Lectures on Power System Analysis

03

(LOAD FLOW STUDY) (Gauss-Seidel method)

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Objective of a Load Flow Study

- For *A specific Loading* (active and reactive)power, The load flow study is applied to *calculate the bus voltage magnitude and angl*e (|V|∟δ) at each bus. Also, the power flow in the transmission lines and the line losses can be calculated.

Description of the problem

A load flow problem is a nonlinear one. A multiple solution can be obtained. Some of the solutions are infeasible solutions.

Example 1

Find the bus voltages at bus 2 for the following two bus DC power system.

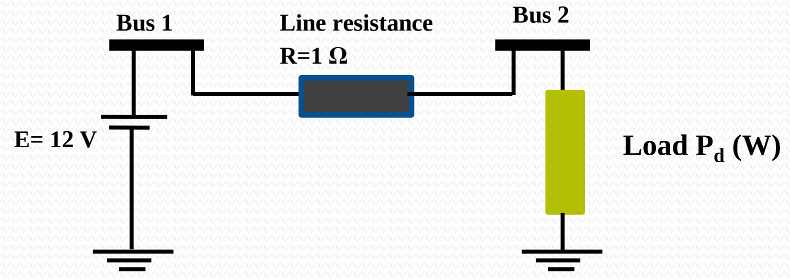
$$I = \frac{V_1 - V_2}{R} \quad \dots\dots\dots\dots\dots \; (1)$$

$$P_d = V_2 * I \dots\dots.(2)$$

Substituting (2) into (1) and rearranging yield:

$$V_2^2 - 12V_2 + P_d = 0 \dots\dots(3)$$

The voltage at bus 2 can be calculated using the formula of equation(4), the results obtained is given in Table 1

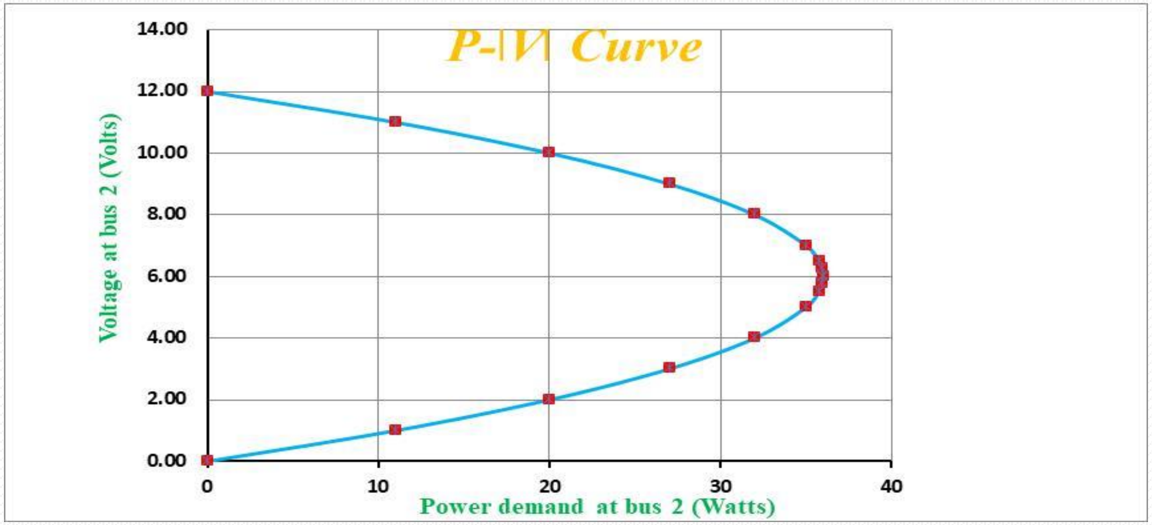


Two bus DC power system

$$V_2 = \frac{-V_1 \pm \sqrt{{V_1}^2 - 4P_d}}{2} \quad \dots\dots (4)$$

Table 1 Voltage solutions for different loading conditions

Loading condition P_d (Watts)	High solution V_2 (Volts)	Low solution V_2 (Volts)	
0	12	0	No load condition
11	11	1	
20	10	2	
27	9	3	
32	8	4	
35	7	5	
35.75	6.5	5.5	
35.9375	6.25	5.75	
36	6	6	Maximum loadability point(one solution)



