Lec. 1.

Components of Electric Power Systems

The intention of this lecture is to lay the groundwork for the study of electric power systems. This is done by developing some basic tools involving concepts, definitions, and some procedures fundamental to electric power systems. The lecture can be considered as a simple review of topics that will be utilized throughout this work. We start by introducing the principal electrical quantities that we will deal with in subsequent lectures.

2.2 Power Concepts

2.2.1 Single-Phase Systems

The electric power systems specialist is in many instances more concerned with electric power in the circuit rather than the currents. To study steady-state behavior of circuits, some further definitions are necessary. Consider a sinusoidal voltage, *v (t)* in given by

$$v(t) = V_m \cos(\omega t) \tag{1}$$

$$i(t) = I_m \cos(\omega t - \phi) \tag{2}$$

Note that in this case, the current lags the voltage by an angle φ. The instantaneous power is defined as

$$p(t) = v(t) \cdot i(t) \tag{3}$$

$$p(t) = V_m \cos(\omega t) \cdot I_m \cos(\omega t - \phi)$$

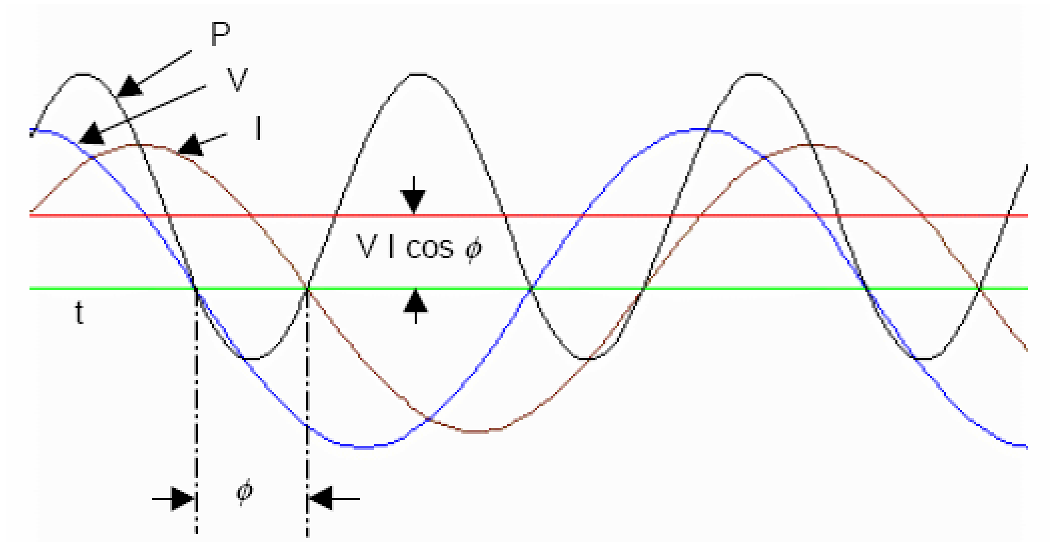


Figure 2.1 Current, Voltage, and Power Plotted Versus Time.

The angle φ in these equations is positive for current lagging the voltage and negative for current leading the voltage. Using the trigonometric identity

$$\cos\alpha\cdot\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right] ,$$

the instantaneous power can be written as:

$$p(t) = \frac{V_m\ I_m}{2}\left[\cos\phi + \cos(2\omega t - \phi)\right] \tag{4}$$

A more useful quantity is the average power that is being delivered. This can be obtained by averaging the instantaneous power over a specified time-period, typically for one cycle. Since the average of cos ($2\omega t - \varphi$) is zero, through one complete cycle, the average power, P, becomes

$$P = \frac{V_m\ I_m}{2}\cos\phi \tag{5}$$

It is more convenient to use the effective (rms) values of voltage and current than the maximum values. Substituting $V_m = \sqrt{2}\ V_{rms}$ and $I_m = \sqrt{2}\ I_{rms}$, we get

$$P = V_{rms}\ I_{rms}\ \cos\phi \qquad (6)$$

where, cos (φ) is called the power factor (PF).

Lagging power factor means the current lags the voltage by an angle φ, see Figure 2.2. Leading power factor means the current leads the voltage by an angle φ, see Figure 2.3.

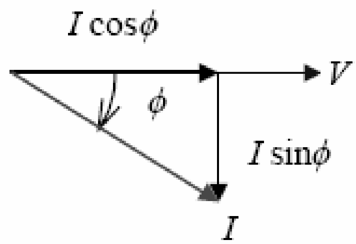


Figure 2.2 Phasor Diagram for Lagging Power Factor.

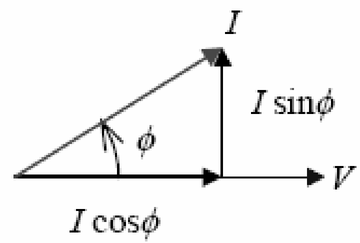


Figure 2.3 Phasor Diagram for Leading Power Factor.

