3. Time Scaling

Let x[n]denote a DT signal, then the signal y[n] obtained by scaling the independent variable, time **n** , by a factor **a** is defined by **y[n]=x[an], a>0**

- ✓ If **a>1,** the signal is a compressed version of x[n]and some values of DT signal y[n] are lost.
- ✓ If **0<a <1,** then the signal y[n] is an expanded version of x[n].

Example : For a=2 ; in x[2n], the samples x[n] for n= ±1, ±3, ±5..... are lost.

Example : x[n] $= \begin{cases} n & for\ n\ odd \\ 0 & otherwise \end{cases}$

Determine y[n]=x[2n]?

Solution: y[n]=0 for all integer values of n

Precedence Rules for time shifting and time scaling:

- **C-T case:**

Suppose that $y(t) = x(ct - s)$, this relation between $y(t)$ and $x(t)$ satisfies the conditions:

$$y(0) = x(-s); \text{ and } y\left(\frac{s}{c}\right) = x(0)$$

which provide useful checks on $y(t)$ in terms of corresponding values of $x(t)$.

The correct order for time shifting and scaling operations:

a) The **time shifting operation is performed first** on $x(t)$, we get an intermediate signal $v(t) = x(t - s)$; the time shift has replaced t by $t - s$.

b) The **time scaling operation is performed on** $v(t)$, replacing t by ct and the result

$$y(t) = v(ct) = x(ct - s).$$

Examples:

1. Voice signal recorded on a tape recorder:

- ☒ **Compression**: if the tape is played back at a rate faster than the original recording rate.
- ☒ **Expansion**: if the rate is slower than the original.

Example : Consider the rectangular pulse of unit amplitude and a duration of 4 units, depicted in figure. Find y(t)= x(3t-4).

Solution: $c=3, \quad s=4 \Rightarrow y(0)=x(-4)=0;\ y(\frac{s}{c})=y(\frac{4}{3})=x(0)=1$, the graphical solution is