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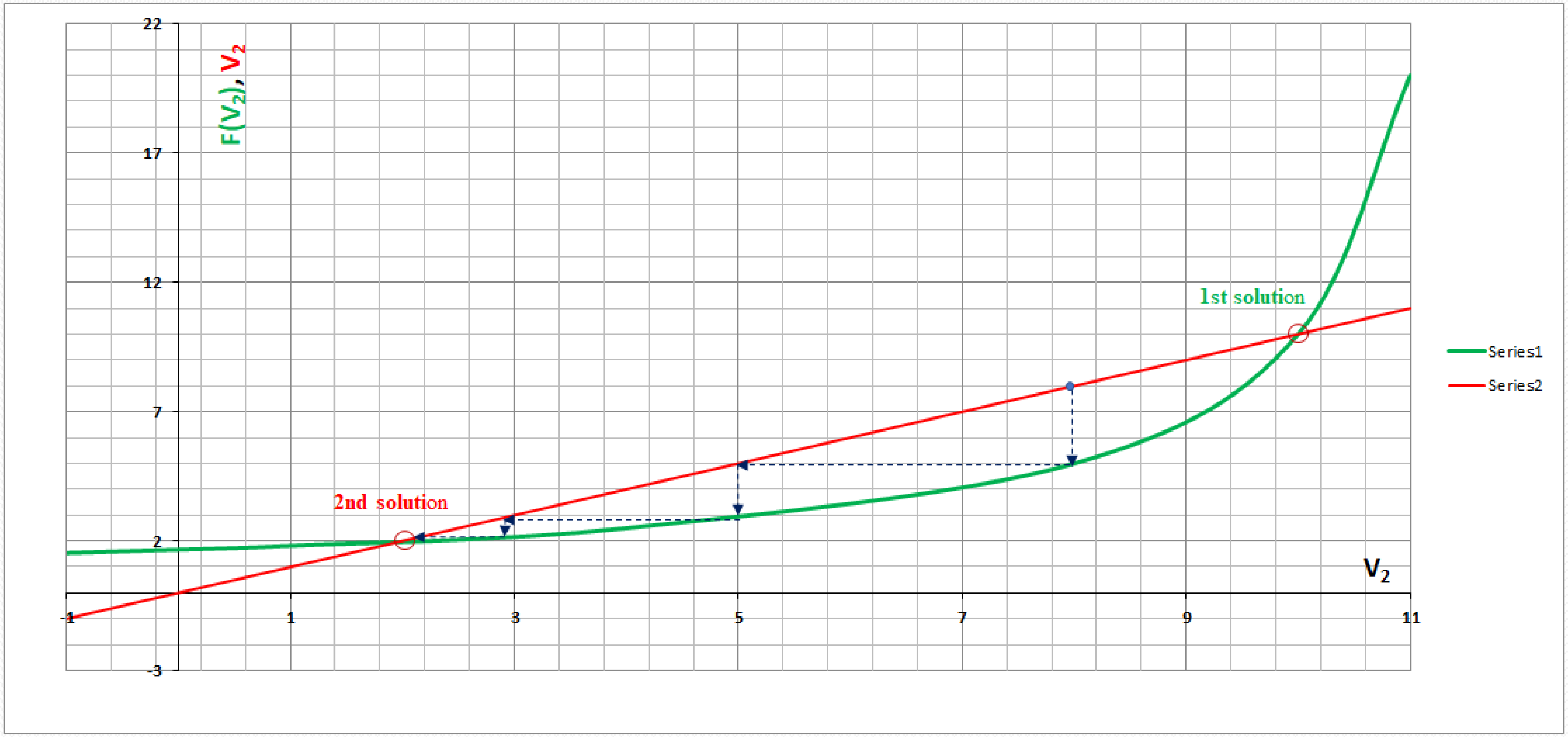
$$V_2^2 - 12V_2 + 20 = 0 \quad for\ P_d = 20\ watts$$

$$The\ nonlinear\ algebraic\ equation\ above\ can\ be\ rewritten\ as\ follow$$

$$V_2 = F(V_2) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. (5)$$

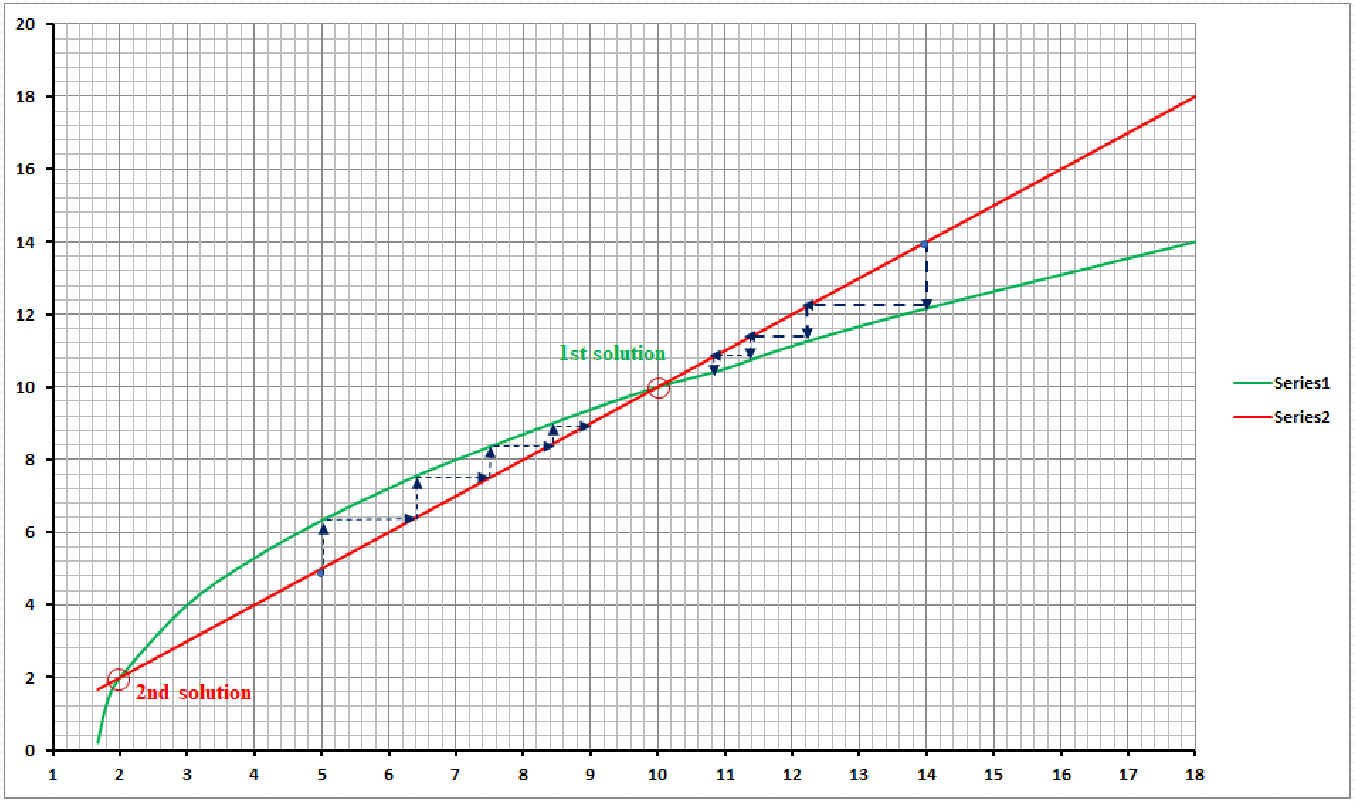
$$V_2 = \frac{-20}{V_2 - 12} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. (6)$$

