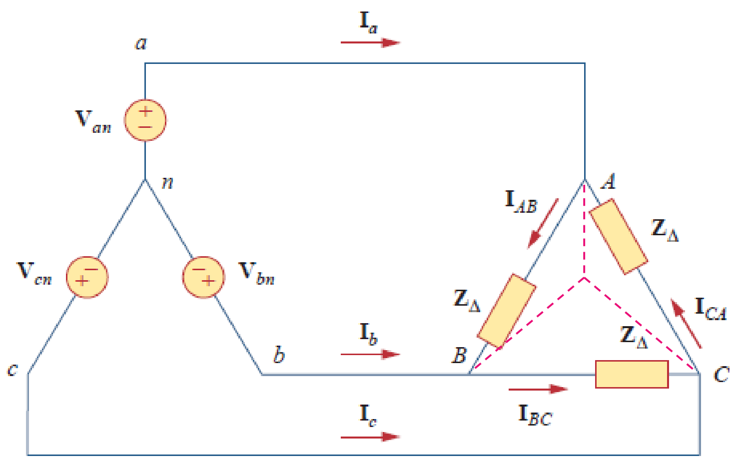
Lecture -9-

➢ Balanced Wye-Delta Connection (Y-Δ)



- $V_{AB} = V_{Line} = V_\phi$
- $V_{ab} = \sqrt{3} \angle 30\ V_{an}$ ***or*** $V_{an} = \frac{1}{\sqrt{3}} \angle -30\ V_{ab}$
- $V_{AB} = \sqrt{3} \angle 30\ V_{AN}$ ***or*** $V_{AN} = \frac{1}{\sqrt{3}} \angle -30\ V_{AB}$
- $I_{AB} = \frac{V_{AB}}{Z_\Delta} = I_\phi$
- $I_{AB} = \frac{1}{\sqrt{3}} \angle 30\ I_{aA}$ ***or*** $I_{aA} = \sqrt{3} \angle -30\ I_{AB}$
- $Z_Y = \frac{Z_\Delta}{3}$

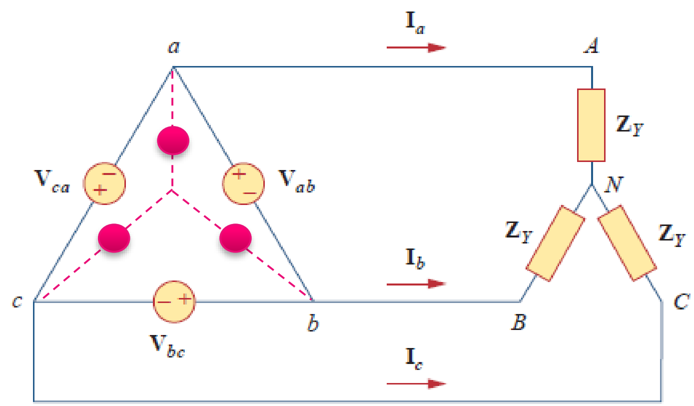
Same as for the Balanced Delta -Wye Connection (Δ -Y)

When the Δ-connected source is transformed to a Y-connected source,

- $V_{ab} = \sqrt{3} \angle 30\ V_{an}$

or

$$V_{an} = \frac{1}{\sqrt{3}} \angle -30\ V_{ab}$$



Example/

A balanced Y-connected load with a phase impedance of 40+j25 Ω/ϕ is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents at the Load. Use Vab as reference.

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\underline{/32^\circ}\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\underline{/0^\circ}\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\underline{/-30^\circ} = 121.2\underline{/-30^\circ}\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\underline{/-30^\circ}}{47.12\underline{/32^\circ}} = 2.57\underline{/-62^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 2.57\underline{/-178^\circ}\ \text{A}$$

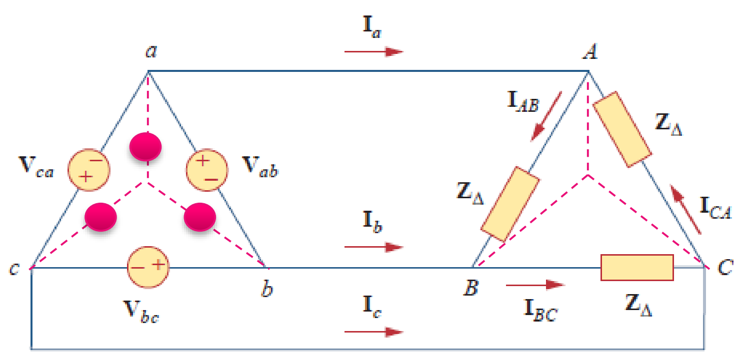
$$\mathbf{I}_c = \mathbf{I}_a\underline{/120^\circ} = 2.57\underline{/58^\circ}\ \text{A}$$

which are the same as the phase currents.