Find y(t) at

t=0 => y(0) =x(-4)=0

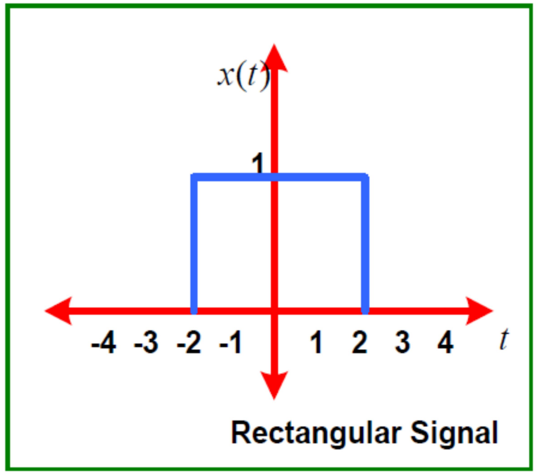
t=1 => y(1) =x(-1)=1

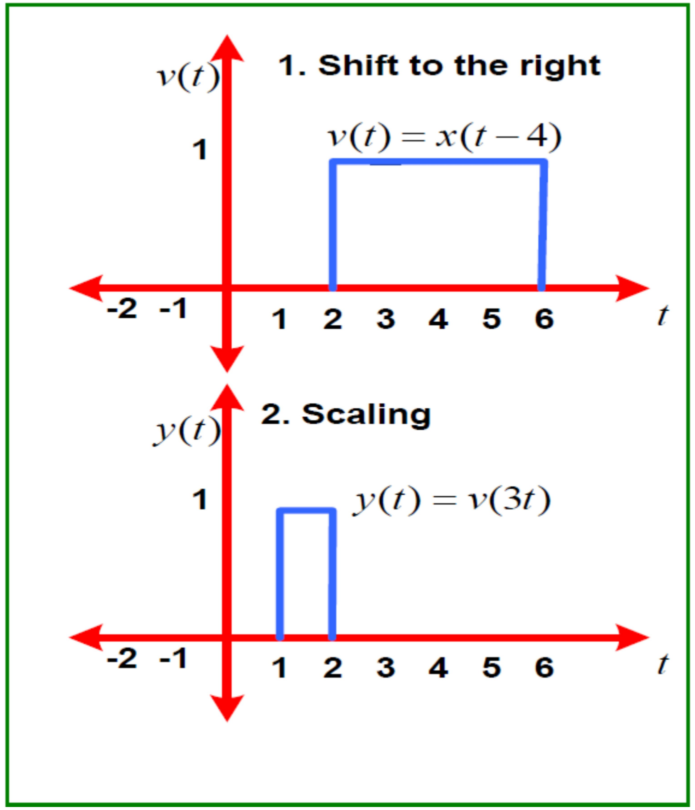
t=2 => y(2) =x(2)=1

t=3 => y(3) =x(5)=0

t=4 => y(4) =x(8)=0

And so on….





D-T case: The same rules are used in the case of DT signals, in the following example, these rules are explained.

Example: Suppose that x[n]= [2, -1, 0, -3, 4]. Find y[n]=x[3n-4]

Solution: c=3, s=4; y[0]=x[-s]=x[-4] =0 and y[s/c]=y[4/3]=x[0]=0

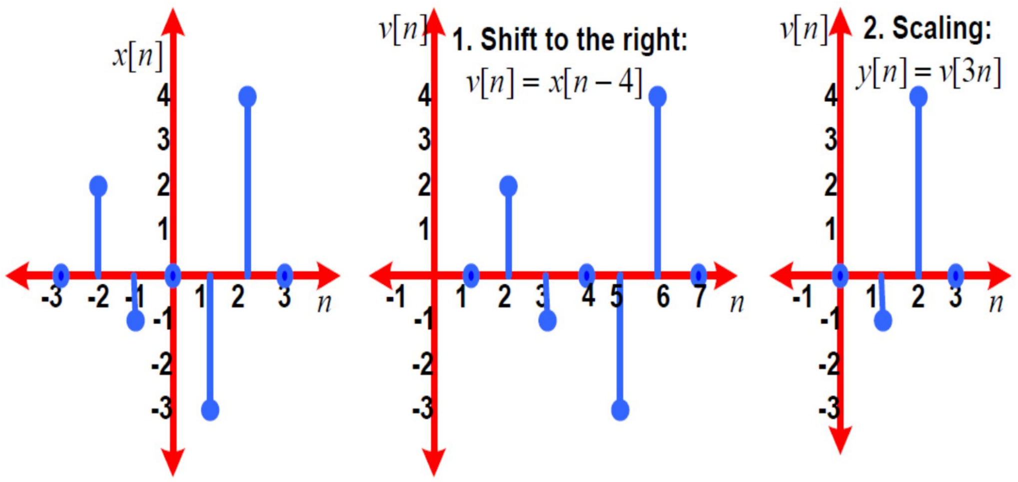
To get y[n]=v[3n], we calculate the following points:

y[0]=v[0]

y[1]=v[3]

y[2]=v[6]

The graphical solution is represented in figure



Manipulation involving the signal amplitude (dependent variable):

Transformations performed on amplitude (dependent variable) are shown in table

Transformation performed on amplitude			
Operation	**D-T signals**	**C-T signals**	**Physical device**
1. Amplitude scaling	$y[n]=cx[n]$	$y(t)=cx(t)$	Electronic amplifier
	c - scaling factor		
2. Addition	$y[n]=x_1[n]+x_2[n]$	$y(t)=x_1(t)+x_2(t)$	Audio mixer
3. Multiplication	$y[n]=x_1[n]\cdot x_2[n]$	$y(t)=x_1(t)\cdot x_2(t)$	Modulator
4. Differentiation	Difference equation	$y(t)=d\dfrac{x(t)}{dt}$	Inductor
5. Integration	Summation	$y(t)=\int_{-\infty}^{t} x(\tau)d\tau$	Capacitor

System is a mathematical transformation of an input signal (or signals) into an output signal (or signals). A system is an entity that processes a set of signals (inputs) to yield another sets of signals (outputs). A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.

