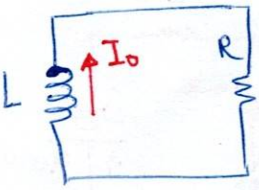
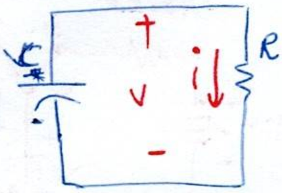
Lect. -4-

« A general Solution for step and Natural Responce »



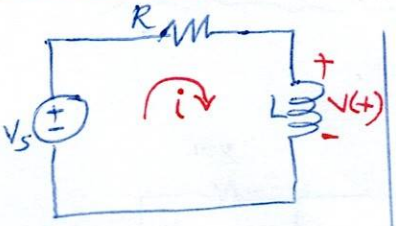
$$L\frac{di}{dt} + Ri = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$



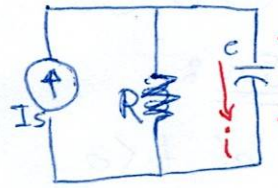
$$c\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{1}{Rc}v = 0$$



$$V_s = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_s}{L}$$



$$I_s = \frac{V}{R} + c\frac{dv}{dt}$$

$$\frac{dv}{dt} + \frac{1}{Rc}\cdot V = \frac{I_s}{c}$$

➤ in general :-

k : constant

$$\frac{dx}{dt} + \frac{1}{\tau}x = k$$

$$\frac{dx}{dt} = \frac{-1}{\tau}x + k$$

$$\frac{dx}{dt} = -\left(\frac{x}{\tau} - k\right)$$

$$\frac{dx}{dt} = -\left(\frac{x - k\tau}{\tau}\right)$$, when $k\tau = x_f$

$$\frac{dx}{dt} = -\left(\frac{x - x_f}{\tau}\right)$$

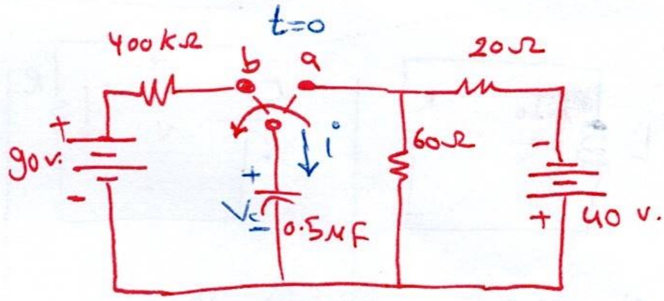
$$\int_{x_o}^{x(t)} \frac{dx}{(x - x_f)} = -\int_{t_o}^{t} \frac{1}{\tau}\, dt$$] by integral two termirals

$$\boxed{x(t) = x_f + [x(t_o) - x_f]\, e^{-(t-t_o)/\tau}}$$

General equation

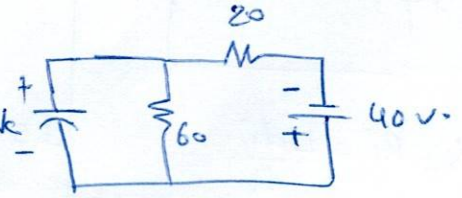
Ex: the switch in the cct has been in position a for along time. At $t=0$, the switch is moved to position b. Find

ⓐ initial value of V_c ?



Sol:-

at $t<0$

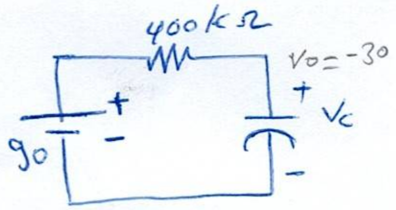


$$V_c(0) = V_0 = -40 * \frac{60}{60+20} \boxed{= -30 \text{ v.}}$$

ⓑ Final value of V_c ?

at $t \geq 0$

$$\boxed{V_f = V_s = 90 \text{ v.}}$$



ⓒ expression of $V_c(t)$ at $t \geq 0$?

$$\tau = RC = 400 * 10^{3} * 0.5 * 10^{-6} = \boxed{0.2 \text{ S}}$$

$$X_c(t) = X_f + [X_0 - X_f] e^{-(t-t_0)/\tau}$$

$$V_c(t) = 90 + (-30-90) e^{-(t-0)/0.2}$$

$$\boxed{V_c(t) = 90 - 120 e^{-5t} \text{ V.}} \quad , t \geq 0$$

ⓓ Find $i(t)$

$$i(t) = c \frac{dv(t)}{dt} = 0.5 * 10^{-6} \left(\frac{d\, 90 - 120 e^{-5t}}{dt} \right) = (0.5 * 10^{-6})(600 e^{-5t})$$