In a balanced Δ-Y circuit, $\mathbf{V}_{ab} = 240\underline{/15^\circ}$ and $\mathbf{Z}_Y = (12 + j15)\ \Omega$. Calculate the line currents.

Answer: $7.21\underline{/-66.34^\circ}$ A, $7.21\underline{/-173.66^\circ}$ A, $7.21\underline{/53.66^\circ}$ A.

➢ Balanced Delta -Delta Connection (Δ-Δ)



- $V_{AB} = V_{Line} = V_\phi$
- $V_{ab} = \sqrt{3}\,\llcorner 30\ V_{an}$
- $I_{AB} = \frac{V_{AB}}{Z_\Delta} = I_\phi = I_{ab}$ تيار الفيز

Example/

A balanced Δ-connected load having an impedance 20-j15 Ω/ϕ is connected to a Δ-connected, positive-sequence generator having **V***ab*=330 v. Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\underline{/-36.87^\circ}\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

(في حال لا توجد ممانعة خط)

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\underline{/0^\circ}}{25\underline{/-36.87}} = 13.2\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 13.2\underline{/-83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 13.2\underline{/156.87^\circ}\ \text{A}$$

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = (13.2\underline{/36.87^\circ})(\sqrt{3}\underline{/-30^\circ}) = 22.86\underline{/6.87^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 22.86\underline{/-113.13^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 22.86\underline{/126.87^\circ}\ \text{A}$$

A positive-sequence, balanced Δ-connected source supplies a balanced Δ-connected load. If the impedance per phase of the load is $18 + j12\ \Omega$ and $\mathbf{I}_a = 19.202\underline{/35^\circ}$ A, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $11.094\underline{/65^\circ}$ A, $240\underline{/98.69^\circ}$ V.

Example/

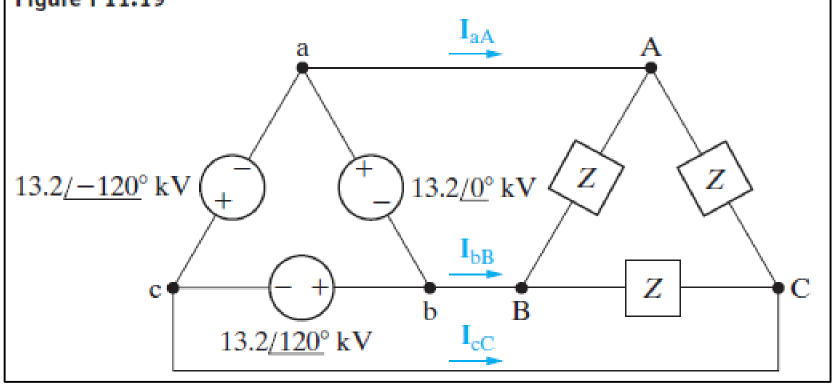
The impedance Z in the balanced three-phase circuit in Fig. P11.19 is $100 - j75\ \Omega$. Find

a) $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$,

b) $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$,

c) $\mathbf{I}_{ba}$, $\mathbf{I}_{cb}$, and $\mathbf{I}_{ac}$.

Figure P11.19



$$[a]\quad \mathbf{I}_{AB} = \frac{13{,}200\angle 0^\circ}{100 - j75} = 105.6\angle 36.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{BC} = 105.6\angle 156.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{CA} = 105.6\angle -83.13^\circ \text{ A (rms)}$$

$$[b]\quad \mathbf{I}_{aA} = \sqrt{3}\angle -30^\circ \mathbf{I}_{AB} = 182.9\angle 66.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{bB} = 182.9\angle -173.13^\circ \text{ A (rms)}$$