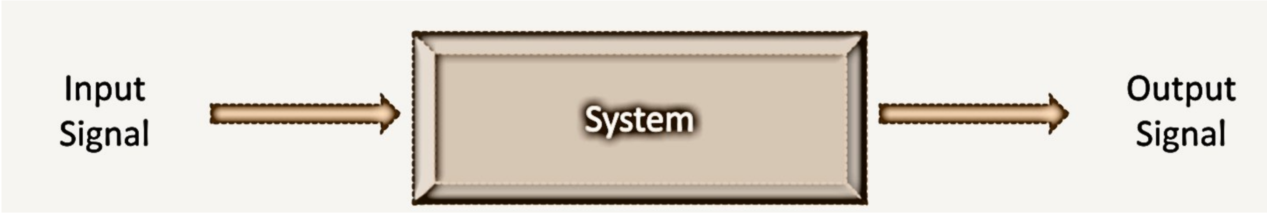


Fig. (a) Elementary block diagrams of (a) general system

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of x into y. The mathematical notation represents this transformation

$$y = f(x) = T\,x$$

where ***f*** *or* ***T*** is the operator representing some well-defined rule by which ***x*** is transformed into ***y***.

Introductory textbooks on signals and systems often begin by restricting the input, the system, and the output to be continuous-time quantities, as shown in Fig. b. Restricting the input, the system, and the output to be discrete-time (DT) quantities, as shown in Fig. c, leads to the topic of discrete-time signals and systems.

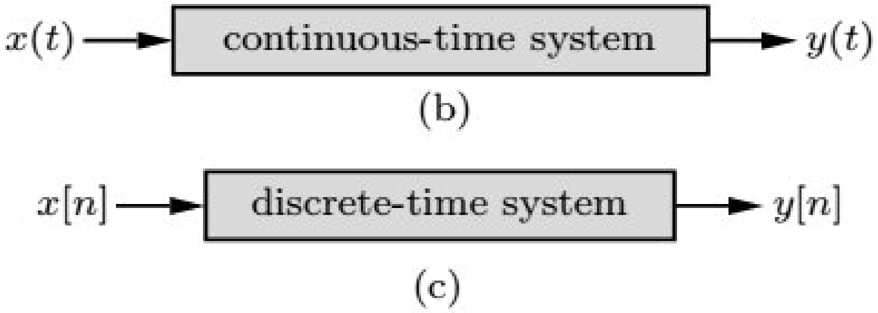


Figure (b) CT System
(c) DT systems.

Typical digital signal processing (DSP) systems are hybrids of continuous-time and discrete-time systems. Ordinarily, DSP systems begin and end with continuous-time signals, but they process signals using a digital signal processor of some sort. Specialized hardware is required to bridge the continuous-time and discrete-time worlds. As the block diagram of Fig. shows, general DSP systems are more complex and both CT and DT concepts are needed to understand complete DSP systems.

