

*Step Response of RL circuit.

KVL

$$-V_s + i.R + L \frac{di}{dt} = 0$$

$$L \frac{di}{dt} = V_s - i.R$$

$$\frac{di}{dt} = \frac{V_s - i.R}{L}$$

$$\frac{di}{dt} = -\frac{R}{L} \left(i - \frac{V_s}{R} \right) \quad] * dt$$

$$di = -\frac{R}{L} \left(i - \frac{V_s}{R} \right) dt$$

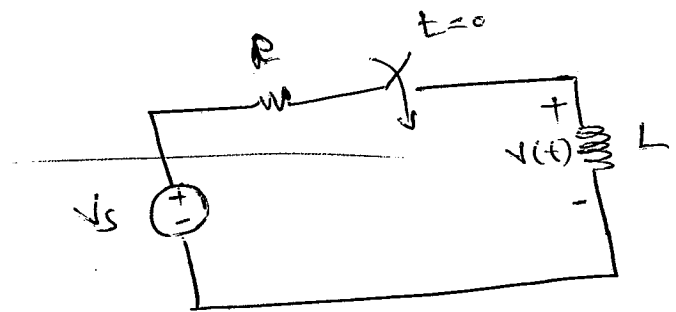
$$\frac{di}{\left(i - \frac{V_s}{R} \right)} = -\frac{R}{L} dt$$

$$\int_{I_0}^{i(t)} \frac{1}{\left(i - \frac{V_s}{R} \right)} di = -\frac{R}{L} \int_0^t dt$$

$$\ln \frac{i(t) - (V_s/R)}{I_0 - (V_s/R)} = e^{-(R/L)t}$$

$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-(R/L)t}}$$

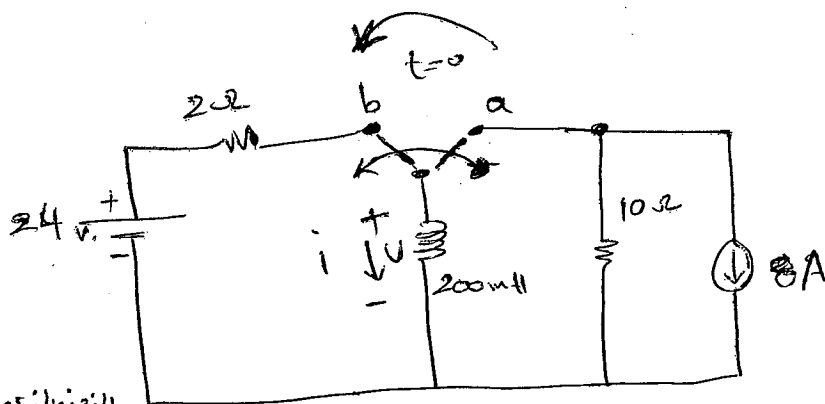
$$v_L = L \frac{di}{dt}$$



step Response of RL cct.

Ex:

- ① $i(t)$
- ② V_L at $t \geq 0$
- ③ does the initial voltage make sense in terms of circuit behavior?
- ④ How many ms after the switch has been moved does the inductor voltage = 24 V. $24 \text{ V}(t)$
- ⑤ Plot both $i(t)$ & $v(t)$ versus t .



Sol

- ① at $t < 0$
at steady state the inductor will be a short act.
So R will ignored

$$\therefore I_0 = -8 \text{ A}$$

at $t > 0$

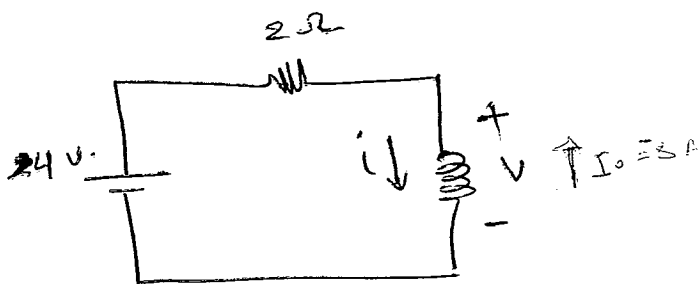
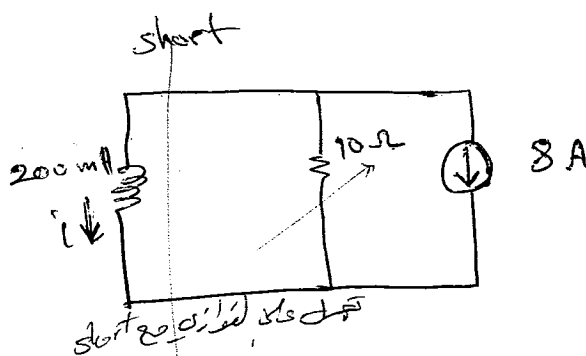
$$\tau = \frac{L}{R} \rightarrow \frac{200 \text{ mH}}{2} = 0.1 \text{ s}$$

