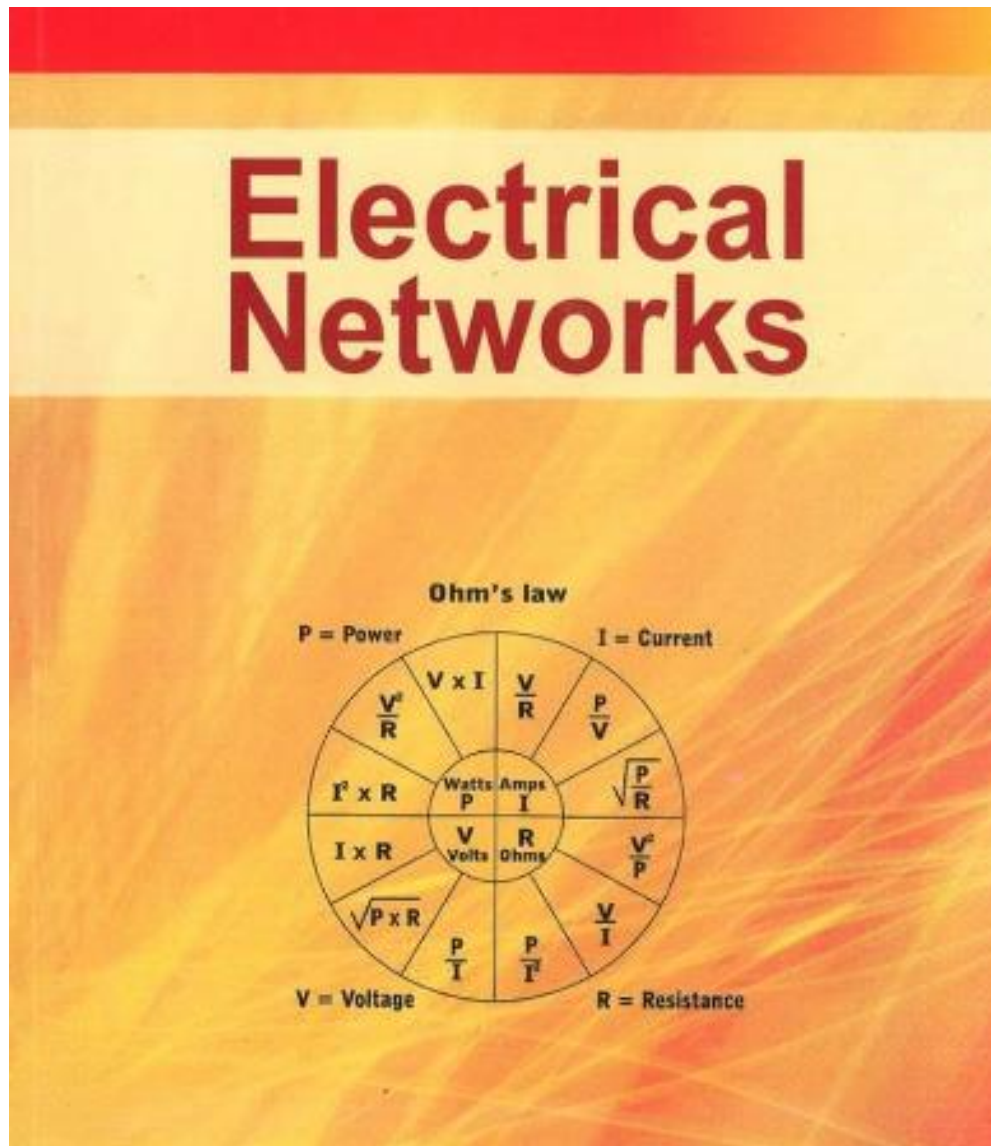


Electrical networks



Prepared by:

Asst. Lect. Sura H. AL-FARAJ

Material Covered
1. Transient response of circuits
2. Natural response of first-order RL and RC circuits.
3. Step Response of first-order RL and RC circuits.
4. sequential switching.
5. parallel & series of second-order RLC circuits (natural & Step).
6. Magnetic coupling.
7. Three-phase circuits .
8. Measuring Average power in 3-phase circuits .
9. Circuit Analysis in the s Domain .
10. Poles and Zeros.
11. The Transfer Function in Partial Fraction Expansions.
12. The Two-Port Parameters and Analysis of the Terminated Two-Port Circuit.
13. Cascade interconnection of two-port circuits.
14. Passive filter .
15. Active filter.