$$\mathbf{I}_{cC} = 182.9\angle -53.13^\circ \text{ A (rms)}$$

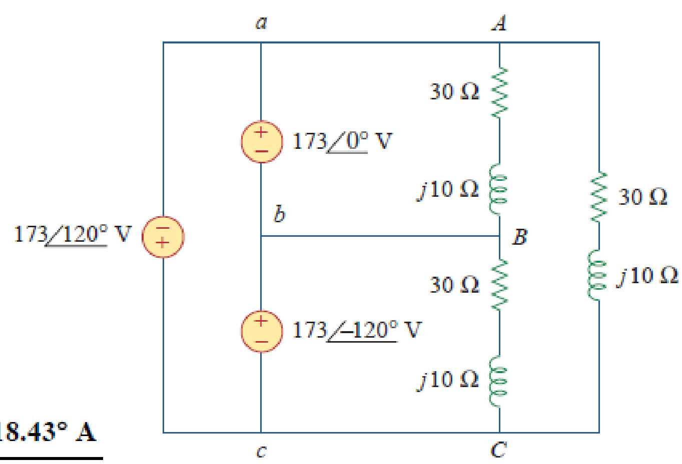
$$[c]\quad \mathbf{I}_{ba} = \mathbf{I}_{AB} = 105.6\angle 36.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{cb} = \mathbf{I}_{BC} = 105.6\angle 156.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{ac} = \mathbf{I}_{CA} = 105.6\angle -83.13^\circ \text{ A (rms)}$$

Example/

For the Δ-Δ circuit, calculate the phase and line currents.



$$\mathbf{Z}_\Delta = 30 + j10 = 31.62\angle 18.43^\circ$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{173\angle 0^\circ}{31.62\angle 18.43^\circ} = \underline{\mathbf{5.47\angle -18.43^\circ\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = \underline{\mathbf{5.47\angle -138.43^\circ\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120^\circ = \underline{\mathbf{5.47\angle 101.57^\circ\ A}}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\mathbf{I}_a = 5.47\sqrt{3}\angle -48.43^\circ = \underline{\mathbf{9.474\angle -48.43^\circ\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = \underline{\mathbf{9.474\angle -168.43^\circ\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = \underline{\mathbf{9.474\angle 71.57^\circ\ A}}$$

Example/

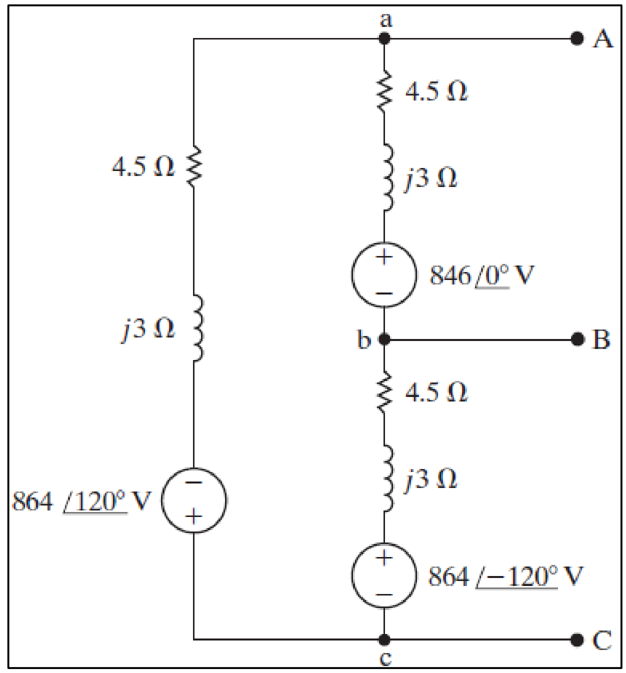
The Δ-connected source of 11.21 is connected to a Y-connected load by means of a balanced three-phase distribution line. The load impedance is $1192 + j1584\ \Omega/\phi$. and the line impedance is $6.5 + j15\ \Omega/\phi$.

a) Construct a single-phase equivalent circuit of the system.

b) Determine the magnitude of the line voltage at the terminals of the load.

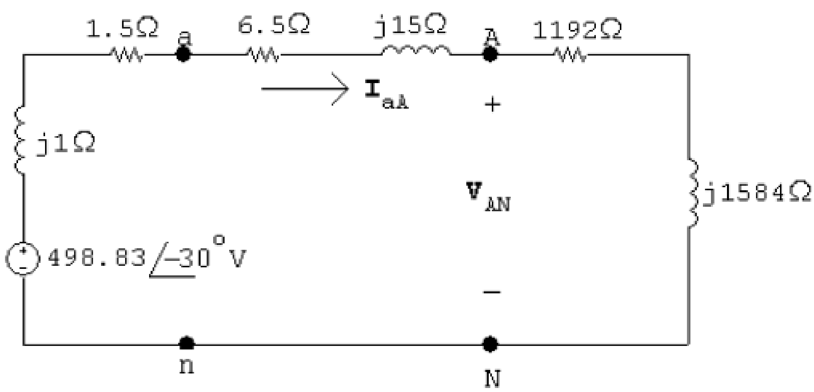
c) Determine the magnitude of the phase current in the Δ-source.

