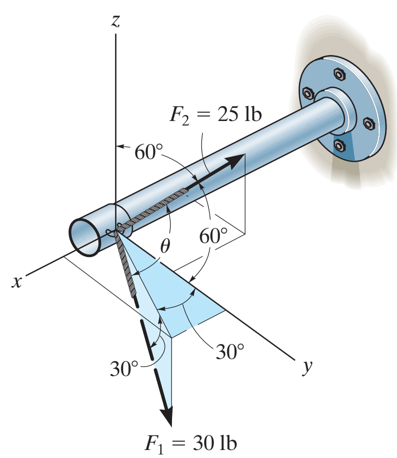
2–119. A force of $\mathbf{F} = \{-40\mathbf{k}\}$ lb acts at the end of the pipe. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which are directed along the pipe's axis and perpendicular to it.



Prob. 2–119

***2–120.** Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.

2–121. Determine the angle θ between the two cables attached to the pipe.

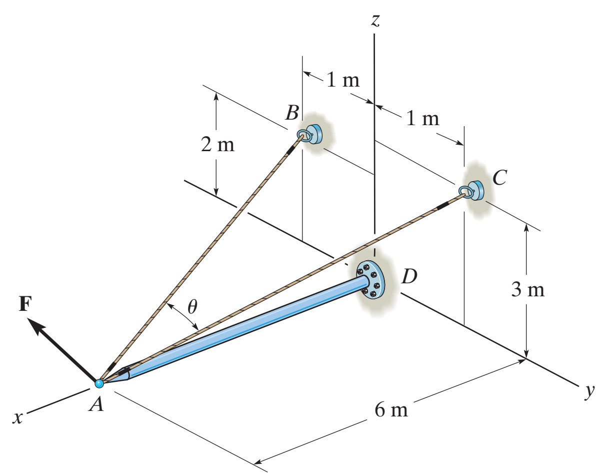


Probs. 2–120/121

2–122. Determine the angle θ between the cables AB and AC.

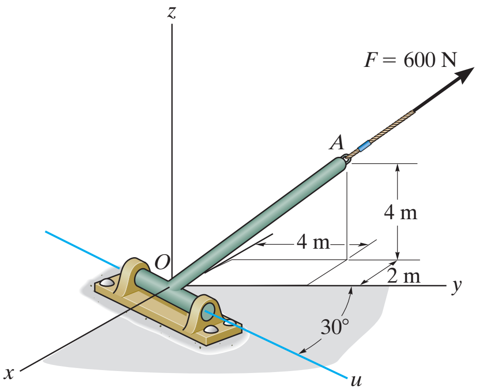
2–123. Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable BA.

***2–124.** Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable CA.



Probs. 2–122/123/124

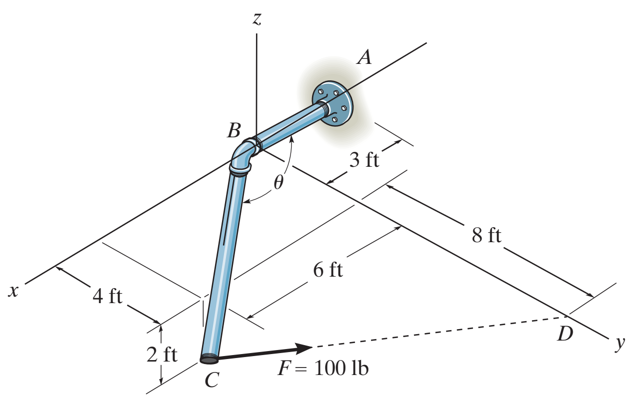
2–125. Determine the magnitude of the projection of force $F = 600$ N along the u axis.



Prob. 2–125

2–126. Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.

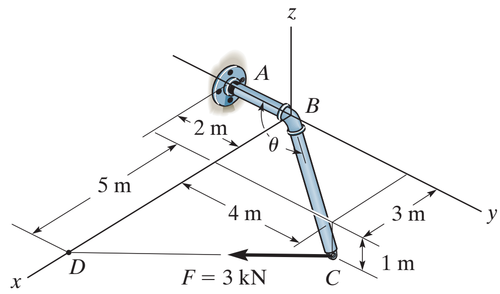
2–127. Determine the angle θ between pipe segments BA and BC.