COMPLEX POWER

If the phasor expressions for voltage and current are known, the calculation of real and reactive power is accomplished conveniently in complex form. For a certain load or part of a circuit, the rms values of the voltage drop and the current flow are expressed as:

$V = |V \angle 0^\circ$ and $I = I \angle -\phi$

It is common convention in the electric power industry to set the voltage angle as the angular reference. The complex power or the *apparent* power *S* is defined as the product of voltage times the conjugate of current, or

$$S = V \cdot I^* = V \cdot I \angle\phi$$
$$S = V \cdot I\cos\phi + jV \cdot I\sin\phi \qquad (7)$$

where φ is the phase angle between the voltage and current. Equation 7 can be written as:

$$S = P + jQ \qquad (VA) \tag{8}$$

Where

$$\begin{aligned} P &= |V \cdot I \cos(\phi) \qquad (W) \\ Q &= V \cdot I \sin(\phi) \qquad (VAr) \end{aligned} \tag{9}$$

The power factor is therefore:

$$\cos\phi = \cos\left[\arctan\left(Q/P\right)\right] = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{P}{|S|} \tag{10}$$

<u>POWER TRIANGLE</u>

Equation 8 suggests a graphical method of obtaining the overall P, Q, and phase angle, φ, for several loads in parallel. A power triangle can be drawn for an inductive load as shown in Figure 2.4. The signs of P and Q are important in knowing the direction of the power flow, that is, whether power is being generated or absorbed when a voltage and current are specified.

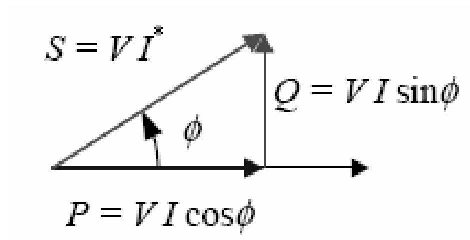


Figure 2.4 Power Triangle for an Inductive Load.

Lec. 2.

EXAMPLE 2.1

Consider the circuit shown in Figure 2.5 with the following parameters:

R = 0.5 Ω L = 2.122 mH

C = 1600 μ F V = 100∠ 0° V

-

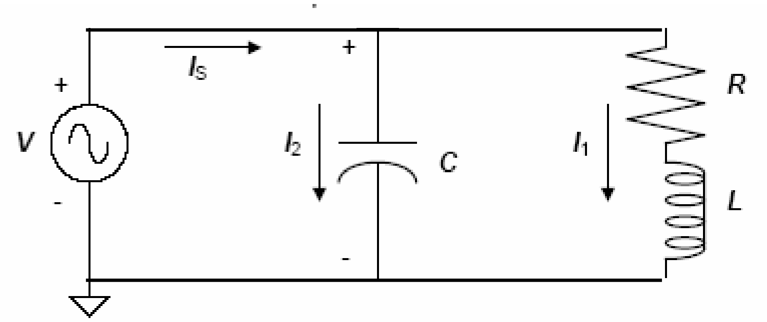


Figure 2.5 Circuit of Example 2.1.

Find the following: (a) the source current, (b) the active, reactive, and apparent power into the circuit, (c) the power factor of the circuit.

a) source current

$$I_1 = \frac{V}{R + j\omega L} = \frac{100\angle 0° \text{ V}}{0.5\ \Omega + j(377 \text{ r/s})(2.122\times 10^{-3} \text{ H})} = 106.0\angle -58.0° \text{ A}$$

$$I_2 = V\cdot(j\omega C) = j(100 \text{ V})(377 \text{ r/s})(1.6\times 10^{-3} \text{ F}) = 60.3\angle 90° \text{ A}$$

$$I_S = I_1 + I_2 = 106.0\angle -58.0 \text{ A} + 60.3\angle 90° \text{ A} = 63.5\angle -27.8° \text{ A}$$

b) power flows

$$S = V\cdot I^* = (100\angle 0° \text{ V})(63.5\angle 27.8° \text{ A}) = 6350\angle 27.8° \text{ VA}$$

$$S = 5617 + j2962 \text{ VA}$$

$$P = 5617 \text{ W}$$

$$Q = 2962 \text{ Var}$$

c) power factor

$$\phi = 27.8°$$
$$PF = \cos(\phi) = \cos(27.8°) = 0.88 \quad \text{lagging}$$

2.2.2 Three-Phase Systems

The major assumption of all the electric power presently used is generated, transmitted, and distributed using balanced three-phase voltage systems. Three-phase operation is preferable to single-phase because a three-phase system is more efficient than a single-phase system, and the flow of power is constant.

A balanced three-phase voltage system is composed of three-single phase voltage sources having the same magnitude and frequency but time-displaced from one another by 120° of a cycle as shown in the phasor diagrams of Figure 2.6.

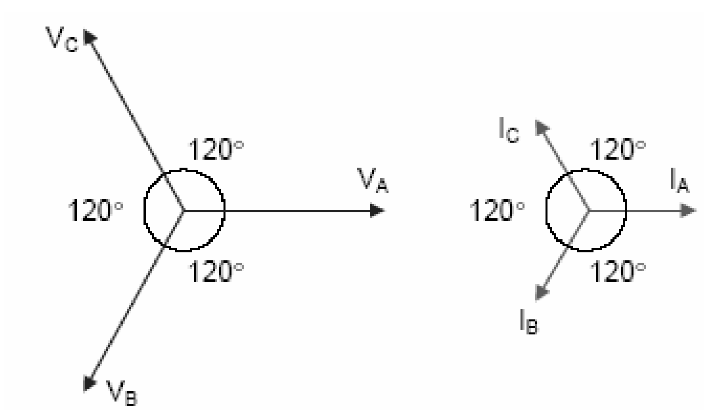


Figure 2.6 Phasor Diagrams of a Balanced Three-Phase System.

There are two possible connections of loads and sources in three-phase systems: wye-connection and delta-connection. Figure 2.7 shows the two types of connections for three-identical impedances.

