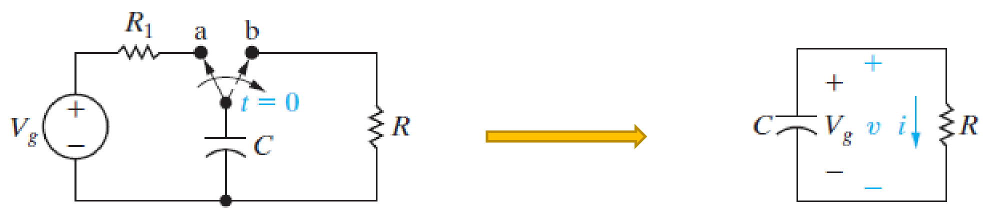
2. Natural Response of an *RC* circuits.



$$i = C\frac{dv}{dt},$$

Applying KCL at the circuit

$$i_C + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0.$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

$$\frac{dv}{v} = -\frac{1}{RC}dt$$

Integrating both sides, we get

$$\int_{Vo}^{v(t)} \frac{dv}{v} = -\int_{0}^{t} \frac{1}{RC}\, dt$$

$$\ln v(t) - \ln Vo = \frac{1}{RC}\, t$$

$$\ln\frac{v(t)}{Vo} = -\frac{t}{RC} \qquad by \text{ exp both sides}$$

$$\frac{v(t)}{Vo} = e^{-\frac{t}{RC}}$$

$$v(t) = V_0 e^{-t/RC}$$

$$\boxed{\boldsymbol{v(t) = Vo\, e^{-\frac{t}{\tau}}}} \qquad \boxed{\boldsymbol{\tau = RC}}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R}e^{-t/\tau}$$

The power dissipated in the resistor is

$$\boxed{p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}}$$

The energy absorbed by the resistor up to time t is

$$w_R(t) = \int_0^t p\,dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau}\,dt$$
$$= -\frac{\tau V_0^2}{2R} e^{-2t/\tau}\Big|_0^t = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}), \qquad \tau = RC$$

The energy stored in the capacitor is

$$\boxed{w = \frac{1}{2} C v^2}$$

→→→→ **w of capacitor**

Notice that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2} C V_0^2$, which is the same as $w_C(0)$, the energy initially stored in the capacitor. The energy that was initially stored in the capacitor is eventually dissipated in the resistor.

$$\boxed{i = C\frac{dv}{dt}} \quad \& \quad \boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

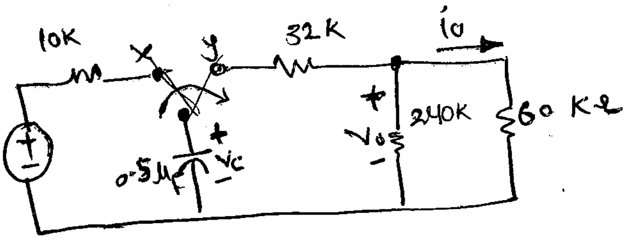
→→→→ **I & v of capacitor**

خطوات الحل:#

1. يبدأ الحل عند زمن اصغر من صفر($t < 0$) اي قبل تغيير حالة المفتاح (الدائرة الرئيسية كاملة),لإيجاد الفولتية الابتدائية Vo المارة خلال المتسعة.
2. بعد تغيير حالة المفتاح عند زمن اكبر من او يساوي صفر ($t \geq 0$), ترسم دائرة جديدة ثم حساب ثابت الزمن τ .
3. ايجاد الفولتية v(t) وبقية العناصر المطلوبة في السؤال جميعها (at $t \geq 0$) من خلال القوانين اعلاه .

①

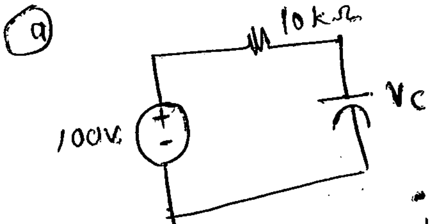
Example :- The switch in the circuit has been in position x for along
At $t=0$, the switch moves to position y. Find
a) $V_c(t)$, b) $V_o(t)$, c) $i_o(t)$, d) total energy dissipated in the 60kΩ



Sol :