Probs. 2–126/127

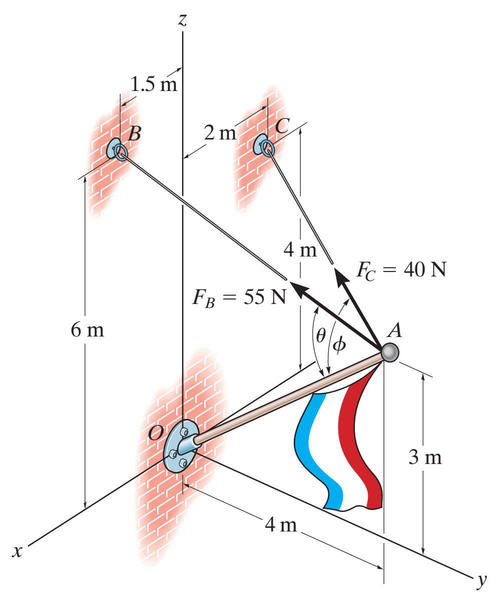
***2–128.** Determine the angle θ between BA and BC.

2–129. Determine the magnitude of the projected component of the 3 kN force acting along the axis BC of the pipe.



Probs. 2–128/129

2–130. Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.

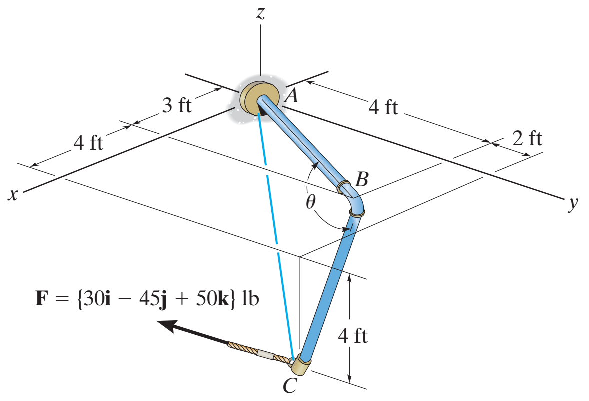


Prob. 2–130

2–131. Determine the magnitudes of the components of **F** acting along and perpendicular to segment BC of the pipe assembly.

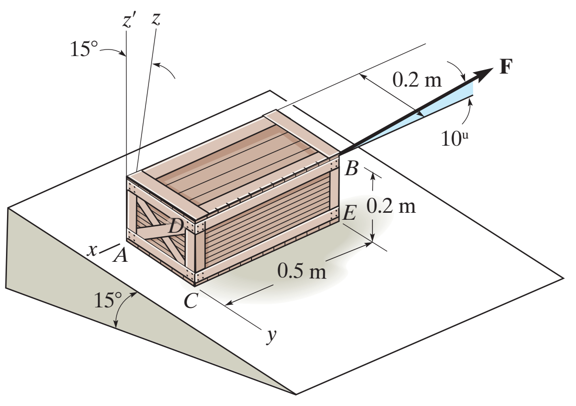
***2–132.** Determine the magnitude of the projected component of **F** along AC. Express this component as a Cartesian vector.

2–133. Determine the angle θ between the pipe segments BA and BC.



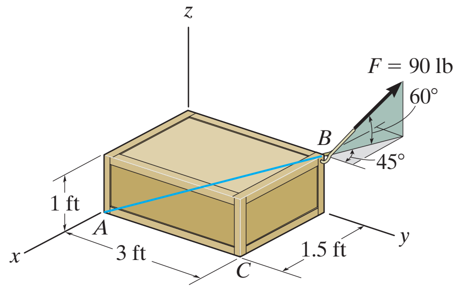
Probs. 2–131/132/133

2–134. If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x–z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that **F** makes with the diagonal AB of the crate.



Prob. 2–134

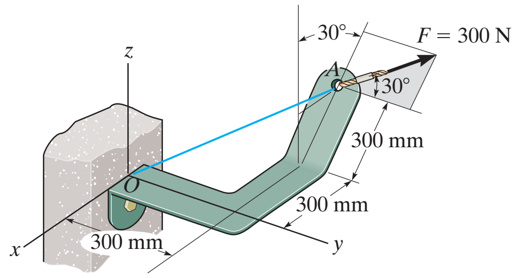
2–135. Determine the magnitudes of the components of the force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



Prob. 2–135

***2–136.** Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.

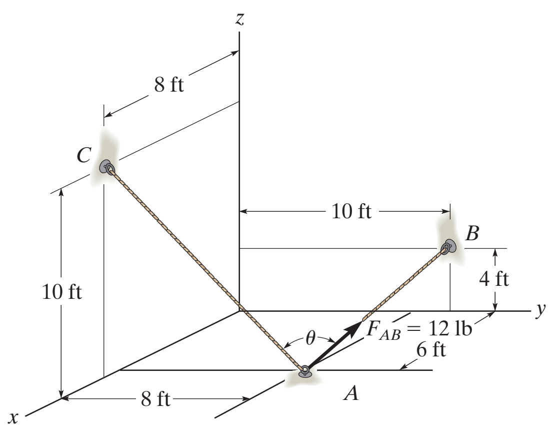
2–137. Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



Probs. 2–136/137

2–138. Determine the angle θ between the two cables.