$$V_2 = \sqrt{12V_2 - 20} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. (7)$$



22
17
12
7
2
-3
-1
1
3
5
7
9
11
F(V2), V2
1st solution
2nd solution
V2
Series1
Series2

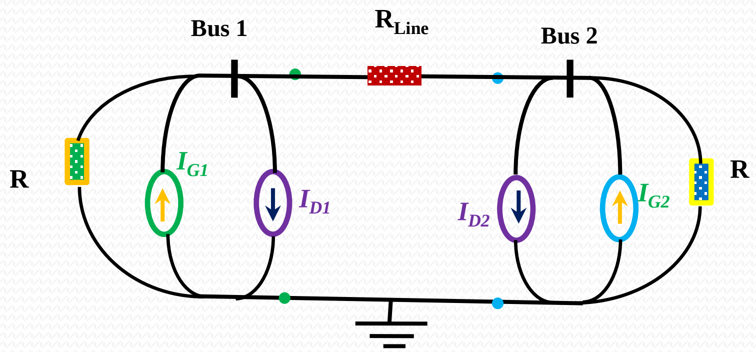
Iteration No. (^{k})	V_2^{k}	V_2^{k+1}
1	8	5
2	5	2.857
3	2.857	2.187
4	2.187	2.038
5	2.038	2.0076
6	2.0076	2.0015
7	2.0015	2.0003
8	2.0003	2.00006
Solution is converged		



1st solution
2nd solution
Series1
Series2

Iteration No. (^{k})	V_2^k	V_2^{k+1}
1	14	12.165
2	12.165	11.224
3	11.224	10.709
4	10.709	10.416
5	10.416	10.247
6	10.247	10.147
7	10.147	10.087
8	10.087	10.052
9	10.052	10.0315
10	10.0315	10.0188
11	10.0188	10.0113
12	10.0113	10.0067
13	10.0067	10.0040
14	10.0040	10.0024
15	10.0024	10.0014
16	10.0014	10.0008
Solution is converged		

Derivation of load flow equations for a DC system



Nodal voltage method by applying KCL

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{G1} - I_{D1} \\ I_{G2} - I_{D2} \end{bmatrix} = \begin{bmatrix} \frac{P_{G1}}{V_1} - \frac{P_{D1}}{V_1} \\ \frac{P_{G2}}{V_2} - \frac{P_{D2}}{V_2} \end{bmatrix} = \begin{bmatrix} \frac{P_{G1} - P_{D1}}{V_1} \\ \frac{P_{G2} - P_{D2}}{V_2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{P_{G1} - P_{D1}}{V_1} \\ \frac{P_{G2} - P_{D2}}{V_2} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\frac{P_{G2} - P_{D2}}{V_2} = Y_{21}V_1 + Y_{22}V_2$$

$$V_2 = \frac{1}{Y_{22}}\left[\frac{(P_{G2} - P_{D2})}{V_2} - \{Y_{21}V_1\}\right]$$

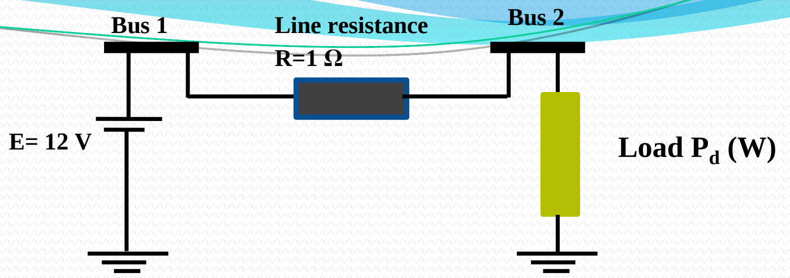
$$V_2 = \frac{1}{Y_{22}}\left[\frac{(P_{G2} - P_{D2})}{V_2} - \{Y_{21}V_1\}\right]$$