Because of the connections, new voltages and current quantities can be defined. Starting with the voltage, the line voltage or line-to-line voltage is that quantity between two supply lines or load terminals. For voltage variables, line-to-line quantities have subscripts with two phases (i.e., ab, bc, or ca). The line-to-neutral or line-to-ground voltage is that quantity between a supply line and the neutral or ground node of the circuit. The voltage variables for these have subscripts of only one phase or one phase and n for neutral or g for ground. For currents, a similar convention is used. A current flowing through an impedance or source between two lines is given a double subscript notation denoting the two phases. A current flowing though an impedance or source between a line and the neutral point may be either a single subscript or one followed by an n. A line current flowing from a source to a load may be a single or a double 4

subscript notation denoting the particular phase or the symbols representing the nodes at each end of the line (i.e., a or Aa).

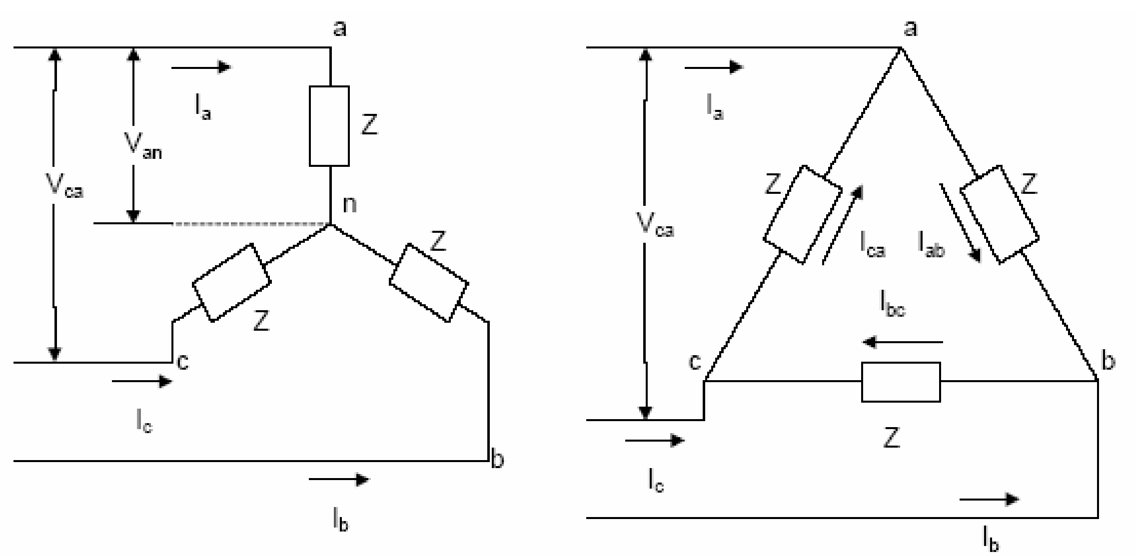


Figure 2.7 (a) Wye-Connected Load (b) Delta-Connected Load

CURRENT AND VOLTAGE RELATIONS IN THE WYE CONNECTION

The voltage appearing between any two of the line terminals a, b, and c have different relationships in magnitude and phase to the voltages appearing between any one terminal and the neutral point n. The set of voltages V_{ab}, V_{bc}, and V_{ca} are called the line voltages, and the set of voltages V_{an}, V_{bn}, and V_{cn} are referred to as the phase voltages. Analysis of a phasor diagram provides the required relationships.

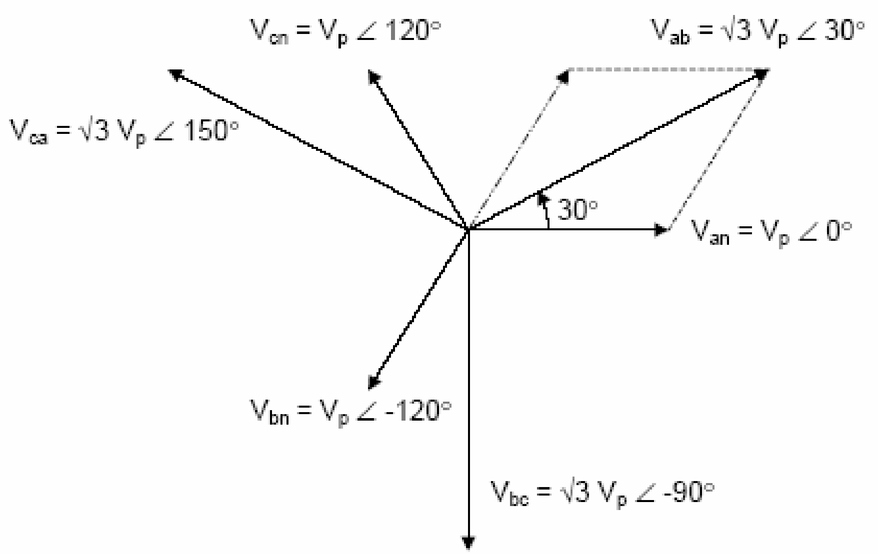


Figure 2.8 Phasor Diagram of the Phase and Line Voltages of a Wye-Connection.