2–139. Determine the projected component of the force $F = 12$ lb acting in the direction of cable AC. Express the result as a Cartesian vector.

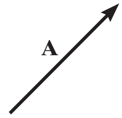


Probs. 2–138/139

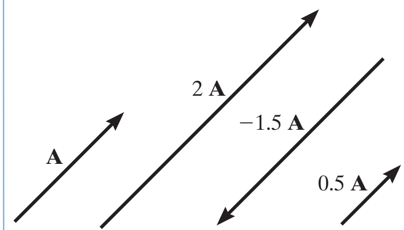
CHAPTER REVIEW

A scalar is a positive or negative number; e.g., mass and temperature.

A vector has a magnitude and direction, where the arrowhead represents the sense of the vector.

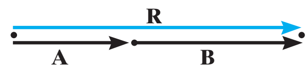


Multiplication or division of a vector by a scalar will change only the magnitude of the vector. If the scalar is negative, the sense of the vector will change so that it acts in the opposite sense.



If vectors are collinear, the resultant is simply the algebraic or scalar addition.

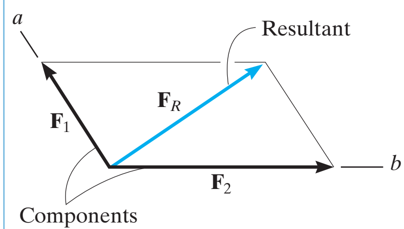
$$R = A + B$$



Parallelogram Law

Two forces add according to the parallelogram law. The *components* form the sides of the parallelogram and the *resultant* is the diagonal.

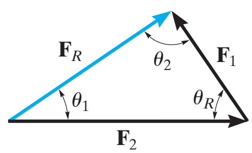
To find the components of a force along any two axes, extend lines from the head of the force, parallel to the axes, to form the components.



To obtain the components of the resultant, show how the forces add by tip-to-tail using the triangle rule, and then use the law of cosines and the law of sines to calculate their values.

$$F_R = \sqrt{F_1^2 + F_2^2 - 2\,F_1F_2\cos\theta_R}$$

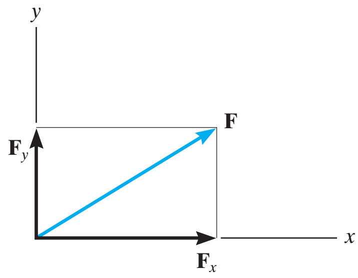
$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_R}{\sin\theta_R}$$



2

Rectangular Components: Two Dimensions

Vectors $\mathbf{F}_x$ and $\mathbf{F}_y$ are rectangular components of $\mathbf{F}$.



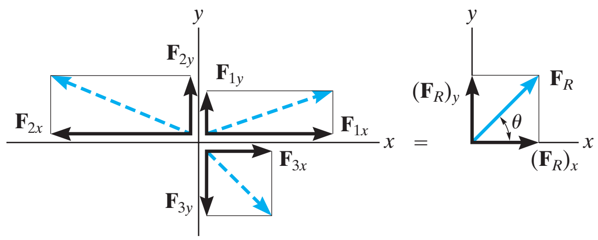
The resultant force is determined from the algebraic sum of its components.

$$(F_R)_x = \Sigma F_x$$

$$(F_R)_y = \Sigma F_y$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

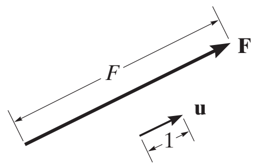
$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$



Cartesian Vectors

The unit vector $\mathbf{u}$ has a length of 1, no units, and it points in the direction of the vector $\mathbf{F}$.

$$\mathbf{u} = \frac{\mathbf{F}}{F}$$



A force can be resolved into its Cartesian components along the x, y, z axes so that $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$.

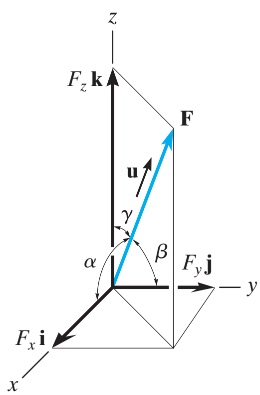
The magnitude of $\mathbf{F}$ is determined from the positive square root of the sum of the squares of its components.