$P_{G2} = 0 \quad P_{D2} = 20 \quad Y_{21} = -1 \quad Y_{22} = 1$

$$V_2 = \frac{1}{1}\left[\frac{(0 - 20)}{V_2} - \{-V_1\}\right]$$

$$V_2 = \frac{-20}{V_2} + V_1$$

$$V_2^{itr+1} = \frac{-20}{V_2^{itr}} + 12$$

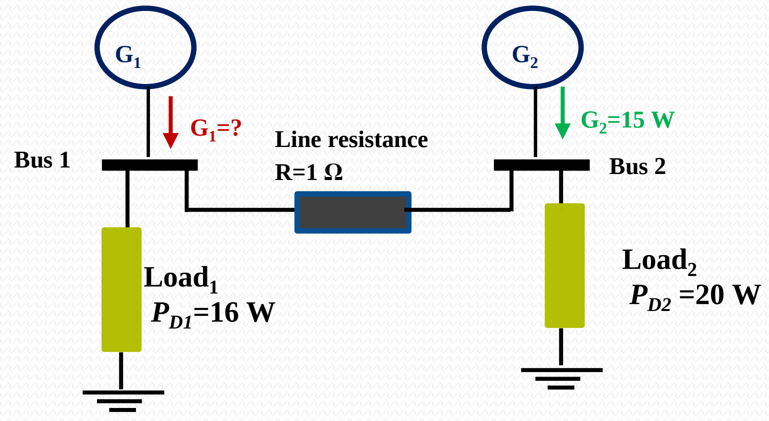


Two bus DC power system

Iteration no.	V_2^{itr}	V_2^{itr+1}
1	3	5.333
2	5.333	8.25
3	8.25	9.575
4	9.575	9.9113
5	9.9113	9.9821
6	9.982	9.9963
7	9.9963	9.99927
8	9.99927	9.99985
Solution converged to $V_2 = 9.99985$		

starting the iterative procees with a guess value $V_2^0 = 12$

Iteration no.	V_2^{itr}	V_2^{itr+1}
1	12	10.333
2	10.333	10.06
3	10.06	10.012
4	10.012	10.0025
5	10.0025	10.0005
Solution converged to $V_2 = 10.0005$		



Two bus DC power system

$$\sum P_G = \sum P_D + \sum P_{Losses}$$

I_1	=	$I_{G1} - I_{D1}$	$\dfrac{P_{G1} - jQ_{G1}}{V_1^*} - \dfrac{P_{D1} - jQ_{D1}}{V_1^*}$	$\dfrac{(P_{G1} - P_{D1}) - j(Q_{G1} - Q_{D1})}{V_1^*}$
I_2		$I_{G2} - I_{D2}$	$\dfrac{P_{G2} - jQ_{G2}}{V_2^*} - \dfrac{P_{D2} - jQ_{D2}}{V_2^*}$	$\dfrac{(P_{G2} - P_{D2}) - j(Q_{G2} - Q_{D2})}{V_2^*}$

- In an AC three-phase power system of *n* bus, the state variables are:
- $|V|_k^p$ $stnad\ for\ the\ voltage\ magnitude\ of\ phase\ p\ at\ bus\ k$
- δ_k^p $stnad\ for\ the\ voltage\ angle\ of\ phase\ p\ at\ bus\ k$

- The given (specified) data from a forecasting process are:
- P_{dk}^p $stnad\ for\ the\ active\ power\ demand\ of\ phase\ p\ at\ bus\ k$
- Q_{dk}^p $stnad\ for\ the\ reactive\ power\ demand\ of\ phase\ p\ at\ bus\ k$
- P_{gk}^p $stnad\ for\ the\ active\ power\ generation\ of\ phase\ p\ at\ bus\ k$
- Q_{dk}^p $stnad\ for\ the\ reactive\ power\ demand\ of\ phase\ p\ at\ bus\ k$

- The specified power at bus *i* is given as:
- $\boldsymbol{P_i + jQ_i = \left(P_{gi} - P_{di}\right) + j\left(Q_{gi} - Q_{di}\right) \ldots \ldots (8)}$

- On the assumptions of a balanced system and adoption of an appropriate Per unit system, the steady state behavior of *n* bus power system can be described by the following system of equations:
- $\boldsymbol{I_1 = Y_{11}V_1 + Y_{12}V_2 + \cdots + Y_{1k}V_k + \cdots + Y_{1n}V_n}$
- . . . .
- $\boldsymbol{I_k = Y_{k1}V_1 + Y_{k2}V_2 + \cdots + Y_{kk}V_k + \cdots + Y_{kn}V_n}$
- . . . .
- $\boldsymbol{I_n = Y_{n1}V_1 + Y_{n2}V_2 + \cdots + Y_{nk}V_k + \cdots + Y_{nn}V_n}$
- Where,
- $\boldsymbol{I_k}$ stands for the complex injected current at bus $\boldsymbol{k}$
- $\boldsymbol{Y_{ik}}$ stands for the $\boldsymbol{i, k\ th}$ element of the bus admittance matrix
- $\boldsymbol{V_k}$ stands for the complex bus voltage at bus $\boldsymbol{k}$

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$$I_k = \sum_{i=1}^{n} Y_{ki} V_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Using Polar Coordinates

$$Y_{ki} = |Y_{ki}| \angle \gamma_{ki}$$

$$V_k = |V_k| \angle \delta_k$$

$$S_k = V_k I_k^*$$

$$I_k = \frac{S_k^*}{V_k^*} = \frac{P_k - jQ_k}{V_k^*}$$

$$P_k - jQ_k = V_k^* \sum_{i=1}^{n} Y_{ki} V_i \ \ldots\ldots\ldots ... (10)$$

$$P_k = \sum_{i=1}^{n} |V_k|\,|V_i||Y_{ki}| cos(\gamma_{ki} + \delta_i - \delta_k) \ldots (11)$$

$$Q_k = -\sum_{i=1}^{n} |V_k|\,|V_i||Y_{ki}| sin(\gamma_{ki} + \delta_i - \delta_k) \ldots (12)$$