$$i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R}) e^{-t/\tau}$$

$$= \frac{24}{2} + (-8 - \frac{24}{2}) e^{-t/0.1}$$

$$= 12 + (-8 - 12) e^{-10t}$$

$$i(t) = 12 - 20 e^{-10t}$$



②

$$2) V_L = L \frac{di(t)}{dt}$$

$$= L \frac{d(12 - 20e^{-10t})}{dt}$$

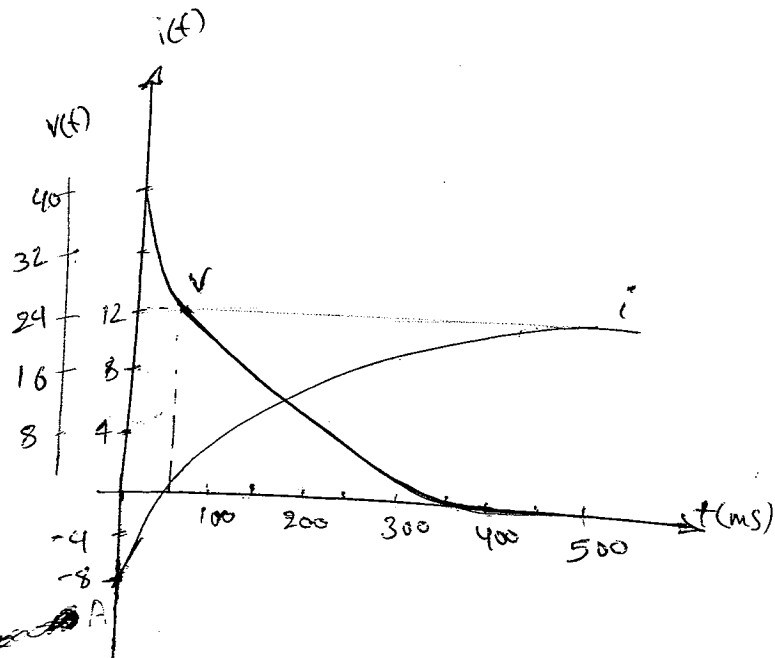
$$= L(2e^{-10t})$$

$$= (200 \times 10^{-3})(2e^{-10t})$$

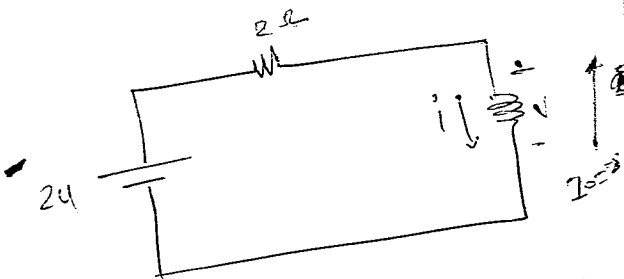
$$\boxed{V_L(t) = 40e^{-10t}} \text{ v. at } t > 0$$

$$\underline{V_L = 40 \text{ v.}}$$

5



3



Yes, كما في تصميم الدارة

لأن في تيار المخزون في القابض $8A$ سوف يكون
 $V_R = (I \times R) = (8 \times 2) = 16$ فولتية المقاومة
 $V_R + V_L = 40 \text{ v.}$
 $16 + 24 = 40 \text{ v.}$
 = تيار حالة الدارة

$$4) V_L(t) = 40e^{-10t} \quad t=?$$

$$24 = 40e^{-10t}$$

$$\ln [24 = 40e^{-10t}]$$

$$\ln \frac{24}{40} = -10t$$

$$-0.5 = -10t$$

$$t = 51.08 \text{ ms}$$

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The step Response of an RC circuit.