⇒ at $t<0$

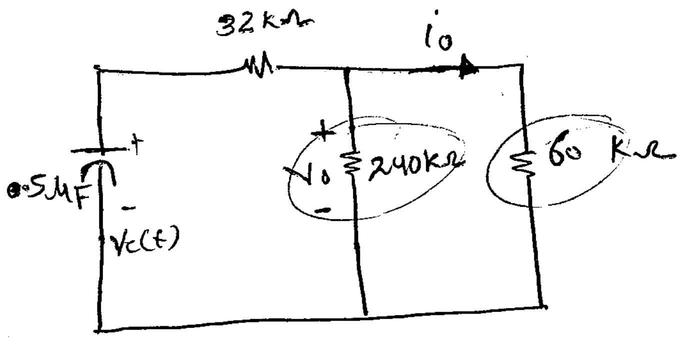
(a)



the capacitor will be open circuit. → at steady state C will be open

$\therefore V_C = V_o = V_S = 100$ volt

⇒ at $t \geq 0$

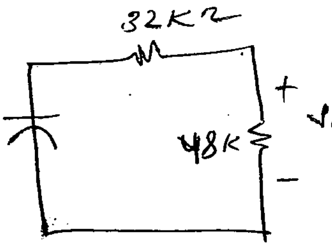


$Req = [(60 // 240) + 32]$ kΩ

$Req = 48 + 32 = 80$ kΩ

$\tau = Rc = 80K * 0.5\mu$

$= 40$ mS



$\boxed{V_c(t) = V_o\, e^{-t/\tau}}$

$= 100\, e^{-t/0.04}$

$\boxed{V_c(t) = 100\, e^{-25t} \text{ v.}}$ for $t \geq 0$

b) $V_o(t) = V_c(t) * \dfrac{48}{48+32}$

$= 100\, e^{-25t} * \dfrac{48}{80} = \boxed{60\, e^{-25t} \text{ volt}}$ for $t \geq 0$

c) $i_o(t) = \dfrac{V_o(t)}{R_{60\Omega}} = \dfrac{60}{60k\Omega} e^{-25t}$ $\boxed{= e^{-25t} \text{ mA}}$ $t \geq 0$

d) power dissipated at 60 kΩ

$$P_{60k\Omega}(t) = i_o^2(t) * R$$

$$= (e^{-25t})^2 * 60 = \boxed{60\, e^{-50t} \ W}$$

∴ the total energy is:

$$W_{60k\Omega} = \int_0^\infty P \cdot dx$$

$$= \int_0^\infty 60\, e^{-50t}\, dt$$

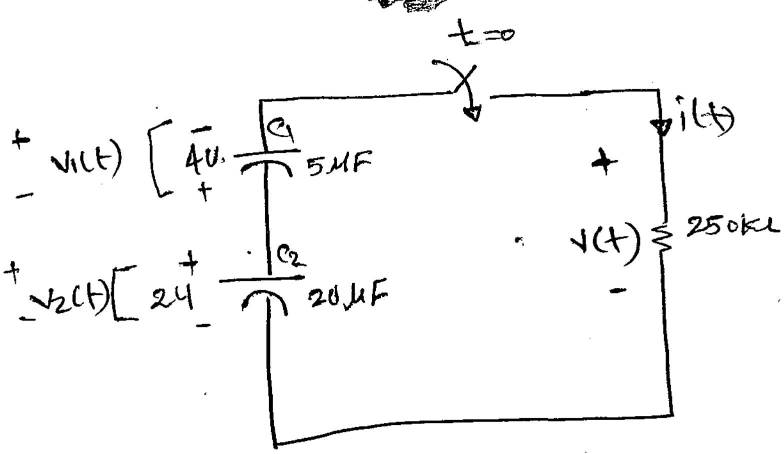
$$\boxed{= 1.2 \ mJ}$$

Example (2)