Learning and Teaching Resources

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Lecture -1-

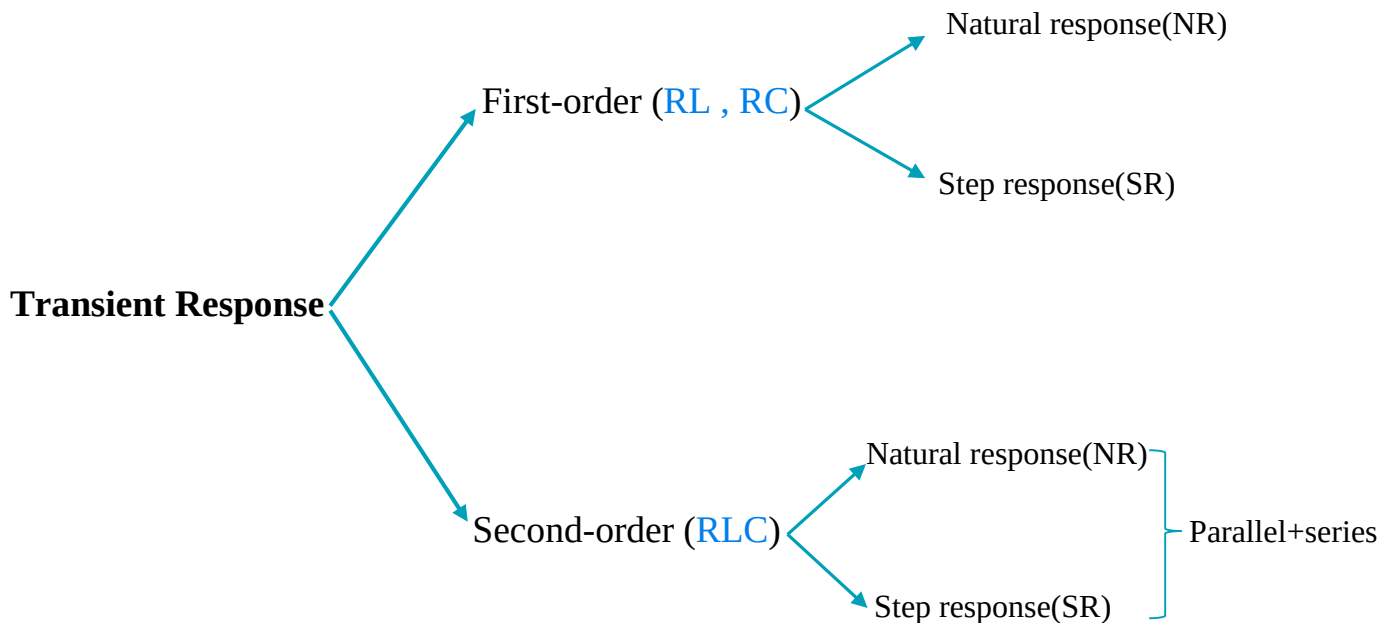
Transient Response of Circuits

- If the output of an electric circuit for an input varies with respect to time, then it is called as time response. After applying an input to an electric circuit, the output takes certain time to reach steady state. So, the output will be in transient state till it goes to a steady state. Therefore, the response of the electric circuit during the transient state is known as transient response.
- A **transient response** of a circuit is a temporary change in the way that it behaves due to an external excitation, that will disappear with time.

• إذا تغير خرج الدائرة الكهربائية لمدخل ما بالنسبة للزمن، فإن ذلك يسمى استجابة زمنية. بعد تطبيق مدخل على الدائرة الكهربائية، يستغرق الخرج وقتاً معيناً للوصول إلى حالة مستقرة. لذا، سيكون الخرج في حالة عابرة حتى ينتقل إلى حالة مستقرة. لذلك، تُعرف استجابة الدائرة الكهربائية أثناء الحالة العابرة بالاستجابة العابرة.

- الاستجابة العابرة للدائرة هي تغير مؤقت في طريقة تصرفها بسبب إثارة خارجية، والتي سوف تختفي مع مرور الوقت.