Classification of Buses

Bus type	***Specified variables***	***Unknown variables***
Load bus *(Major of buses)*	**Active power P** **Reactive power Q**	**Voltage magnitude $\|V\|$** **Voltage angle δ**
Voltage controlled bus *(equipped with a VAR source)*	**Active power P** **Voltage magnitude $\|V\|$**	**Voltage angle δ** **Reactive power Q**
Slack (Reference) bus *(one bus with larger units)*	**Voltage magnitude $\|V\|$** **Voltage angle $\delta=0^o$**	**Active power P** **Reactive power Q**

Solution of a load flow problem by *Gauss-Seidel*

Equation (10) can be written as:

$$\frac{P_k - jQ_k}{V_k^*} = \sum_{i=1}^{n} Y_{ki} V_i \;\ldots\ldots\ldots\ldots\ldots (13)$$

For bus k

$$\frac{P_k - jQ_k}{V_k^*} = Y_{kk} V_k + \sum_{\substack{i=1 \\ i\neq k}}^{n} Y_{ki} V_i \;\ldots\ldots\ldots\ldots\ldots (14)$$

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{i=1 \\ i\neq k}}^{n} Y_{ki} V_i \right] \;\ldots\ldots\ldots\ldots\ldots\ldots. (15)$$

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Solved example of Gauss-Seidel

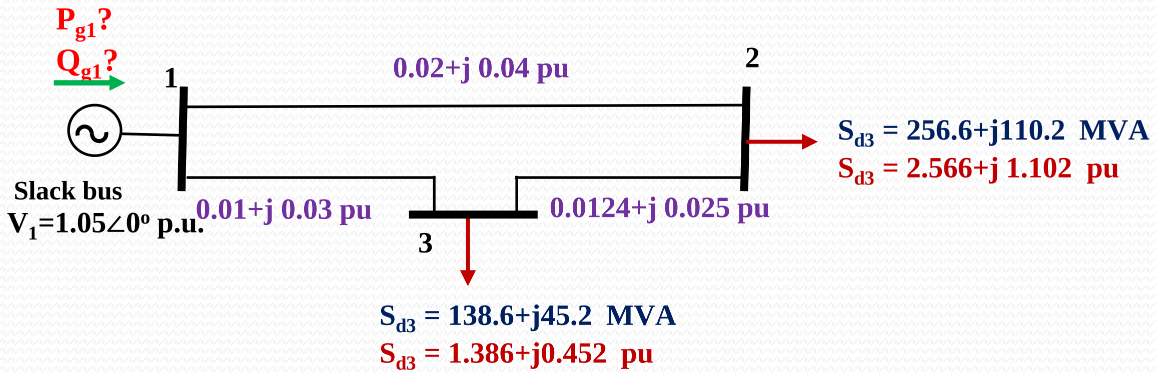

Fig. (1)

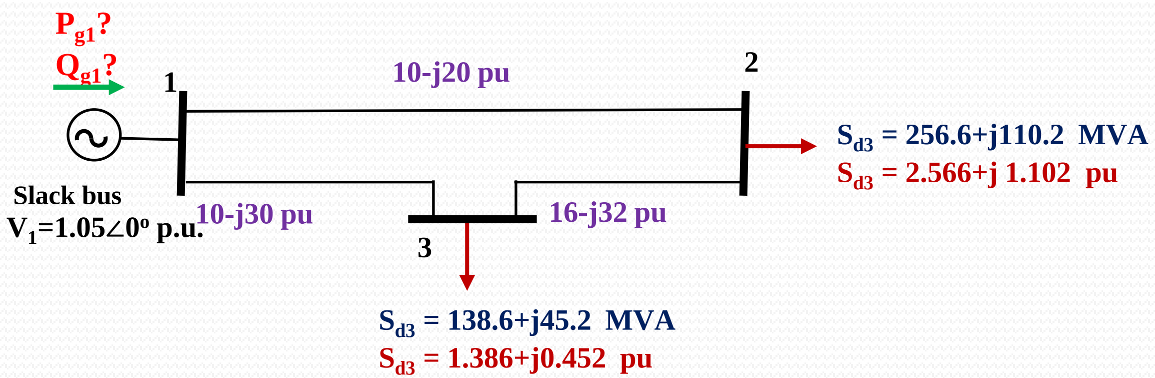

Fig. (1)

$Y_{bus} =$	20 -j50 pu	-10+j20 pu	-10+j30 pu
	-10+j20 pu	26-j52 pu	-16+j32 pu
	-10+j30 pu	-16+j32 pu	26-j62 pu

By assuming flat starting (guess) values for V_2 & V_3

$$V_2^{(0)} = 1.0 + j0\ pu; V_3^{(0)} = 1.0 + j0\ pu$$

$$V_k^{(itr+1)} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^{itr}} - \sum_{\substack{j=1\\ j\neq k}}^{n} Y_{kj}V_j\right]$$ general formula to calculate the voltage at bus k