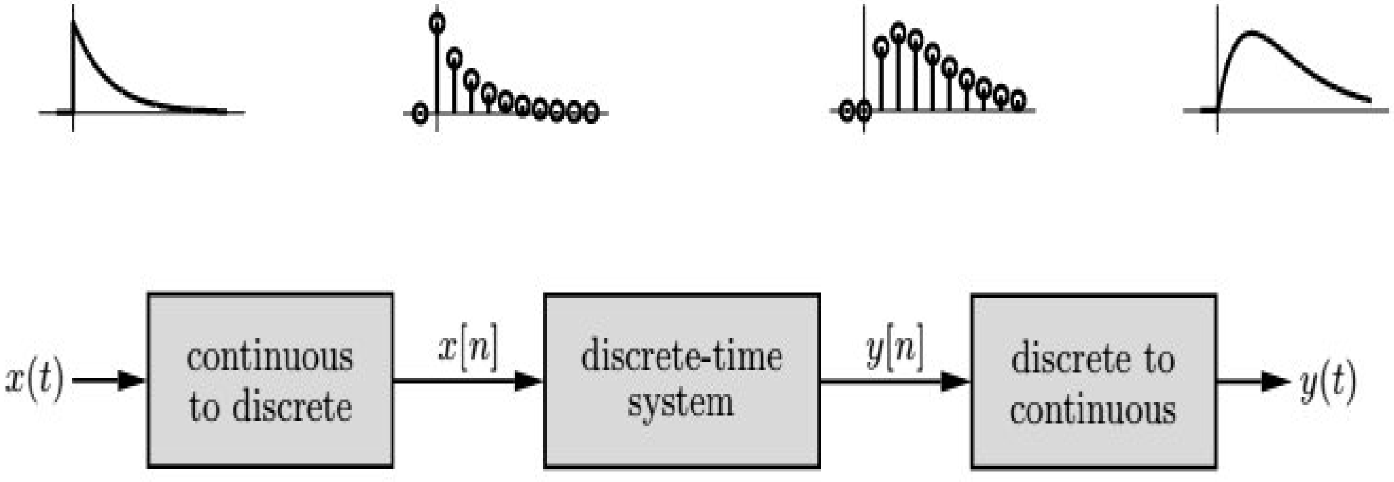


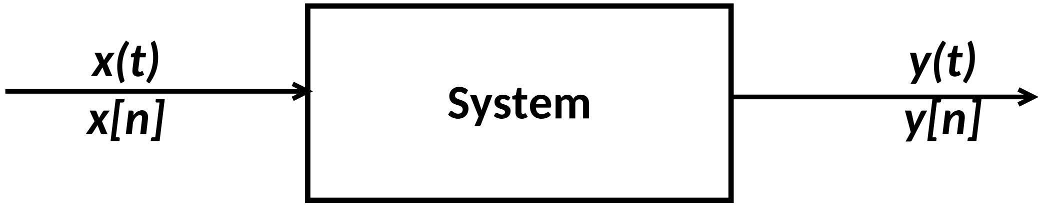
Figure: Block diagram of a typical DSP system

Based on the properties of the functional ***relationship between the input and output***, system can be classified as follow

1. **Systems with and without Memory**
2. **Causal and Noncausal Systems**
3. **Stable and unstable systems**
4. **Linear and nonlinear system**
5. **Time-Invariant and Time-Varying Systems**

Systems with and without Memory (Static and Dynamic Systems)

A system is classified as ***memoryless*** or ***instantaneous or static*** if its output at any instant ***t*** depends at most on the input values at the same instant. If the output of the system at any instant depends also on some of the past or future input values, the system is said to have memory and it is sometimes referred to as a ***dynamic*** system. Thus, the **Systems with memory** is a systems whose output $y(t_0)$ at time t_0 depends on values of the input other than just $x(t_0)$ have memory.



Consider a time-domain system with input *x* and output *y*

$$\boldsymbol{y(t) = x^2(t).}$$

This example defines a simple system, where the output signal at each time depends only on the input at that time. Such systems are said to be **memoryless** because you do not have to remember previous values (or future values, for that matter) of the input in order to determine the current value of the output.

Note that

1- Any discrete-time system described by a difference equation is a dynamic system.

2- A purely resistive electrical circuit is a static system, whereas an electric circuit having inductors and/or capacitors is a dynamic system.

3- A summer or accumulator is an example of a discrete time system with memory.

4- A delay is also a discrete time system with memory.

Example : An example of a memoryless system is a resistor ***v(t) = R × i(t)***

Example : the system ***y(t) =a x²(t) +b x(t)*** is memoryless system.

Example : Check if the below Discrete time (DT) system has memory or not

y[n]=n x[n].

Sol: Since the output value at ***n*** depends on only the input value at ***n* ,** the system is **memoryless.**

Example : An example of a system with memory is a capacitor **C** with the current as the input ***x(t)*** and the voltage as the output ***y(t)* ;** then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau$$

Example: An example of a discrete system with memory is

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Example Find whether the following systems are dynamic or not: **(a)** $y(n) = x(n + 2)$. **(b)** $y(n) = x^2(n)$. **(c)** $y(n) = x(n - 2) + x(n)$

Solution:

(a) Given $y(n) = x(n + 2)$: The output depends on the future value of input. Therefore, the system is **dynamic**.