The effective values of the phase voltage are shown in Figure 2.8 as Van, Vbn, and Vcn. Each has the same magnitude, and each is displaced 120° from the other two phasors. The relation between the line voltage and the phase voltage at the terminal a and b can be written as:

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} \\ &= V_p\angle 0° - V_p\angle -120° \\ &= \sqrt{3}\cdot V_p\angle 30° \end{aligned} \tag{11}$$

Similarly,

$$V_{bc} = \sqrt{3} \cdot V_p \angle -90°$$
$$V_{ca} = \sqrt{3} \cdot V_p \angle 150°$$

Thus the relation between line-to-line voltage, V_L and phase voltage Vp for a balanced wye-connected, three-phase voltage system is

$$V_L = \sqrt{3} \cdot V_p \angle +30° \tag{12}$$

The current flowing out of a line terminal is the same current that is flowing through the phase terminal. Thus the relation between the line current I_L and phase current Ip for a wye-connected, three-phase system is

$$I_L = I_p \tag{13}$$

CURRENT AND VOLTAGE RELATIONS IN THE DELTA CONNECTION

Consider now the case when three single-phase sources are rearranged to form a three-phase delta connection as shown in Figure 2.9. It is clear from the circuit shown that the line and phase voltages are the same. Thus:

$$V_L = V_p \tag{14}$$

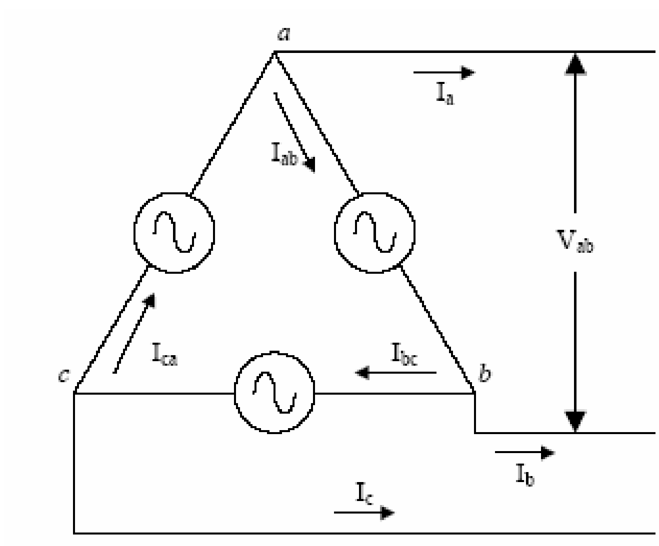


Figure 2.9 Delta-Connected, Three-Phase Source.

The phase and line currents, however, are not identical and the relationships between them can be obtained as:

$$I_{ab} = I_p \angle 0°$$
$$I_{bc} = I_p \angle -120°$$
$$I_{ca} = I_p \angle 120°$$

Also, from Figure 2.9, the relation between the line and phase currents can be obtained as:

$$\begin{aligned} I_a &= I_{ca} - I_{ab} = I_p \angle 120° - I_p \angle 0° \\ &= \sqrt{3} \cdot I_p \angle 150° \end{aligned}$$

Similarly,

$$I_b = \sqrt{3} \cdot I_p \angle 30°$$
$$I_c = \sqrt{3} \cdot I_p \angle -90°$$

The phasor diagram in Figure 2.10 illustrates these relations. Thus the relation between line and phase currents for a balanced delta-connected system is:

$$I_L = \sqrt{3} \cdot I_p \angle +30° \qquad (15)$$

Note that in the equations above, V_L, Vp, I_L, and Ip are the rms or effective values of voltages and currents.

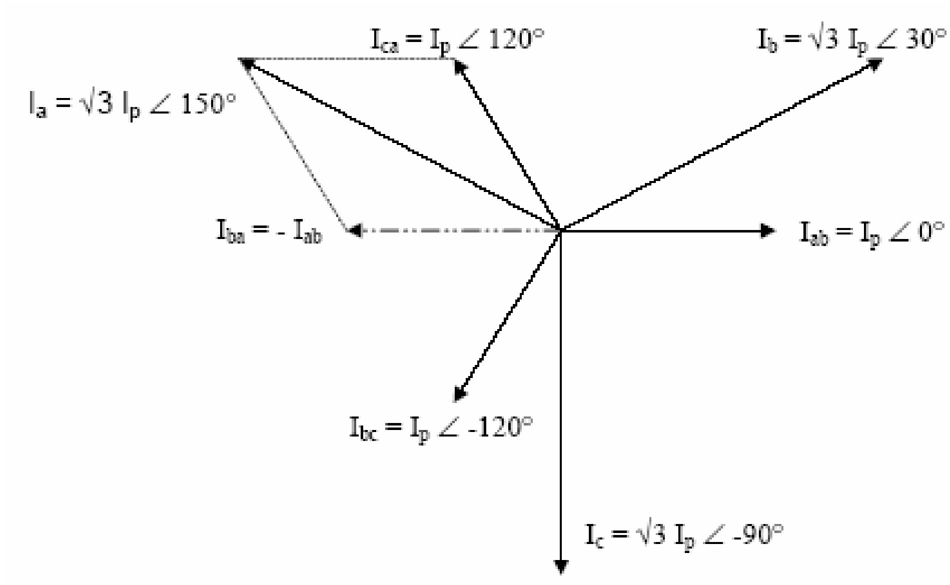


Figure 2.10 Relations between Phase and Line Currents in a Delta Connection.

POWER RELATIONSHIPS

Assume that a balanced three-phase voltage source is supplying a balanced load. The three sinusoidal phase voltages can be written as:

$$V_a(t) = \sqrt{2} \cdot V_p \sin(\omega t)$$
$$V_b(t) = \sqrt{2} \cdot V_p \sin(\omega t - 120°)$$
$$V_c(t) = \sqrt{2} \cdot V_p \sin(\omega t + 120°)$$

with the currents given by

$$I_a(t) = \sqrt{2} \cdot I_p \sin(\omega t - \phi)$$
$$I_b(t) = \sqrt{2} \cdot I_p \sin(\omega t - 120° - \phi)$$
$$I_c(t) = \sqrt{2} \cdot I_p \sin(\omega t + 120° - \phi)$$

where φ is the phase angle between the current and voltage in each phase or the power factor angle.