* The first iteration

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \{Y_{21}V_1 + Y_{23}V_3\}\right]$$

$$V_2^{(1)} = \frac{1}{26 - j52}\left[\frac{-2.566 - (-j\,1.102\,)}{(1 - j0)} - \{(-10 + j20\,)(1.05 + j0) + (-16 + j32\,)(1 + j0)\}\right]$$

$$V_2^{(1)} = 0.9825 - j0.0310\ pu$$

$$V_3^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - \{Y_{31}V_1 + Y_{32}V_2\}\right]$$

$$V_3^{(1)} = \frac{1}{26 - j62}\left[\frac{-1.386 - (-j0.452)}{(1 - j0)} - \{(-10 + j30\,)(1.05 + j0) + (-16 + j32\,)(0.9825 + j0.0310\,)\}\right]$$

$$V_3^{(1)} = 1.0011 - j0.0353\ pu$$

* The second iteration

$$V_2^{(2)} = \frac{1}{26-j52}\left[\frac{-2.566-(-j\,1.102\ )}{(0.9825+j0.0310)} - \{(-10+j20\ )(1.05+j0)+(-16+j32\ )(1.0011-j0.0353\ )\}\right]$$

$$V_2^{(2)} = 0.9816 - j0.0520\,pu$$

$$V_3^{(2)} = \frac{1}{26-j62}\left[\frac{-1.386-(-j0.452)}{(1.0011+j0.0353\ )} - \{(-10+j30\ \ )(1.05+j0)+(-16+j32\ )(0.9816+j0.0520\ )\}\right]$$

$$V_3^{(2)} = 1.0008 - j0.0459pu$$

* The third iteration

$$V_2^{(3)} = \frac{1}{26-j52}\left[\frac{-2.566-(-j\,1.102\ )}{(0.9816+j0.0520)} - \{(-10+j20\ )(1.05+j0)+(-16+j32\ )(1.0008-j0.0459\ )\}\right]$$

$$V_2^{(3)} = 0.9808 - j0.0578\,pu$$

$$V_3^{(3)} = \frac{1}{26-j62}\left[\frac{-1.386-(-j0.452)}{(1.0008+j0.0459\ )} - \{(-10+j30\ \ )(1.05+j0)+(-16+j32\ )(0.9808-j0.0578\ )\}\right]$$

$$V_3^{(3)} = 1.0004 - j0.0488pu$$

The iterative process is conrinued and a solution is converged with an accuracy of $5x10^{-5}$ ***pu in seven iterations as follow***:

$$V_2^{(4)} = 0.9803 - j0.0594\,pu \quad V_3^{(4)} = 1.0002 - j0.0497pu$$

$$V_2^{(5)} = 0.9801 - j0.0598\,pu \quad V_3^{(5)} = 1.0001 - j0.0499pu$$

$$V_2^{(6)} = 0.9801 - j0.0599\,pu \quad V_3^{(6)} = 1.0000 - j0.0500pu$$

$$V_2^{(7)} = 0.9800 - j0.0600\,pu \quad V_3^{(7)} = 1.0000 - j0.0500pu$$

Calculation of the slack power

$$S_1^* = P_1 - jQ_1 = V_1^* \sum_{k=1}^{n} Y_{1k} V_k$$

$$P_1 - jQ_1 = (1.05 - j0)[(20 \ - j50\,)(1.05 + j0) + (-10 + j20\,)(0.9800 - j0.0600)$$
$$+ (-10 + j30\,)(1.0000 - j0.0500)]$$

$$P_1 - jQ_1 = 4.095 - j1.89\ pu = 409.5\ MW - j189\ MVAr$$

Calculation of the Line power flow & losses

* For line connecting bus 1 to bus 2

$$S_{12} = V_1 I_{12}^*$$

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{(1.05 - j0) - (0.9800 - j0.0600)}{0.02 + j\,0.04}$$