KIL:

$$I_s = C \frac{dV_c}{dt} + \frac{V_c}{R} \quad] \div C$$

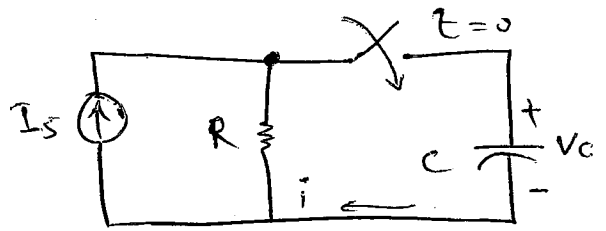
$$\frac{dV_c}{dt} + \frac{V_c}{R_c} = \frac{I_s}{C} \quad] \times dt$$

$$dV_c = \left(\frac{I_s}{C} - \frac{V_c}{R_c} \right) dt$$

$$dV_c = \frac{1}{R_c} (I_s \cdot R - V_c) dt \quad \left[\frac{dV_c}{I_s \cdot R - V_c} = \frac{1}{R_c} \cdot dt \right] \int \text{integral}$$

$$V_c = I_s \cdot R + (V_0 - I_s \cdot R) e^{-t/R_c} \quad \text{for } t \geq 0$$

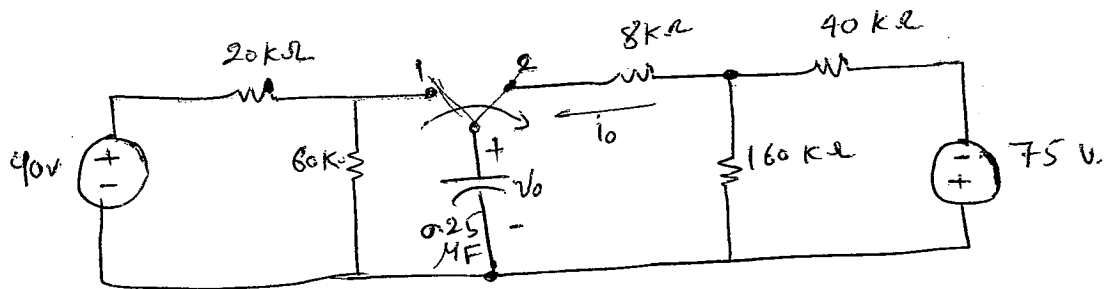
$$i_c = C \frac{dV}{dt}$$



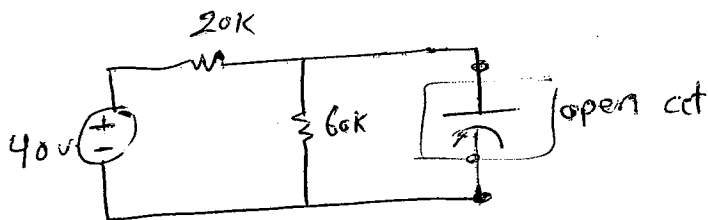
Ex:

a) $V_0(t)$

b) $i_0(t)$



at $t < 0$



$$V_0 = V_{60k\Omega} \Rightarrow V_0 = 40 \times \left(\frac{60}{60 + 20} \right) k\Omega$$

$$V_0 = 30 \text{ V.}$$

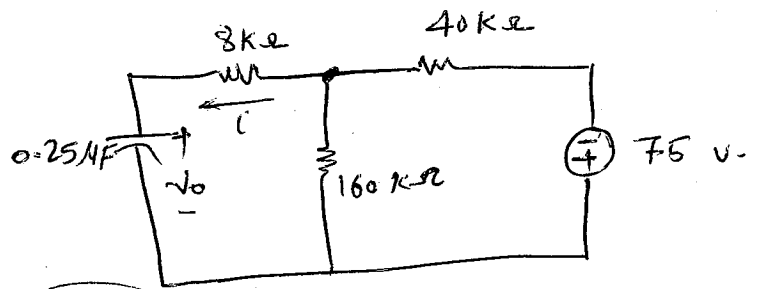
(4)

at $t > 0$

$$R_{eq} = (160 // 40) + 8 \text{ k}\Omega$$

$$= 40 \text{ k}\Omega$$

$$V_{o.c} = (-75) + \frac{160 \cdot 10^3}{(160 + 40) \cdot 10^3} = -60 \text{ V.}$$



$t > 0$ is

$$\tau = RC = (40 \cdot 10^3) \cdot 0.25 \cdot 10^{-6} = 10 \text{ ms}$$

$$V_o(t) = I_s R + (V_o - I_s R) e^{-t/\tau}$$

$$= -60 + [30 + 60] e^{-100t}$$

$$V_o(t) = -60 + 90 e^{-100t} \quad t \geq 0$$

$$b) i_o(t) = i_c = C \frac{dV_o(t)}{dt}$$

$$= (0.25 \cdot 10^{-6}) \cdot \frac{d(-60 + 90 e^{-100t})}{dt}$$

$$= 0.25 \cdot 10^{-6} \cdot (-9000 e^{-100t})$$

$$i_o(t) = -2.25 e^{-100t} \text{ mA}$$