(1) find, $v_1(t)$, $v_2(t)$, $v(t)$, $i(t)$

(2) $w_o C_1$, $w_o C_2$

(3) W_{eq}

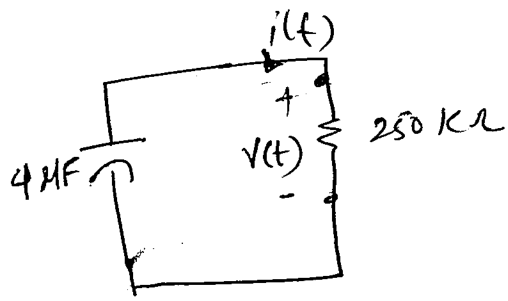


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{5 * 20}{5+20} = 4\ \mu F$$

$$\tau = RC = 250 * 10^3 * 4 * 10^{-6}$$
$$= 1 \text{ sec}$$

⇒



$$V_o = 24 - 4 = 20 \ V.$$

$$v(t) = V_o\, e^{-t/\tau}$$

$$\boxed{v(t) = 20\, e^{-t} \ V.}$$ for $t \geq 0$

$$i(t) = \frac{v(t)}{R} = \frac{20 \cdot e^{-t}}{250 \cdot 10^3} = 80\, e^{-t} \ \mu A \ ] \text{ for } t \geqslant 0$$

$$\boxed{V_1(t) = \frac{-1}{C_1}\int_0^t i(t) \cdot dt - V_{0_1}}$$

$$= \frac{-1}{5 \cdot 10^{-6}} \int_0^t 80 \cdot 10^{-6}\, e^{-t} \cdot dt - 4$$

$$= (16 e^{-t} - 20)\ V \ ] \text{ at } t \geqslant 0$$

$$\boxed{V_2(t) = \frac{-1}{C_2}\int_0^t i(t) \cdot dt - V_{0_2}}$$

$$= \frac{1}{20 \cdot 10^{-6}} \int_0^t 80 \cdot 10^{-6}\, e^{-t}\, dt + 24$$

$$= (4 e^{-t} + 20)\ V \ ] \quad t \geqslant 0$$

(2) The initial energy storde in C_1 is

$$w_1 = \frac{1}{2} C_1 V_{0_1}^2$$

$$= \frac{1}{2}(5 \times 10^{-6})(4)^2 = \underline{40 \mu J}$$

$$W_2 = \frac{1}{2} C_2 V_{0_2}^2 = \frac{1}{2}(20 \times 10^{-6})(24)^2 = \underline{5760\ \mu J}$$

(3) The total energy stordin the 2 capacitor is

$$W = 40 + 5760 = \boxed{5800\ \mu J}$$

(4) . As $t \to \infty$

$$V_1(t) = (16 \cancel{e^{-t}} - 20)\ V \Rightarrow V_1 \to -20$$

$$V_2(t) = (4 \cancel{e^{-t}} + 20)\ V \Rightarrow V_2 \to +20$$

$$\therefore W_\infty = \frac{1}{2} C_e V_2^2$$

$$= \frac{1}{2}(5 + 20) 10^{-6} \times (400) = 5000\ \mu J$$

Total energy delivered to the 250 kΩ resistor is

$$w = \int_0^{\infty} p \, dt = \int_0^{\infty} \frac{400e^{-2t}}{250 \cdot 10^3} \, dt = 800 \, \mu J$$

∴ comparing the results obtainen in (b) and (c) show that

$$800 \, \mu J = (5800 - 5000) \, \mu J$$

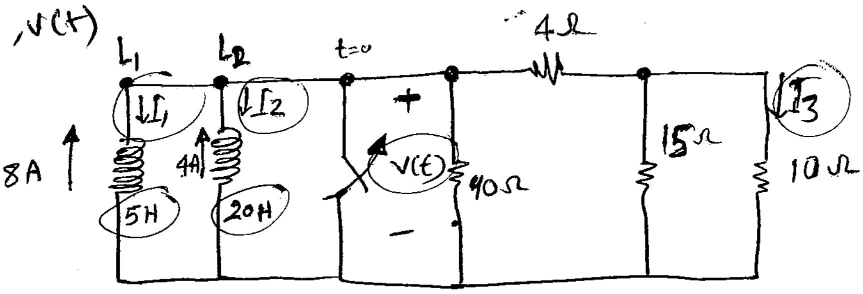
Example for NR of RL (Serieal inductors)

Find (a) i_1, i_2, i_3 at $t \geq 0$, $i(t)$, $v(t)$

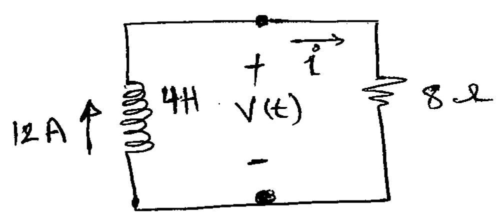
(b) w_o for parallel inductors.