$$I_{12} = 1.9 - j0.8\ pu$$

$$S_{12} = P_{12} + jQ_{12} = (1.05 + j0)(1.9 + j0.8)$$

$$S_{12} = 1.995 + j0.84\ pu = 199.5\ MW + j84\ MVAr$$

$$S_{21} = V_2 I_{21}^*$$

$$I_{21} = -I_{12} = -1.9 + j0.8\ pu$$

$$S_{21} = P_{21} + jQ_{21} = (0.9800 - j0.0600)(-1.9 - j0.8)$$

$$S_{21} = -1.91 - j0.67\ pu = -191\ MW - 67\ MVAr$$

$$S_{12}^{losses} = P_{12}^{losses} + jQ_{12}^{losses} = S_{12} + S_{21}$$

$$S_{12}^{losses} = 199.5\ MW + j84\ MVAr + (-191\ MW - 67\ MVAr)$$

$$S_{12}^{losses} = 7.5\ MW + j17\ MVAr$$

* *For line connecting bus* **1** *to bus* **3**

$$I_{13} = 2.0 - j1.0\ pu$$

$$S_{13} = V_1 I_{13}^*$$

$$S_{13} = P_{13} + jQ_{13} = (1.05 + j0)(2.0 + j1.0)$$

$$S_{13} = 2.1 + j1.05 pu = 210\ MW + j\ 105\ MVAr$$

$$S_{31} = V_3 I_{31}^*$$

$$S_{31} = P_{31} + jQ_{31} = (1.0000 - j0.0500)(-2.0 - j1.0)$$

$$S_{31} = P_{31} + jQ_{31} = -2.05 - j0.9\ pu = -205\ MW - j90\ MVAr$$

$$S_{13}^{losses} = P_{13}^{losses} + jQ_{13}^{losses} = S_{13} + S_{31}$$

$$S_{13}^{losses} = 210\ MW + j\ 105\ MVAr + (-205\ MW - j90\ MVAr)$$

$$S_{13}^{losses} = 5\ MW + j15\ MVAr$$

* *For line connecting bus* **2** *to bus* **3**

$$I_{23} = -0.64 + j0.48\ pu$$

$$S_{23} = V_2 I_{23}^*$$

$$S_{23} = P_{23} + jQ_{23} = (0.9800 - j0.0600)(-0.64 - j0.48)$$

$$S_{32}^{losses} = P_{32}^{losses} + jQ_{32}^{losses} = S_{32} + S_{23}$$

$$S_{32} = V_3 I_{32}^*$$

$$S_{32} = P_{32} + jQ_{32} = (1.0000 - j0.050)(0.64 + j0.48)$$

$$S_{32} = P_{32} + jQ_{32} = 0.664 + j0.448 = 66.4\ MW + j44.8\ MVAr$$

$$S_{23} = P_{23} + jQ_{23} = -0.656 - j0.432 = -65.6\ MW - j43.2\ MVAr$$

$$S_{23}^{losses} = 66.4\ MW + j44.8\ MVAr + (-65.6\ MW - j43.2\ MVAr)$$

$$S_{23}^{losses} = 0.8\ MW + j1.6\ MVAr$$

Treatment of voltage controlled bus (P|V| bus)

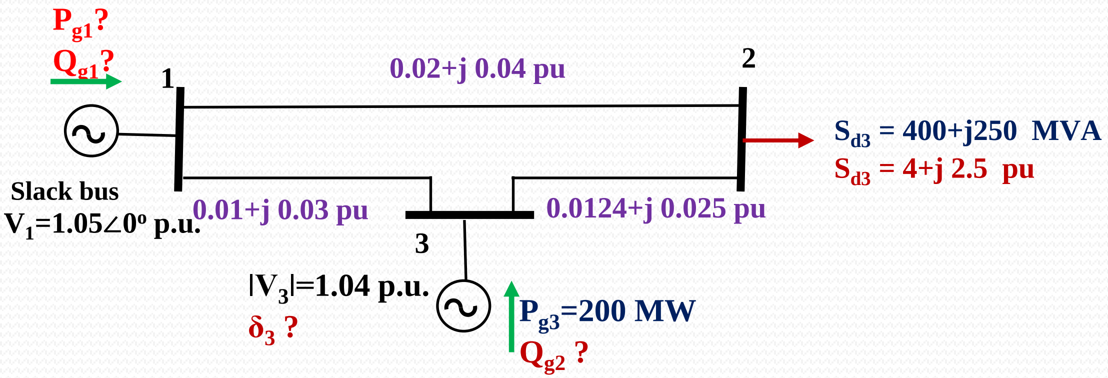

Fig. (2)

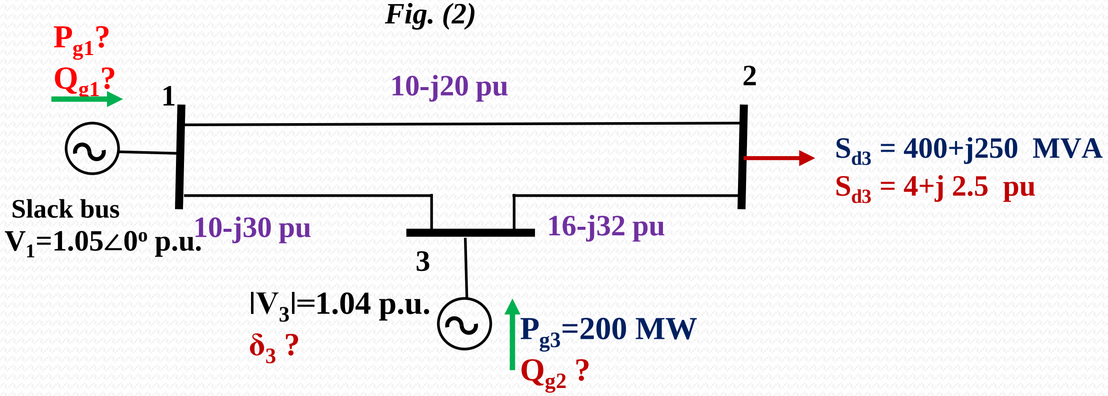

Fig. (2)

$$Y_{bus} = $$

20 -j50 pu	-10+j20 pu	-10+j30 pu
-10+j20 pu	26-j52 pu	-16+j32 pu
-10+j30 pu	-16+j32 pu	26-j62 pu

By assuming starting value(guess)value for $V_2^{(0)} = 1.0 + j0\ pu$

For the voltage controlled bus the angle δ_3 ***should be assign a starting value*** $\delta_3 = 0$

$$V_3^{(0)} = 1.04 + j0\ pu$$

$*$ ***the first iteration***

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \{Y_{21}V_1 + Y_{23}V_3\}\right]$$

$$V_2^{(1)} = \frac{1}{26 - j52}\left[\frac{-4 - (-j\,2.5\ )}{(1 - j0)} - \{(-10 + j20\ )(1.05 + j0) + (-16 + j32\ )(1.04 + j0)\}\right]$$

$$V_2^{(1)} = 0.97462 - j0.042307\ pu$$

$$V_3^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - \{Y_{31}V_1 + Y_{32}V_2\}\right]$$ Q_3 ***unknown for*** $P|V|$ ***bus?***