d) Determine the magnitude of the line voltage at the terminals of the source.



solution

[a]



[b] $\mathbf{I}_{aA} = \dfrac{498.83/\underline{-30^\circ}}{1200 + j1600} = 249.42/\underline{-83.13^\circ}\ \text{mA (rms)}$

$$\mathbf{V}_{AN} = (1192 + j1584)(0.24942/\underline{-83.13^\circ}) = 494.45/\underline{-30.09^\circ}\ \text{V (rms)}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}(494.45) = 856.41\ \text{V (rms)}$$

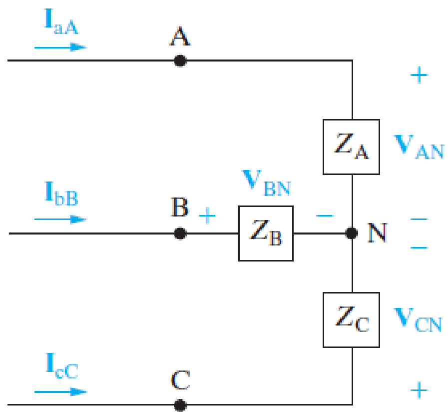
[c] $|\mathbf{I}_{ab}| = \dfrac{0.24942}{\sqrt{3}} = 144\ \text{mA (rms)}$

[d] $\mathbf{V}_{an} = (1198.5 + j1599)(0.24942/\underline{-83.13^\circ}) = 498.42/\underline{-29.98^\circ}\ \text{V (rms)}$

$$|\mathbf{V}_{ab}| = \sqrt{3}(498.42) = 863.29\ \text{V (rms)}$$

➢ <u>Power Calculations in Balanced Three-Phase Circuits</u>

1) Average Power in a Balanced Wye Load



$$P_A = |V_{AN}||I_{aA}|\cos(\theta_{vA} - \theta_{iA}),$$

$$P_B = |V_{BN}||I_{bB}|\cos(\theta_{vB} - \theta_{iB}),$$

$$P_C = |V_{CN}||I_{cC}|\cos(\theta_{vC} - \theta_{iC}).$$

$$V_\phi = |V_{AN}| = |V_{BN}| = |V_{CN}|,$$

$$I_\phi = |I_{aA}| = |I_{bB}| = |I_{cC}|,$$

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}.$$

Moreover, for a balanced system, the power delivered to each phase of the load is the same, so

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi,$$

where P_ϕ represents the average power per phase.

The total average power delivered to the balanced Y-connected load is simply three times the power per phase, or

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi.$$

$$V_\phi = \frac{V_L}{\sqrt{3}} \qquad \textit{for Wye connection}$$

Total real power in a balanced three-phase load ▶

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi$$

$$= \sqrt{3} V_L I_L \cos\theta_\phi.$$

Total reactive power in a balanced three-phase load

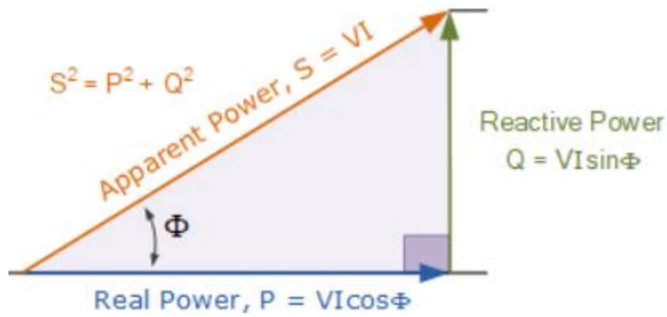
$$Q_\phi = V_\phi I_\phi \sin\theta_\phi,$$

$$Q_T = 3Q_\phi = \sqrt{3} V_L I_L \sin\theta_\phi.$$

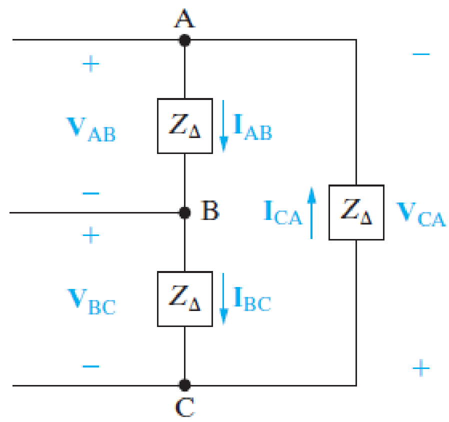
Total complex power in a balanced three-phase load