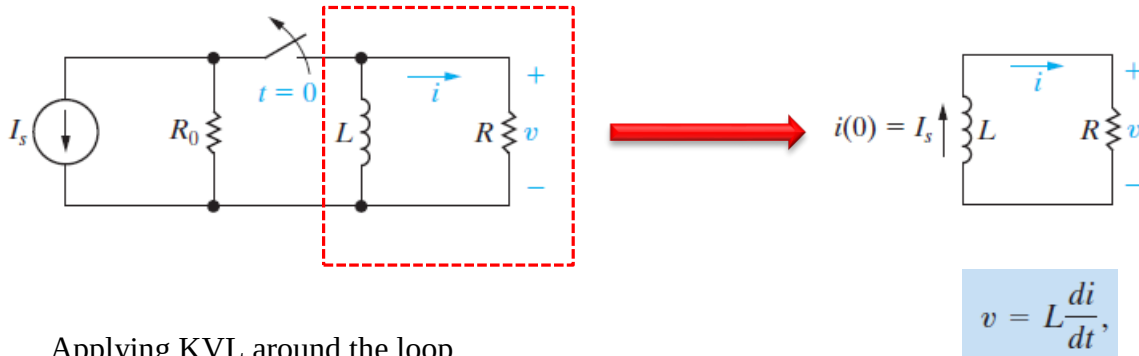
In electrical engineering, a transient response is the response of a system to a change from an equilibrium or a steady state. The transient response is tied to any event that affects the equilibrium of the system. The natural response (NR) and step response (SR) are a transient response to a specific input.



1. Natural Response of **RL** circuits.

The natural response depends on the nature of the circuit alone or refers to the circuit's behavior (in terms of voltages and currents), with no external sources. The circuit has a response only because of the energy initially stored in the capacitor and inductor.

تعتمد الاستجابة الطبيعية على طبيعة الدائرة وحدها أو تشير إلى سلوك الدائرة (من حيث الفولتية والتيارات)، دون وجود مصادر خارجية. لا تستجيب الدائرة إلا بسبب الطاقة المخزنة في البداية في المكثف والمحث.



Applying KVL around the loop

$$v_L + v_R = 0$$

$$L \frac{di}{dt} + Ri = 0,$$

$$\frac{di}{dt} dt = -\frac{R}{L} i dt.$$

$$\frac{di}{i} = -\frac{R}{L} dt.$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L} dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L} \Big|_0^t \Rightarrow \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

$$i(t) = I_0 e^{-Rt/L}$$

$$\tau = \frac{L}{R}$$

$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

$$p = vi, \quad p = i^2 R, \quad \text{or} \quad p = \frac{v^2}{R}.$$

$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

$$w_R(t) = \int_0^t p \, dt = \int_0^t I_0^2 R e^{-2t/\tau} \, dt = -\frac{1}{2} \tau I_0^2 R e^{-2t/\tau} \Big|_0^t, \quad \tau = \frac{L}{R}$$

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

with the corresponding energy stored in the inductor as

$$w(0) = \frac{1}{2} L I_0^2$$

Note that as $t \rightarrow \infty$, $w_R(\infty) \rightarrow \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the initial energy stored in the inductor. Again, the energy initially stored in the inductor is eventually dissipated in the resistor.

Calculating the natural response of an RL circuit can be summarized as follows:

1. Start at $t < 0$, Find the initial current I_0 , through the inductor.
2. at $t \geq 0$, Find the time constant of the circuit τ .
3. Find $i(t)$ and the other required items $t \geq 0$.

$$\begin{aligned} i(t) &= I_0 e^{-t/\tau}, \quad t \geq 0, \\ v(t) &= I_0 R e^{-t/\tau}, \quad t \geq 0^+, \\ p &= I_0^2 R e^{-2t/\tau}, \quad t \geq 0^+, \\ w &= \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}), \quad t \geq 0. \end{aligned}$$

خطوات الحل:

1. يبدأ الحل عند زمن اصغر من صفر ($t < 0$) اي قبل تغيير حالة المفتاح (الدائرة الرئيسية كاملة), لإيجاد التيار الابتدائي I_0 المار خلال المحاثه.
2. بعد تغيير حالة المفتاح عند زمن اكبر من او يساوي صفر ($t \geq 0$), ترسم دائرة جديدة ثم حساب ثابت الزمن τ .
3. ايجاد التيار $i(t)$ وبقية العناصر المطلوبة في السؤال جميعها ($t \geq 0$) من خلال القوانين اعلاه.

⑤

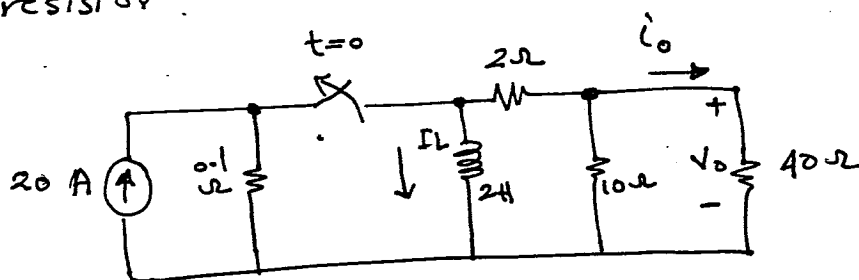
Calculating the natural Response of RL circuit.