(c) w.stored in inductors as $t \to \infty$

(d) show the total energy deliverd to the R network equals the difference between the results in (b) and (c).



at $t \geq 0$



Sol:

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} = \frac{5 * 20}{5 + 20} = \boxed{4H}$$

$$I_o = I_{o_1} + I_{o_2} = 8 + 4 = \boxed{12 \, A}$$

$\rightarrow R_{eq} = [(10 // 15) + 4] // 40$
$= (6 + 4) // 40$
$= 8 \, \Omega$

$$\therefore \tau = \frac{L}{R} = \frac{4}{8} = \boxed{0.5 \, s}$$

$$\therefore i(t) = 12 e^{-t/0.5}$$

$$\boxed{i_L(t) = 12 e^{-2t} \, A}, \; t \geq 0$$

$$v(t) = i(t) * R$$

$$= 8 * 12e^{-2t} = 96e^{-2t} \text{ volt}, \text{ at } t \geq 0$$

$$\boxed{i_1 = \frac{1}{L_1} \int_0^t v(t) \, dt - I_{o_1}} \Rightarrow i_1 = \frac{1}{5} \int_0^t 96 e^{-2t} dt - 8$$

$$= \frac{1}{5} \left[\frac{-96}{2} e^{-2t} \right]_0^t - 8$$

$$= \frac{1}{5} \left[-48 (e^{-2t} - e^0) \right] - 8$$

$$= -9.6e^{-2t} + 9.6 - 8$$

$$= -9.6e^{-2t} + 1.6$$

$$= 1.6 - 9.6e^{-2t} \; A, \; t \geq 0$$

$$i_2 = \frac{1}{L_2}\int_0^t v(t)\,dt - I_{o_2}$$

$$= \frac{1}{20}\int_0^t 96e^{-2t}\,dt - 4$$

$$= \underline{-1.6 - 2.4e^{-2t}} \quad A, \quad t \geq 0$$

$$i_3 \Rightarrow i(t) = \frac{v(t)}{R_{10\Omega}} = \frac{96e^{-2t}}{10} = \underline{9.6\,e^{-2t}}$$

$$i_3 = i(t) * \frac{15}{15+10} = 5.76e^{-2t} \quad A, \quad at\ t \geq 0$$

b) The initial energy stored in the inductors is :-

$$w_{L_1} = \frac{1}{2}L_1 I_{o_1}^2 = \frac{1}{2}(5)(64) = 160\ J$$

$$w_{L_2} = \frac{1}{2}L_2 I_{o_2}^2 = \frac{1}{2}(20)(16) = 160\ J$$

$$w = 160 + 160 = 320\ J$$

نحسب الطاقة المخزونة بكل ملف المأخوذة من المصدر الرئيسي قبل فتح المفتاح

c) energy as at $t \to \infty$

$$i_1 = 1.6 - \cancel{9.6e^{-2t}} \Rightarrow i_1 \longrightarrow 1.6\ A$$

$$i_2 = -1.6 - \cancel{2.4e^{-2t}} \Rightarrow i_2 \longrightarrow -1.6\ A$$

الطاقة المخزونة بكل ملف بعد فتح المفتاح يعني $t \to \infty$ $t \geq 0$ يعني ناخذ قيم جديدة للتيار

$\therefore$ w total at ∞ is

$$w = \frac{1}{2}(5)(1.6)^2 + \frac{1}{2}(20)(-1.6)^2 = 32\ J$$

d) $$w = \int_0^\infty p\,dt = \int_0^\infty \left(i_{(t)}^2 \cdot R\right) dt$$

اثبت ان الطاقة الواصلة للمقاومات هي الفرق بين الطاقة المخزونة بالملفات قبل $t=0$ وعند $t \to \infty$

$$= \int_0^\infty [12e^{-2t}]^2 * 8\,dt = \int_0^\infty 1152e^{-4t}\,dt$$

$$= 1152\frac{e^{-4t}}{-4}\Big]_0^\infty = \underline{288\ J}$$

$$\therefore \boxed{288 = 320 - 32}$$

هذا يعني ان الطاقة المخزونة بالملف المثالي تمثل كمية الطاقة التي سوف تصل الى شبكة المقاومات خلال اطراف الملفات الاصلية.