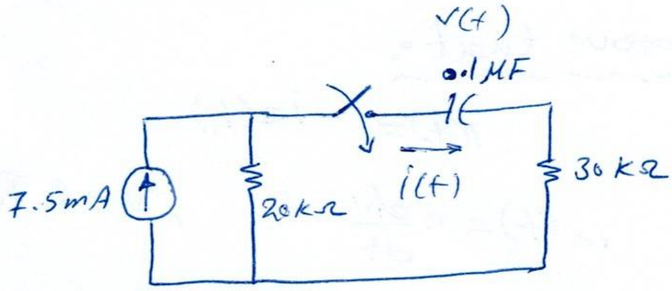
$$= \boxed{300\, e^{-5t} \text{ μA}}$$

ⓔ How long after the S in position b does $V_C = 0$?

$$0 = 90 - 120 e^{-5t}$$

$$90 = 120 e^{-5t} \implies \boxed{t = 57.54} \text{ mS}$$

Ex: Find $v(t), i(t)$?



at $t < 0$

$V_0 = 0$

at $t \geq 0$

$\tau = RC = 50 \cdot 10^3 * 0.1 * 10^{-6}$

$= 5 \, mS$

$V_f = 150 \, V$

$x(t) = X_f + (X_0 - X_f) e^{-t/\tau}$

$v(t) = 150 + (0 - 150) e^{-t/0.005}$

$$\boxed{v(t) = 150 - 150 e^{-200t} \, v.}$$

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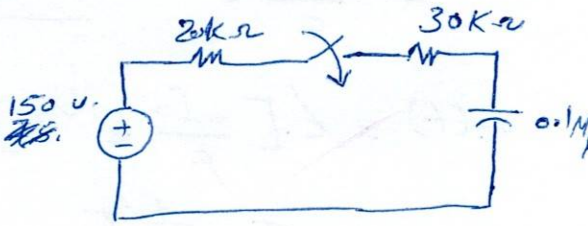
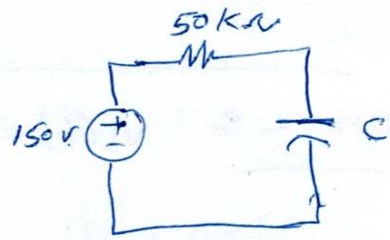
⇓

$7.5 \cdot 10^{-3} * 20 \cdot 10^3 = 150 \, V.$



⇓



$i(t) = i_C(t) = C \frac{dv(t)}{dt}$

$= C \cdot \frac{d(150 - 150 e^{-200t})}{dt}$

$= 0.1 * 10^{-6} (-150 * -200 e^{-200t})$

$$\boxed{i(t) = 3e^{-200t} \, mA}$$

$V_{30k\Omega} = i(t) * R_{30}$

$= 3e^{-200t} * 10^{-3} * 30 * 10^3 = 90 e^{-200t}$

prove that:

$$i(t) = -i_c(t)$$

$$i_c(t) = c\frac{dv}{dt} \quad , v = R.I_o e^{-t/\tau} \quad , \tau = R.C$$

$$\frac{dv}{dt} = R.I_o\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= \cancel{R}.I_o\left(-\frac{1}{\cancel{R}C}\right)e^{-t/\tau}$$

$$= -\frac{I_o}{c}e^{-t/\tau}$$ — sub in $i_c(t)$ equation

$$i_c(t) = \cancel{c}.\left[\frac{-I_o}{\cancel{c}}e^{-t/\tau}\right]$$

$$i_c(t) = -I_o e^{-t/\tau} = -i(t)$$

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prove: $V_L(t) = -V(t)$

$$V_L(t) = L\frac{di}{dt} \quad , i = \frac{V_o}{R}e^{-t/\tau} \quad , \tau = \frac{L}{R}$$

$$\frac{di}{dt} = \frac{V_o}{\cancel{R}}\cdot\left(-\frac{\cancel{R}}{L}\right)e^{-t/\tau}$$

$$= -\frac{V_o}{L}e^{-t/\tau}$$ — sub in eq. of $V_L(t)$

$$V_L(t) = \cancel{L}\left[-\frac{V_o}{\cancel{L}}e^{-t/\tau}\right]$$

$$V_L(t) = -V_o e^{-t/\tau} = -V(t)$$