- 1- Find the initial current I_0
- 2- time constant $(\tau) = L/R$
- 3- Use eqn of $i(t)$

example : ①

The switch in the circuit has been closed for long time before it is opened at $t=0$, find

- a) $i_L(t)$ for $t \geq 0$
- b) $i_o(t)$ for $t \geq 0$
- c) $v_o(t)$ for $t \geq 0$
- d) The percentage of the total energy stored in the 2 H inductor that is dissipated in the 10 Ω resistor.

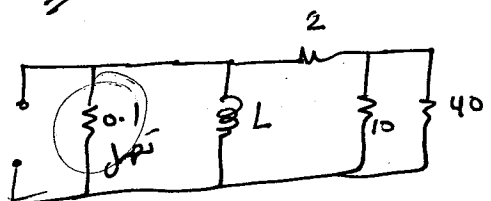


Sol:

at $t < 0$ → the L is short that means I_0 is same to current of the source.

$$\therefore I_0 = 20 \text{ A}$$

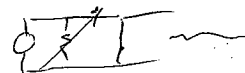
at $t \geq 0$



$$\Rightarrow R_{eq} = (40 // 10) + 2 = 10 \Omega$$

$$\therefore \tau = \frac{L}{R} = \frac{2}{10} = 0.2 \text{ sec}$$

at $t < 0$.
 قبل ان يفتح المفتاح I_0 في الدارة هو نفس التيار في المصدر
 because the inductor is short circuit



(6)

$$\therefore a) i_L(t) = I_0 e^{-t/\tau}$$

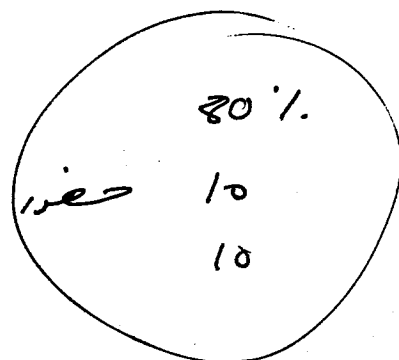
$$= 20 e^{-5t} \text{ A, at } t \geq 0$$

b) $i_o(t)$ / جولتاجر الحقا 40Ω
 تسند current divider

$$i_o(t) = -i_L(t) \times \frac{10}{10+40} \rightarrow \text{على اليمين}$$

$$= (-20 e^{-5t}) \times 0.2$$

$$i_o(t) = -4 e^{-5t} \text{ See, } t \geq 0$$



c) $V_o(t)$ / جولتاجر 40Ω

$$V_o(t) = i_o(t) \times R_{40\Omega}$$

$$= -4 e^{-5t} \times 40$$

$$= -160 e^{-5t} \text{ V.}$$

d) w

$$P_{10\Omega}(t) = \frac{V_o^2(t)}{R} = \frac{(160 e^{-5t})^2}{10} = 2560 e^{-10t} \text{ W, } t \geq 0$$

* total energy dissipated in the 10Ω is:

$$w_{10\Omega}(t) = \int_0^{\infty} P_{10\Omega}(t) dt = 256 \text{ J}$$

$$\int 2560 e^{-10t} dt = \frac{2560}{10} = 256 \text{ J}$$

* The initial energy stored in the $2H$ inductor is

$$w(0) = \frac{1}{2} L i_L^2(0)$$

$$= \frac{1}{2} (2) \left(\frac{20}{10} \right)^2 = 400 \text{ J}$$

$$w(0) = \frac{1}{2} L (10)^2 = \frac{1}{2}$$

$$\therefore \frac{256}{400} \times 100 = 64\%$$

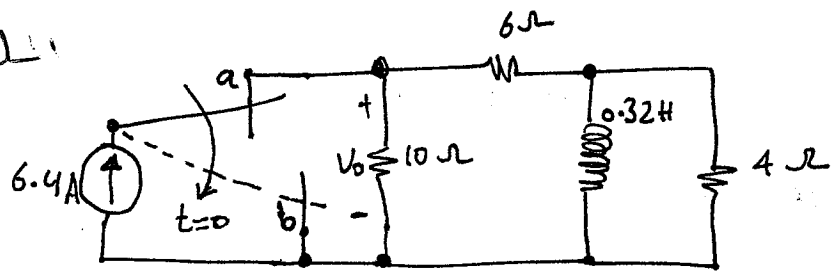
$$= \frac{w_{10\Omega}}{w(0)}$$



Example (2) : At $t=0$, the switch in the ckt moves instantaneously from position a to b.

a) Find V_0 for $t \geq 0^+$.

b) what percentage of the initial energy stored in the inductor is eventually dissipated in the 4Ω resistor?