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

The coordinate direction angles α, β, γ are determined by formulating a unit vector in the direction of $\mathbf{F}$. The x, y, z components of $\mathbf{u}$ represent $\cos\alpha, \cos\beta, \cos\gamma$.

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$\mathbf{u} = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$



The coordinate direction angles are related so that only two of the three angles are independent of one another.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

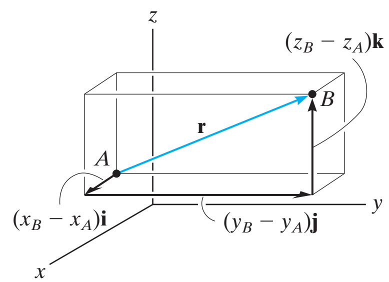
To find the resultant of a concurrent force system, express each force as a Cartesian vector and add the **i**, **j**, **k** components of all the forces in the system.

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

Position and Force Vectors

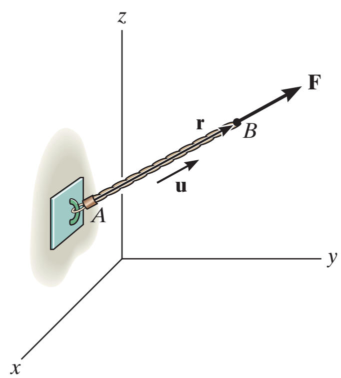
A position vector locates one point in space relative to another. The easiest way to formulate the components of a position vector is to determine the distance and direction that one must travel along the x, y, and z directions—going from the tail to the head of the vector.

$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$



If the line of action of a force passes through points A and B, then the force acts in the same direction as the position vector **r**, which is defined by the unit vector **u**. The force can then be expressed as a Cartesian vector.

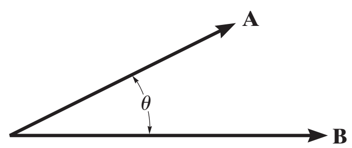
$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$



Dot Product

The dot product between two vectors **A** and **B** yields a scalar. If **A** and **B** are expressed in Cartesian vector form, then the dot product is the sum of the products of their x, y, and z components.

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

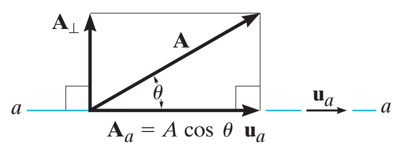


The dot product can be used to determine the angle between **A** and **B**.

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$

The dot product is also used to determine the projected component of a vector **A** onto an axis aa defined by its unit vector $\mathbf{u}_a$.

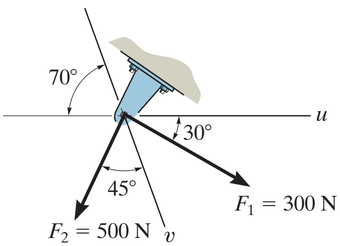
$$\mathbf{A}_a = A\cos\theta\,\mathbf{u}_a = (\mathbf{A}\cdot\mathbf{u}_a)\mathbf{u}_a$$



REVIEW PROBLEMS

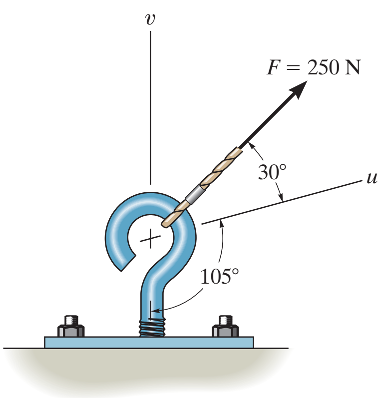
Partial solutions and answers to all Review Problems are given in the back of the book.

R2–1. Determine the magnitude of the resultant force $\mathbf{F}_R$ and its direction, measured clockwise from the positive u axis.



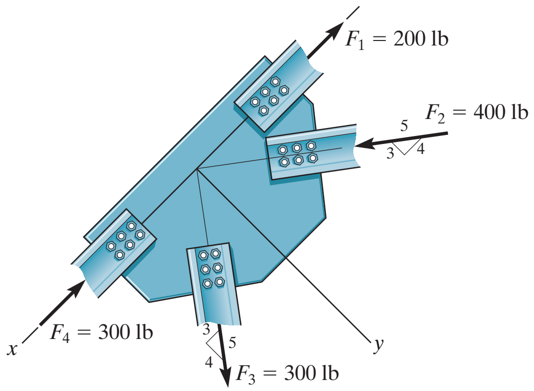
Prob. R2–1

R2–2. Resolve **F** into components along the u and v axes and determine the magnitudes of these components.