(b) Given $y(n) = x^2(n)$: The output depends on the present value of input alone. Therefore, the system is **static**.

(c) Given $y(n) = x(n - 2) + x(n)$: The system is described by a difference equation. Therefore, the system is **dynamic**.

Systems with and without Memory

Ex 1. Does $y(t) = x(t) + 5$ have memory?

Yes or No

Ex 2. Does $z(t) = x(t+5)$ have memory?

Yes or No

Ex 3. Does $y(t) = (t+5)x(t)$ have memory?

Yes or No

Ex 4. Does $z(t) = [x(t+5)]^2$ have memory?

Yes or No

Ex 5. Does $a(t) = x(5)$ have memory?

Yes or No

Ex 6. Does $v(t) = x(2t)$ have memory?

Yes or No

Ex 1. Does $y(t) = x(t) + 5$ have memory?

Solution: $y(t) = x(t) + 5 \Rightarrow$ memoryless.

Ex 2. Does $z(t) = x(t+5)$ have memory?

Solution: $z(t) = x(t+5) \Rightarrow$ memory.

Ex 3. Does $y(t) = (t+5)x(t)$ have memory?

Solution: $y(t) = (t+5)\,x(t) \Rightarrow$ memoryless.

Ex 4. Does $z(t) = [x(t+5)]^2$ have memory?

Solution: $z(t) = [x(t+5)]^2 \Rightarrow$ memory.

Ex 5. Does $a(t) = x(5)$ have memory?

Solution: $a(t) = x(5) \Rightarrow$ memory.

Ex 6. Does $v(t) = x(2t)$ have memory?

Solution: $v(t) = x(2t) \Rightarrow$ memory.

Causal and Noncausal Systems

A causal system is one whose response does not begin before the input signal is applied. If the output of the system *y(t)* at any time depends only on the input at present and/or previous times, we say that the system is causal. On other words, **A system is said to be causal system if its output depends on present and past inputs only and not on future inputs.** A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n\ \ 1)$, $x(n\ \ 2)$,.. Otherwise, if a system output depends on future input values such as $x(n + 1)$, $x(n+2)$,., the system is noncausal. The noncausal system cannot be realized in real time.

CT :	$y(t_1) = f(x(t_0))$	$t_0 \leq t_1$

DT:	$y[n_1]= f(x[n_0]$	$n_0 \leq n_1$

A non-causal (or Anti causal) system response will begin before the input signal is applied. It can be made realizable by introducing a positive time delay into the system.

Since causal system does not include future inputs; such system is practically realizable. That mean such system can be implemented practically. Generally all real time systems are causal systems; because in real time applications only present and past samples are present.

Example : is ***y(t) =x(t) - x(t-2) causal or not?***

Solution: Since the output does not depend on the future input values, the system is causal.

Example: Determine the Causality of the following systems:

$$y[n] = x[-n]$$

Solution: System is not causal, since when n < 0 , e.g. n = -4 , we see that y[-4] = x[4], so that the output at this time depends on a future value of input.

Example: Are the systems below causal or noncausal?

$$y[n] = x[n]- x[n+1] \quad \text{and} \quad y(t) = x(t+1)$$

Solution/ both systems are not causal systems because their outputs at this time depends on a future value of inputs.

Example: The accumulator system $y(n) = \sum_{k=-\infty}^{n} x(k)$ is causal because the value of **y[.]** at any instant ***n*** depends only on the previous (past) values of ***x[.].***

Example: The system y(t)= [cos (3t)] x(t) is

a) Causal and has memory; **b)** Causal and memoryless; **c)** Noncausal and has memory ; **d)** Noncausal and memoryless

----

Check yourself for $y(t) = x(\frac{t}{3})$ and $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

Note that *all memoryless systems are causal*, since the output responds only to the current value of input.