The three-phase power can be defined as:

$$P_{3\phi} = V_a \cdot I_a \cdot \cos\phi + V_b \cdot I_b \cdot \cos\phi + V_c \cdot I_c \cdot \cos\phi$$

Using a trigonometric identity, we get the following:

$$P_{3\phi} = V_p I_p \{3\cos\phi - [\cos(2\omega t - \phi) + \cos(2\omega t - 240 - \phi) + \cos(2\omega t + 240 - \phi)]\}$$

The summation of the last three terms, in the above equation, is zero. Thus the three-phase power can be obtained as:

$$P_{3\phi} = 3 \cdot V_p I_p \cos(\phi) \tag{16}$$

In wye-connected systems, $I_p = I_L$ and $V_p = V_L/\sqrt{3}$, and in delta-connected systems, $I_p = I_L/\sqrt{3}$ and $V_p = V_L$. Thus, the power equation, Equation 16, reads in terms of line quantities:

$$P_{3\phi} = \sqrt{3} \cdot V_L I_L \cos(\phi) \tag{17}$$

Note that Equations 16 and 17 apply for both wye-and delta-connected systems.

COMPLEX POWER

The above analysis can be extended to include the reactive power, Q, or to get the apparent power, S, for a three-phase system.

$$\begin{aligned} S_{3\phi} &= 3 \cdot V_p \cdot I_p^* \\ &= \sqrt{3} \cdot V_L \cdot I_L^* \end{aligned} \tag{18}$$

If

$$V_p = |V_p|\angle 0^\circ \text{ and } I_p = |I_p|\angle -\phi \text{, then}$$

$$S_{3\phi} = 3|V_p||I_p|\angle\phi$$

or in complex notation:

$$\begin{aligned} S_{3\phi} &= 3|V_p||I_p|(\cos\phi + j\sin\phi) \\ &= P_{3\phi} + jQ_{3\phi} \end{aligned}$$

where

$$P_{3\phi} = 3|V_p||I_p|\cos\phi = \sqrt{3}|V_L||I_L|\cos\phi \quad (19)$$

$$Q_{3\phi} = 3|V_p||I_p|\sin\phi = \sqrt{3}|V_L||I_L|\sin\phi \quad (20)$$

<u>EXAMPLE 2.2</u>

A wye-connected, balanced three-phase load consisting of three impedances of $10\Omega\angle 30°$ each as shown in Figure 2.11, is supplied with a balanced set of line-to-neutral voltages:

$$V_{an} = 220V\angle 0°$$
$$V_{bn} = 220V\angle 240°$$
$$V_{cn} = 220V\angle 120°$$

(a) Calculate the phasor currents in each line, b) calculate the line-to-line phasor voltages and show the corresponding phasor diagram, and c) calculate the apparent power, active power, and reactive power supplied to the load.

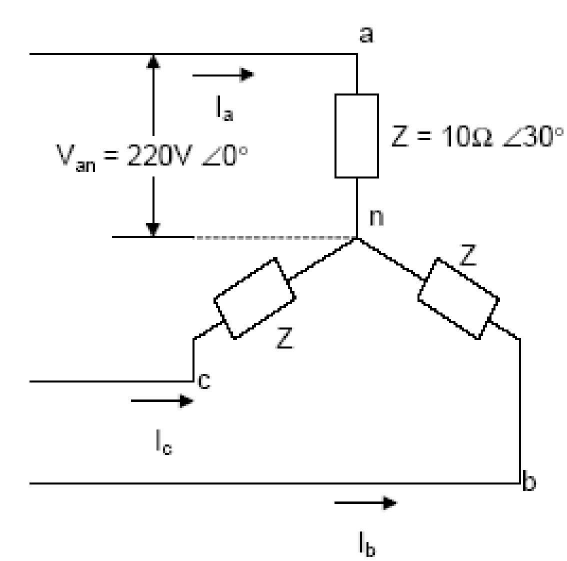


Figure 2.11 Load Connection for Example 2.2.

a) The phase currents of the loads are obtained as:

$$I_{an} = \frac{220V\angle 0^\circ}{10\Omega\angle 30^\circ} = 22A\angle -30^\circ$$
$$I_{bn} = \frac{220V\angle 240^\circ}{10\Omega\angle 30^\circ} = 22A\angle 210^\circ$$
$$I_{cn} = \frac{220V\angle 120^\circ}{10\Omega\angle 30^\circ} = 22A\angle 90^\circ$$

b) The line voltages are obtained as

$$\begin{aligned} V_{ab} &= V_a - V_b \\ &= 220V\angle 0^\circ - 220V\angle 240^\circ \\ &= 220 \cdot \sqrt{3}V\angle 30^\circ \end{aligned}$$

Or

$$\begin{aligned} V_{ab} &= \sqrt{3} \cdot |V_{an}|\angle(\delta_{an} + 30^\circ) \\ &= 220 \cdot \sqrt{3}V\angle 0^\circ + 30^\circ \\ &= 220 \cdot \sqrt{3}V\angle 30^\circ \end{aligned}$$

Similarlry

$$\begin{aligned} V_{bc} &= \sqrt{3} \cdot |V_{bn}|\angle(\delta_{bn} + 30^\circ) \\ &= 220 \cdot \sqrt{3}V\angle 240^\circ + 30^\circ \\ &= 220 \cdot \sqrt{3}V\angle -90^\circ \end{aligned}$$

$$\begin{aligned} V_{ca} &= \sqrt{3} \cdot |V_{cn}|\angle(\delta_{cn} + 30^\circ) \\ &= 220 \cdot \sqrt{3}V\angle 120^\circ + 30^\circ \\ &= 220 \cdot \sqrt{3}V\angle 150^\circ \end{aligned}$$