First we have to calculate $Q_3^{(1)}$

$$S_3^* = P_3 - jQ_3 = V_3^* \sum_{k=1}^{n} Y_{3k} V_k$$

$$Q_3^{(1)} = -Imag(V_3^* \sum_{k=1}^{n} Y_{3k} V_k)$$

$$Q_3^{(1)} = -Imag[V_3^* \{Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3\}]$$

$$Q_3^{(1)} = -Imag[(1.04 - j0)[(-10 + j30)(1.05 + j0) + (-16 + j32)(0.97462 - j0.042307) + (26 - j62)(1.04 + j0.0)]])$$

$$Q_3^{(1)} = 1.16\,pu$$

$$V_3^{(1)} = \frac{1}{26 - j62}\left[\frac{2 - (j1.6)}{(1.4 - j0)} - \{(-10 + j30)(1.05 + j0) + (-16 + j32)(0.97462 - j0.042307)\}\right]$$

$$V_3^{(1)} = 1.03783 - j0.00517\,pu$$

The above value for $V_3^{(1)}$ does not give $|V_3| = 1.04$. It may be corrected

Only the imaginary part of $V_3^{(1)}$ is retained, the real part is recalculated by the following

$$e_3^{(1)} = \sqrt{\left(|V_3^{(1)}|\right)^2 - {f_3^{(1)}}^2}$$

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.00517)^2} = 1.039987\,pu$$

$$V_3^{(1)} = 1.039987 - j0.00517\,pu$$

$*$ ***the second iteration***

$$V_2^{(2)} = \frac{1}{26-j52}\left[\frac{-4-(-j\,2.5\ )}{(0.97462+j0.042307)} - \{(-10+j20\ )(1.05+j0)+(-16+j32\ )(1.039987-j0.00517)\}\right]$$

$$V_2^{(2)} = 0.971057 - j0.043432\ pu$$

$$Q_3^{(2)} = -Imag(V_3^* \sum_{k=1}^{n} Y_{3k}V_k)$$

$$Q_3^{(2)} = -Imag[(1.039987+j0.00517)[(-10+j30\ )(1.05+j0) + (-16+j32\ )(0.971057-j0.043432\ ) + (26-j62\ )(1.039987-j0.00517)]])$$

$$Q_3^{(2)} = 1.38796\ pu$$

$$V_3^{(2)} = \frac{1}{26-j62}\left[\frac{2-(j1.38796)}{(1.039987+j0.00517)} - \{(-10+j30\ )(1.05+j0)+(-16+j32\ )(0.971057-j0.043432\ )\}\right]$$

$$V_3^{(2)} = 1.03908 - j0.00730\ pu\ \ < this\ value\ should\ be\ corrected >$$

$$e_3^{(2)} = \sqrt{\left(|V_3^{(2)}|\right)^2 - {f_3^{(2)}}^2}$$

$$e_3^{(2)} = \sqrt{(1.04)^2-(0.00730\ )^2} = 1.039974\ pu$$

$$V_3^{(2)} = 1.039974 - j0.0073\ pu$$

$\boldsymbol{The\ iterative\ process\ is\ conrinued\ and\ a\ solution\ is\ converged\ with\ an}$

$\boldsymbol{accuracy\ of\ 5x10^{-5}\ pu\ in\ seven\ iterations\ as\ follow}$:

$$V_2^{(3)} = 0.97073 - j0.04479\ pu\ \ Q_3^{(3)} = 1.42904\ \ V_3^{(3)} = 1.03996 - j0.00833pu$$

$$V_2^{(4)} = 0.97065 - j0.04533\ pu\ Q_3^{(4)} = 1.44833\ \ V_3^{(4)} = 1.03996 - j0.00873$$

$$V_2^{(5)} = 0.97062 - j0.04555\ pu\ \ \ Q_3^{(5)} = 1.45621\ \ \ V_3^{(5)} = 1.03996 - j0.00893pu$$

$$V_2^{(6)} = 0.97061 - j0.04565\ pu\ \ Q_3^{(6)} = 1.45947\ \ V_3^{(6)} = 1.03996 - j0.0900pu$$

$$V_2^{(7)} = 0.97601 - j0.04569\ pu\ \ Q_3^{(7)} = 1.44682\ \ V_3^{(7)} = 1.03996 - j0.00903pu$$

Final solution

$$V_2 = 0.97168\angle -2.69^0\ pu$$

$$V_3 = 1.04\angle -0.498^0\ pu$$

$$S_3 = 2.0 + j1.467\ pu = 200\ MW + j\ 146.7\ MVAr$$

$$S_1 = 2.1842 + j1.4085\ pu = 218.42\ MW + j\ 140.85\ MVAr$$

Line flow

$S_{12} = 179.36 + j118.734\ pu;\ S_{21} = -170.97 - j\ 101.947$; $Slosses_{12} = 8.39 + j16.79$

$S_{13} = 39.06 + j22.118\ pu;\ S_{31} = -38.88 - j\ 21.569$; $Slosses_{13} = 0.18 + j0.548$

$S_{23} = -229.03 - j148.05\ pu;\ S_{23} = 238.88 + j167.746$; $Slosses_{23} = 9.85 + j19.69$