$$S_\phi = P_\phi + jQ_\phi = V_\phi I_\phi^*,$$

$$S_T = 3S_\phi = \sqrt{3} V_L I_L \angle\theta_\phi^\circ.$$



2) Average Power in a Balanced Delta Load



$$P_A = |\mathbf{V}_{AB}||\mathbf{I}_{AB}| \cos(\theta_{vAB} - \theta_{iAB}),$$

$$P_B = |\mathbf{V}_{BC}||\mathbf{I}_{BC}| \cos(\theta_{vBC} - \theta_{iBC}),$$

$$P_C = |\mathbf{V}_{CA}||\mathbf{I}_{CA}| \cos(\theta_{vCA} - \theta_{iCA}).$$

$$|\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_\phi,$$

$$|\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| = I_\phi,$$

$$\theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA} = \theta_\phi,$$

Moreover, for a balanced system, the power delivered to each phase of the load is the same, so

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi.$$

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi$$

$$I_\phi = \frac{I_L}{\sqrt{3}} \qquad \textit{for Delta connection}$$

$$= 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\theta_\phi$$

$$= \sqrt{3}V_L I_L \cos\theta_\phi.$$

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi;$$

$$Q_T = 3Q_\phi = 3V_\phi I_\phi \sin\theta_\phi;$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*;$$

$$S_T = 3S_\phi = \sqrt{3}V_L I_L \angle\theta_\phi.$$

Example/

A balanced three-phase Y-connected generator with positive sequence has an impedance of $0.2 + j0.5\ \Omega/\phi$ and an internal voltage of $120\ V/\phi$. The generator feeds a balanced three-phase Y-connected load having an impedance of $39 + j28\ \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8 + j1.5\ \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Calculate the average power per phase delivered to the Y-connected load

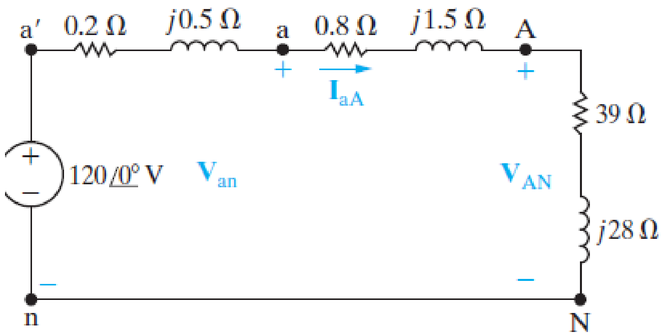
b) Calculate the total average power delivered to the load.

c) Calculate the total average power lost in the line.

d) Calculate the total average power lost in the generator.

e) Calculate the total number of magnetizing vars absorbed by the load.

f) Calculate the total complex power delivered by the source.



$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120\angle 0^\circ}{40 + j30}$$

$$= 2.4\angle -36.87^\circ \text{ A.}$$

$$\mathbf{V}_{AN} = (39 + j28)(2.4\angle -36.87^\circ)$$

$$= 115.22\angle -1.19^\circ \text{ V.}$$

[a]

$V_\phi = 115.22$ V, $I_\phi = 2.4$ A, and $\theta_\phi = -1.19 - (-36.87) = 35.68^\circ$. Therefore

$$P_\phi = (115.22)(2.4)\cos 35.68^\circ$$
$$= 224.64 \text{ W}.$$

The power per phase may also be calculated from $I_\phi^2 R_\phi$, or

$$P_\phi = (2.4)^2(39) = 224.64 \text{ W}.$$

b) The total average power delivered to the load is $P_T = 3P_\phi = 673.92$ W.

$$\mathbf{V}_{AB} = (\sqrt{3}\,\angle 30^\circ)\mathbf{V}_{AN}$$
$$= 199.58\,\angle 28.81^\circ \text{ V},$$
$$P_T = \sqrt{3}(199.58)(2.4)\cos 35.68^\circ$$
$$= 673.92 \text{ W}.$$

c) The total power lost in the line is

$$P_{\text{line}} = 3(2.4)^2(0.8) = 13.824 \text{ W}.$$

d) The total internal power lost in the generator is

$$P_{\text{gen}} = 3(2.4)^2(0.2) = 3.456 \text{ W}.$$

e) The total number of magnetizing vars absorbed by the load is

$$Q_T = \sqrt{3}(199.58)(2.4)\sin 35.68^\circ$$
$$= 483.84 \text{ VAR}.$$

f) The total complex power associated with the source is

$$S_T = 3S_\phi = -3(120)(2.4)\,\angle 36.87^\circ$$
$$= -691.20 - j518.40 \text{ VA}.$$