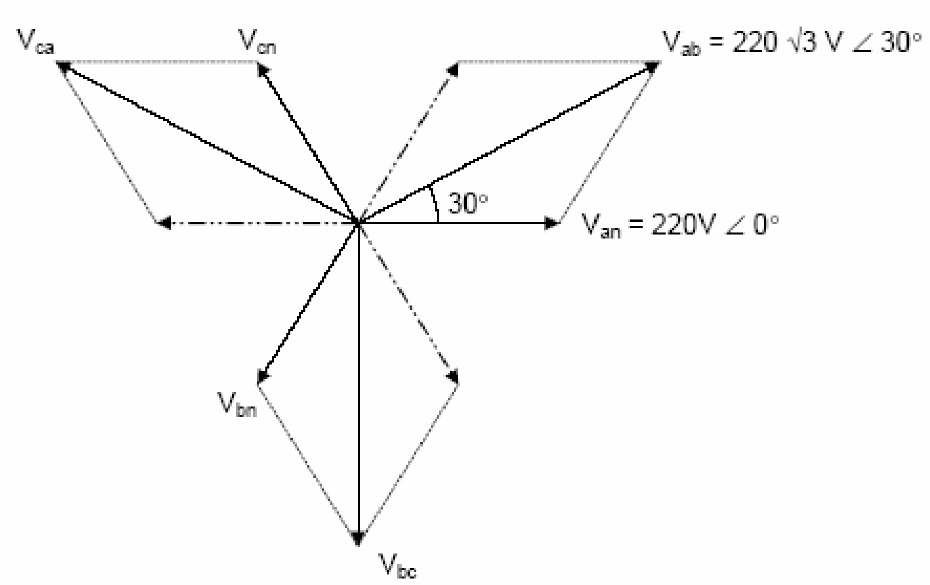


Figure 2.12 Relations between Phase and Line Voltages in a Wye-Connection.

c) The powers are given by:

$$
\begin{aligned}
S_{3\phi} &= 3 \cdot V_p \cdot I_p^* = 3 \cdot V_{an} \cdot I_{an}^* \\
&= 3(220V\angle 0°)(22A\angle 30°) = 14520.0\angle 30° \\
&= 12574.69 + j7260.0 \quad VA \\
P_{3\phi} &= 12574.69 \quad W \\
Q_{3\phi} &= 7260.0 \quad VAr
\end{aligned}
$$

<u>EXAMPLE 2.3</u>

Lec.4.

A delta-connected, balanced three-phase load consisting of three impedances of $10\angle\ 30°\Omega$ each as shown in Figure 2.13, is supplied with a balanced set of line-to-line voltages:

$$
\begin{aligned}
V_{ab} &= \sqrt{3} \cdot 220\angle 30° \text{ V} \\
V_{bc} &= \sqrt{3} \cdot 220\angle -90° \text{ V} \\
V_{ca} &= \sqrt{3} \cdot 220\angle 150° \text{ V}
\end{aligned}
$$

a) Calculate the phasor voltage across each phase load, b) calculate the phase and line currents and show the corresponding phasor diagram, and c) calculate the apparent power, active power, and reactive power supplied to the load.

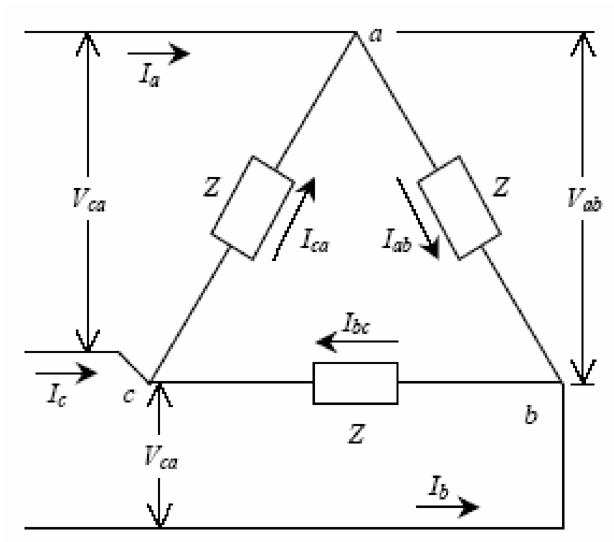


Figure 2.13 Load connection for example 2.3.

a) For a delta connected load,

$$V_{phs} = V_{LL}$$
$$V_{ab} = \sqrt{3}\cdot 220\angle 30^\circ \text{ V}$$
$$V_{bc} = \sqrt{3}\cdot 220\angle -90^\circ \text{ V}$$
$$V_{ca} = \sqrt{3}\cdot 220\angle 150^\circ \text{ V}$$

b) The phase currents in each of the impedances are:

$$I_{ab} = \frac{\sqrt{3}\cdot 220\angle 30^\circ \text{ V}}{10\angle 30^\circ\ \Omega} = \sqrt{3}\cdot 22\angle 0^\circ \text{ A}$$
$$I_{bc} = \sqrt{3}\cdot 22\angle 240^\circ \text{ A}$$
$$I_{ca} = \sqrt{3}\cdot 22\angle 120^\circ \text{ A}$$

The line currents can be obtained as:

$$I_a = I_{ab} - I_{ca}$$
$$= \sqrt{3}\cdot 22\angle 0^\circ - \sqrt{3}\cdot 22\angle 120^\circ = 66\angle -30^\circ \text{ A}$$