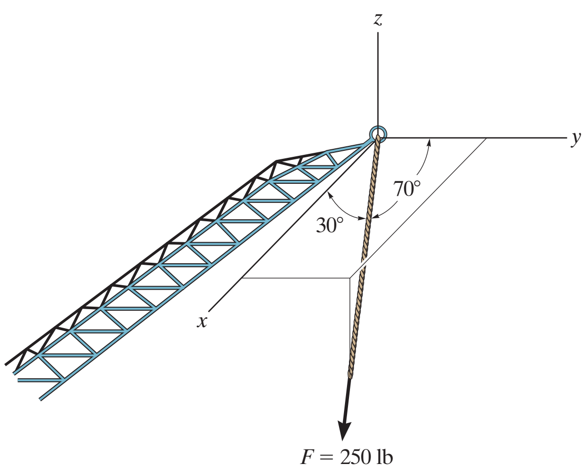
Prob. R2–2

R2–3. Determine the magnitude of the resultant force acting on the *gusset plate* of the bridge truss.



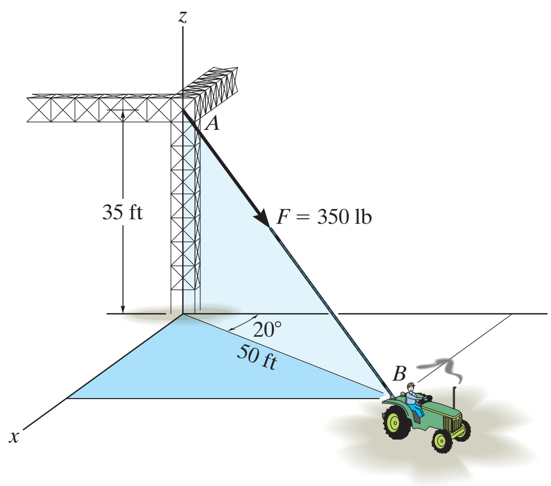
Prob. R2–3

R2–4. The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



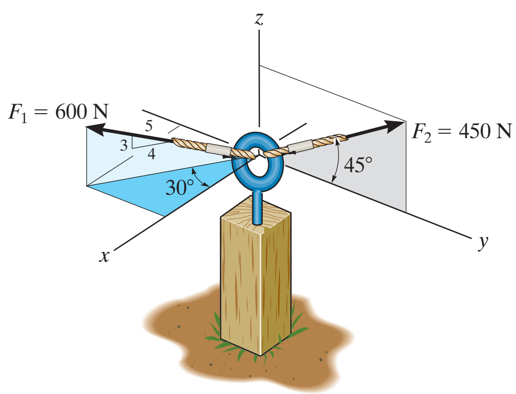
Prob. R2–4

R2–5. The cable attached to the tractor at B exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



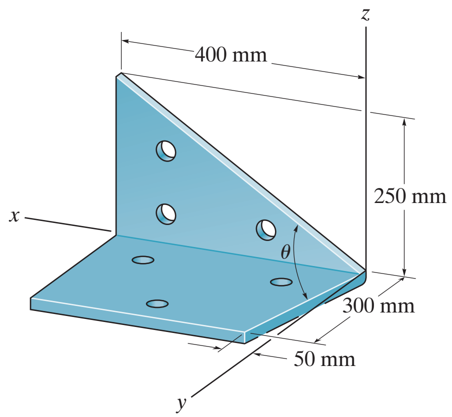
Prob. R2–5

R2–6. Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



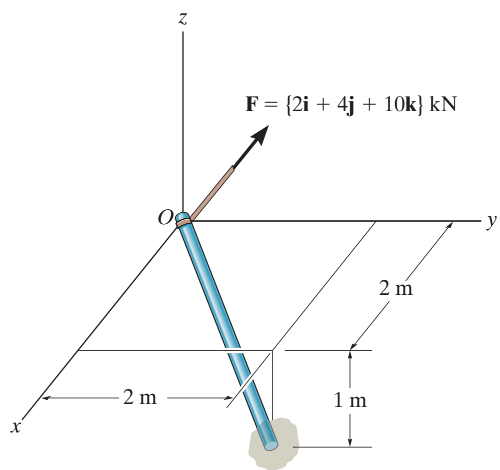
Prob. R2–6

R2–7. Determine the angle θ between the edges of the sheet-metal bracket.



Prob. R2–7

R2–8. Determine the projection of the force $\mathbf{F}$ along the pole.



Prob. R2–8