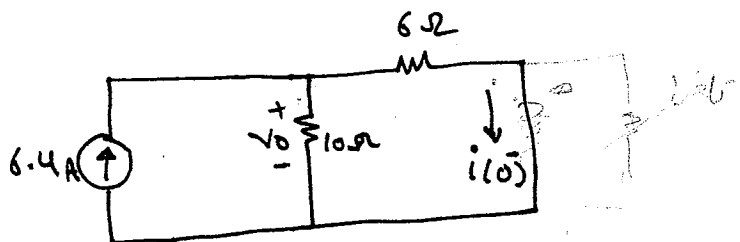


Sol:

at $t < 0$

Short ckt $\Rightarrow$ $i_{10\Omega}$ $\Rightarrow$ $i_{10\Omega}$

الحال $i_{10\Omega}$ $\Rightarrow$ $i_{10\Omega}$



Using current division

$$i(0^-) = i_{10\Omega} = 6.4 \times \frac{10}{10+6} = 4 \text{ A}$$

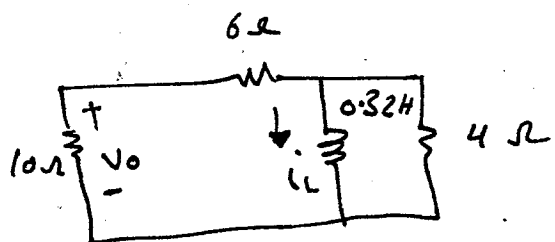
at $t > 0$

$$R_{eq} = (6+10) \parallel 4 = 3.2 \Omega$$

$$\therefore \tau = \frac{L}{R} = \frac{0.32}{3.2} = 0.1 \text{ sec}$$

$$\text{So } i_L(t) = i_0 e^{-t/\tau}$$

$$i_L(t) = 4 e^{-10t} \text{ A}, t \geq 0$$



to find V_0 that is the voltage in 10Ω Resistor.

we have to find the current in 10Ω too.

$$\therefore i_0(t) = i_{10\Omega}(t) = \underbrace{[-i_L(t)]}_{\text{med}} \times \frac{4}{10+6+4} = \frac{4}{20} (-4e^{-10t})$$

$$\therefore i_0(t) = -0.8 e^{-10t} \text{ A}, t \geq 0$$

(8)

$$V_0(t) = i_0(t) \cdot R$$

$$= (-0.8 e^{-10t}) \cdot (10) = -8 e^{-10t} \text{ volt, } t \geq 0^+$$

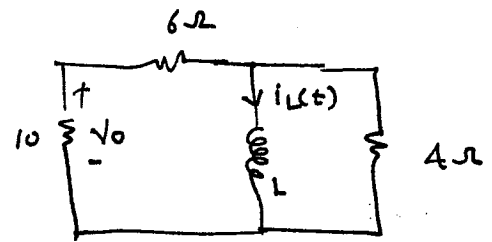
(b) initial energy in ~~the~~ inductor:-

$$W_0 = \frac{1}{2} L i_0^2$$

$$= \frac{1}{2} (0.32) (4)^2 = 2.56 \text{ J}$$

* the energy dissipated in the 4Ω is:

$$W_{4\Omega} = ?$$



$$V_{4\Omega}(t) = V_L = L \frac{di_L(t)}{dt}$$

~~$$V_L = L \cdot i_L(t) \cdot \frac{d}{dt}$$~~

$$V_L = L \cdot i_L(t) \cdot \frac{d}{dt}$$

$$= (0.32) \cancel{(4e^{-10t})} \cdot \frac{d}{dt} = (0.32) (-10) (4e^{-10t})$$

$$V_{4\Omega}(t) = -12.8 e^{-10t} \text{ V, } t \geq 0^+$$

$$P_{4\Omega}(t) = \frac{V_{4\Omega}(t)^2}{4} = 40.96 e^{-20t} \text{ , } t \geq 0$$

$$\therefore W_{4\Omega}(t) = \int_0^{\infty} P \cdot dt$$

$$= \int_0^{\infty} 40.96 e^{-20t} \cdot dt = 2.048 \text{ J}$$

$$\therefore \left(\frac{2.048}{2.56} \right) \times 100\% = 80\%$$