Or

$$I_L = \sqrt{3} \cdot I_p \angle -30^\circ$$
$$I_a = \sqrt{3} \cdot I_{ab} \angle -30^\circ = \sqrt{3}\left(22\sqrt{3}A\right)\angle 0^\circ - 30^\circ = 66A \angle -30^\circ$$

Similarly,

$$I_b = \sqrt{3} \cdot I_{bc} \angle -30^\circ = 66A \angle 210^\circ$$
$$I_c = \sqrt{3} \cdot I_{ca} \angle -30^\circ = 66A \angle 90^\circ$$

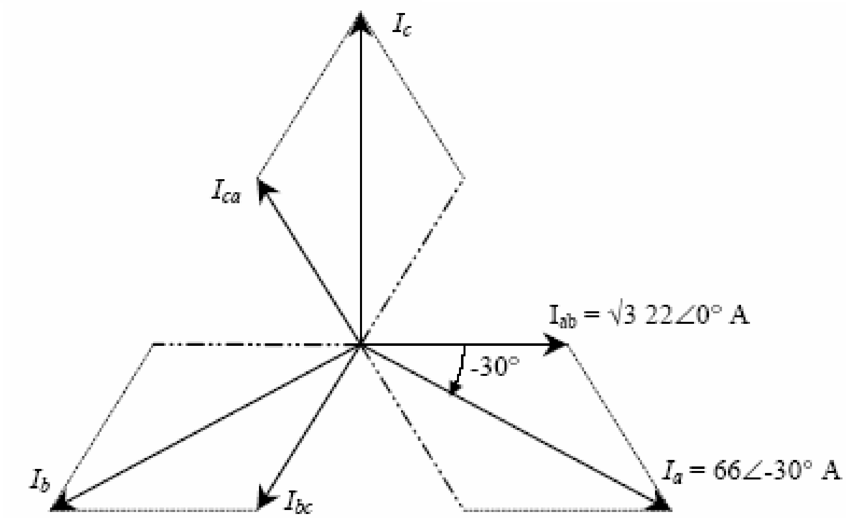


Figure 2.14 Relation between Phase and Line Currents in a Delta Connection.

c) The apparent, active, and reactive powers are:

$$\begin{aligned} S_{3\phi} &= 3 \cdot V_p \cdot I_p^* = 3 \cdot V_{ab} \cdot I_{ab}^* \\ &= 3\left(\sqrt{3} \cdot 220\angle 30^\circ\right)\left(\sqrt{3} \cdot 22\angle 0^\circ\right) = 43560\angle 30^\circ \text{ VA} \\ &= 37724.04 + j21780.0 \text{ VA} \end{aligned}$$
$$P_{3\phi} = 37724.04 \text{ W}$$
$$Q_{3\phi} = 21780.0 \text{ VAR}$$

2.3 Power System Representation

A major portion of the modern power system utilizes three-phase as circuits and devices. A balanced three-phase system is solved as a single-phase circuit made of one line and the neutral return. Standard symbols are used to indicate the various components. The simplified one-line diagram is called the single-line diagram. From the one-line diagram the impedance, or reactance, diagram can be conveniently developed, as shown in the following section. A further advantage of the one-line diagram is in the power flow studies. The one-line diagram rather becomes second nature to power system engineers as they attempt to visualize a widespread complex network.

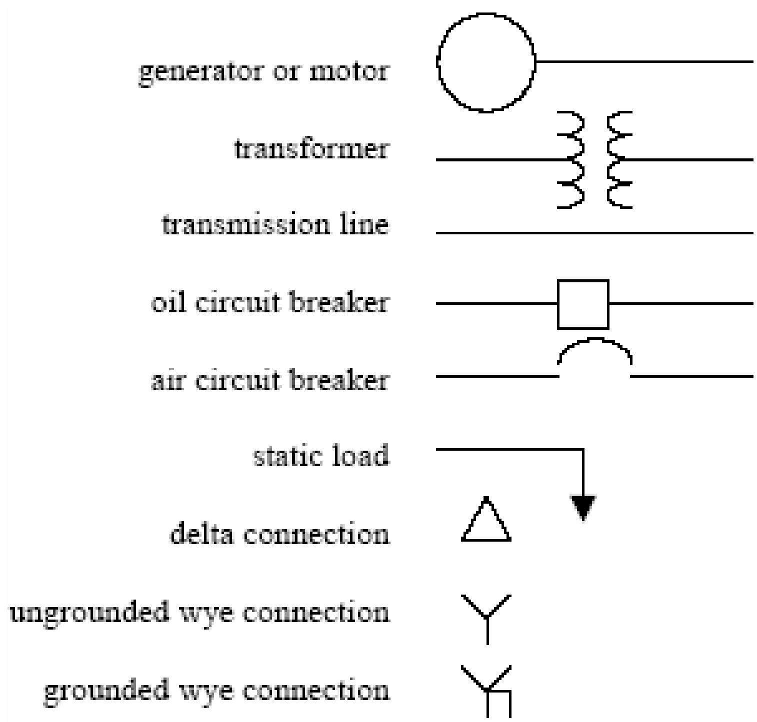


Figure 2.15 Symbolic Representation of Elements of a Power System.

Using the symbols in Figure 2.15, a section of a one-line diagram of a power system is shown in Figure 2.16.