A system is defined as stable if for every absolutely bounded input $|x| \leq k_1$, the output is also absolutely bounded $|y| \leq k_2$ **(BIBO)** where k_1 , and k_2, are finite real constants. To show the system is not stable, we need to find a single bounded input leads to an unbounded output. Finite values such as *DC values*, *sin* t, *cost t*, *u(t)* ...

If B_x = Max $(\,|x|\,) < \infty$ and B_y = Max $(|y|) < \infty$, then the system is stable.

Otherwise system is not stable.

Example / Check the stability of the below systems?

$$y(t) = e^{x(t)}$$

Assume the input is bounded $x(t) < B$, or $-B < x(t) < B$ for all t. We then see that $y(t)$ is bounded $e^{-B} < y(t) < e^{B}$. **System is stable**.

Example/ Check the stability of the below systems?

$$y(t) = t\,x(t)\ ;$$

Here, for a finite input, we cannot expect a finite output. For example, if we will put ***x(t)=2*** $\Rightarrow$ ***y(t)=2t.*** This is not a finite value because we do not know the value of ***t***. So, it can be ranged from anywhere. Therefore, this system is not stable. It is an **unstable system**.

Stable Systems

Example/ Determine whether the below system is stable or not ?

The accumulator

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Sol: ***System is not stable***, since the sum grows continuously even if $x[n]$ is bounded as
Bounded input $B_x = 1$ produces unbounded output $B_y = \sum_{-\infty}^{n} 1 = \infty$

Example/ Check the stability of the below systems?

$$\boldsymbol{f(x([n]) = 5\ x([n-10]}$$

Sol: *Input* $|x| \le k_1 \le \infty$ is bounded for all values of *n*
Also output $|y| \le k_2 = 5\,k_1 \le \infty$ is also bounded for all values of *n*.
It is stable system.

Example / Check if the DT system is stable or not **?**

$$\boldsymbol{y[n]=n\,x[n].}$$

Sol: If the input is 1 or unit step then the bounded input will produce unbounded output
y(n)= n .1 ... The system is **not stable.**

Check yourself for $y(t) = x(\frac{t}{3})$ $\quad y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$

Linear and nonlinear system

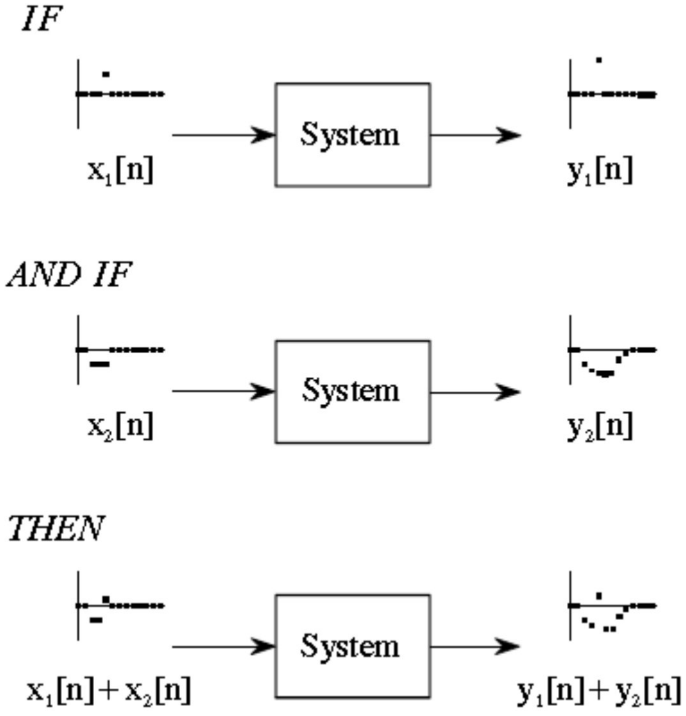
A system is linear if and only if it satisfy the superposition principle (**Homogeneity and Additivity**). A system in which superposition property does not apply is called nonlinear system.

Homogeneity: (Deals with amplitude)

If x[n] → y[n], **then** *k*x[n] → *k*y[n]

K is a constant

Additively or scaling



Homogeneity & Additivity (superposition principle)