The minus sign indicates that the internal power and magnetizing reactive power are being delivered to the circuit. We check this result by calculating the total and reactive power absorbed by the circuit:

$$P = 673.92 + 13.824 + 3.456$$
$$= 691.20 \text{ W (check)},$$
$$Q = 483.84 + 3(2.4)^2(1.5) + 3(2.4)^2(0.5)$$
$$= 483.84 + 25.92 + 8.64$$
$$= 518.40 \text{ VAR(check)}.$$

Example/

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8. The load is fed from a line having an impedance of $0.005 + j0.025\ \Omega/\phi$. The line voltage at the terminals of the load is 600 V.

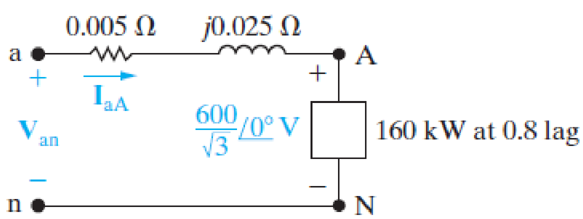
a) Construct a single-phase equivalent circuit of the system.

b) Calculate the magnitude of the line current.

c) Calculate the magnitude of the line voltage at the sending end of the line.

d) Calculate the power factor at the sending end of the line.

a)



b) The line current $\mathbf{I}_{aA}^{*}$ is given by

$$\left(\frac{600}{\sqrt{3}}\right)\mathbf{I}_{aA}^{*} = (160 + j120)10^3,$$

or

$\mathbf{I}_{aA}^{*} = 577.35\,\angle 36.87^\circ$ A.

Therefore, $\mathbf{I}_{aA} = 577.35\,\angle -36.87^\circ$ A. The magnitude of the line current is the magnitude of $\mathbf{I}_{aA}$:

$$I_L = 577.35 \text{ A}.$$

We obtain an alternative solution for I_L from the expression

$$\begin{aligned} P_T &= \sqrt{3}V_L I_L \cos\theta_p \\ &= \sqrt{3}(600)I_L(0.8) \\ &= 480{,}000 \text{ W}; \end{aligned}$$

$$\begin{aligned} I_L &= \frac{480{,}000}{\sqrt{3}(600)(0.8)} \\ &= \frac{1000}{\sqrt{3}} \\ &= 577.35 \text{ A}. \end{aligned}$$

c) To calculate the magnitude of the line voltage at the sending end, we first calculate $\mathbf{V}_{an}$. From Fig. 11.17,

$$\mathbf{V}_{an} = \mathbf{V}_{AN} + Z_{\ell}\mathbf{I}_{aA}$$

$$= \frac{600}{\sqrt{3}} + (0.005 + j0.025)(577.35 \angle -36.87^{\circ})$$

$$= 357.51 \angle 1.57^{\circ} \text{ V}.$$

Thus

$$V_L = \sqrt{3}|\mathbf{V}_{an}|$$

$$= 619.23 \text{ V}.$$

d) The power factor at the sending end of the line is the cosine of the phase angle between $\mathbf{V}_{an}$ and $\mathbf{I}_{aA}$:

$$\text{pf} = \cos[1.57^{\circ} - (-36.87^{\circ})]$$

$$= \cos 38.44^{\circ}$$

$$= 0.783 \text{ lagging}.$$

An alternative method for calculating the power factor is to first calculate the complex power at the sending end of the line:

$$S_{\phi} = (160 + j120)10^3 + (577.35)^2(0.005 + j0.025)$$

$$= 161.67 + j128.33 \text{ kVA}$$

$$= 206.41 \angle 38.44^{\circ} \text{ kVA}.$$

The power factor is

$$\text{pf} = \cos 38.44^{\circ}$$

$$= 0.783 \text{ lagging}.$$

Finally, if we calculate the total complex power at the sending end, after first calculating the magnitude of the line current, we may use this value to calculate V_L. That is,

$$\sqrt{3}V_L I_L = 3(206.41) \times 10^3,$$

$$V_L = \frac{3(206.41) \times 10^3}{\sqrt{3}(577.35)},$$

$$= 619.23 \text{ V}.$$