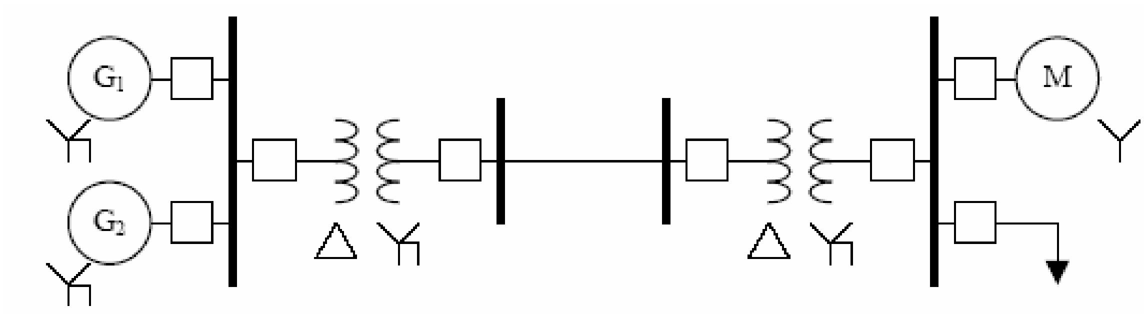


Figure 2.16 A One-Line Diagram of a Portion of a Power System.

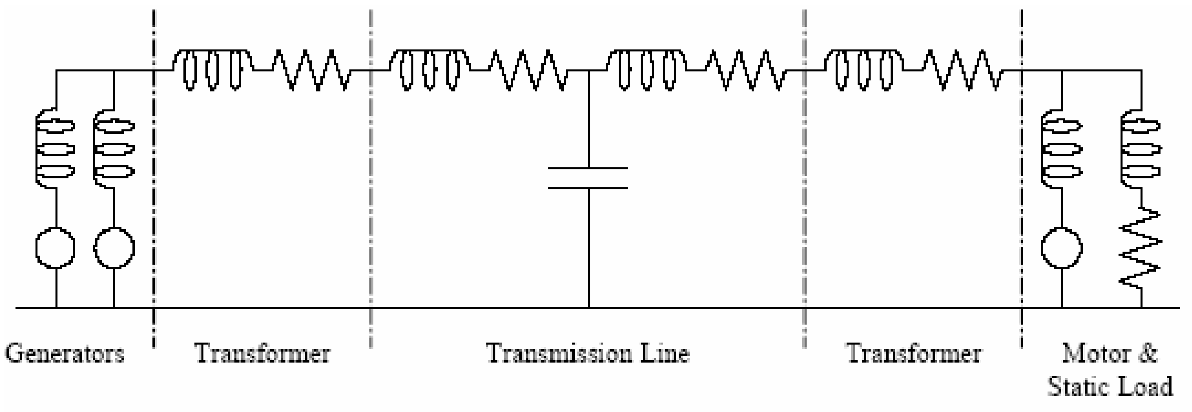
2.3.1 Equivalent Circuit and Reactance Diagram

We note from Figure 2.16 that the power system components are: generators, transformers, transmission lines, and loads. Equivalent circuits of these components may then be interconnected to obtain a circuit representation for the entire system. In other words, the one-line diagram may be replaced by an impedance diagram or a reactance diagram (if resistances are neglected).

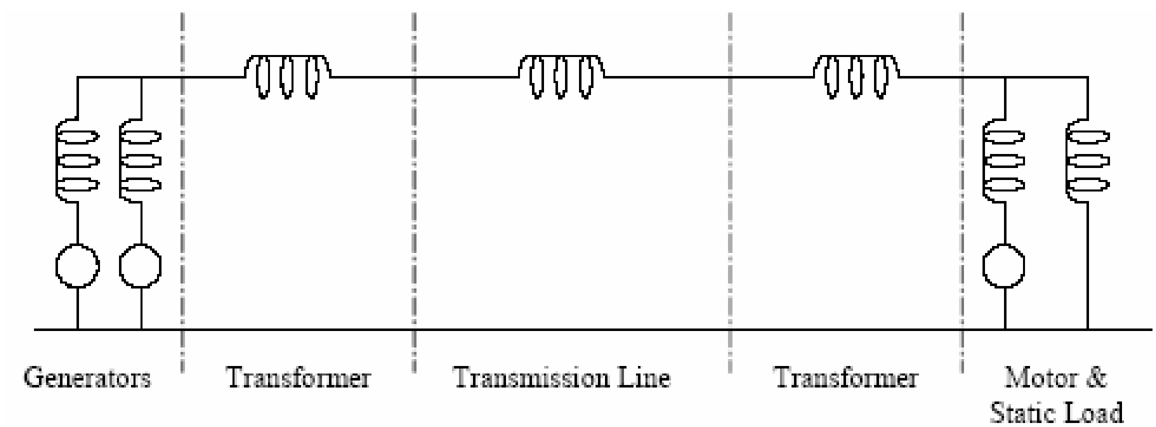
Thus, corresponding to Figure 2.16, the impedance and reactance diagrams are shown in Figures 2.17(a) and 2.17(b), respectively, on a per phase basis. In the equivalent circuit of the components in Figure 2.17(a) is based on the following assumptions:

1. A generator can be represented by a voltage source in series with an inductive reactance. The internal resistance of the generator is negligible compared to the reactance.
2. The motor load is inductive.
3. The static load has a lagging power factor.
4. A transformer is represented by a series impedance on a per phase basis.
5. The transmission line is of medium length and can be represented by a T section.

The reactance diagram, shown in Figure 2.17(b), is drawn by neglecting all the resistances, static loads, and capacitances of the transmission line. Reactance diagrams are generally used for short-circuit calculation, whereas the impedance diagram is used for power-flow studies.



(a) Impedance Diagram



(b) Corresponding Reactance Diagram

Figure 2.17 Electrical Diagrams of the System Illustrated in Figure 2.16.