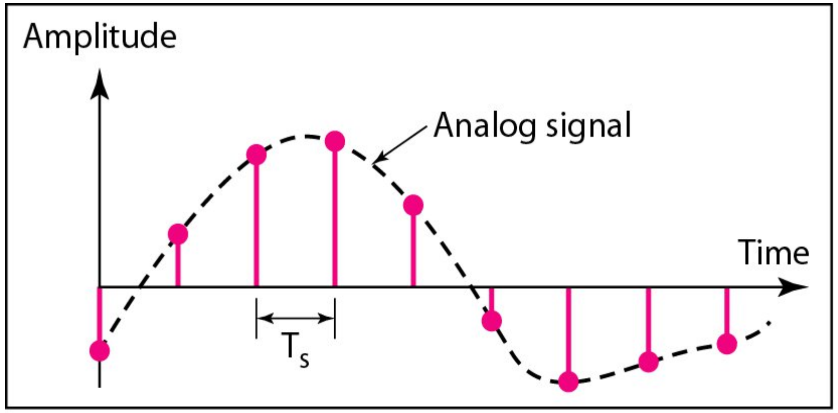
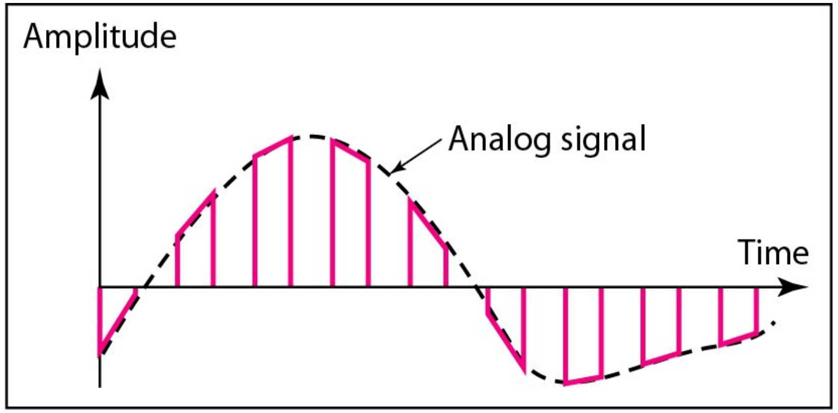
There are three sampling methods:

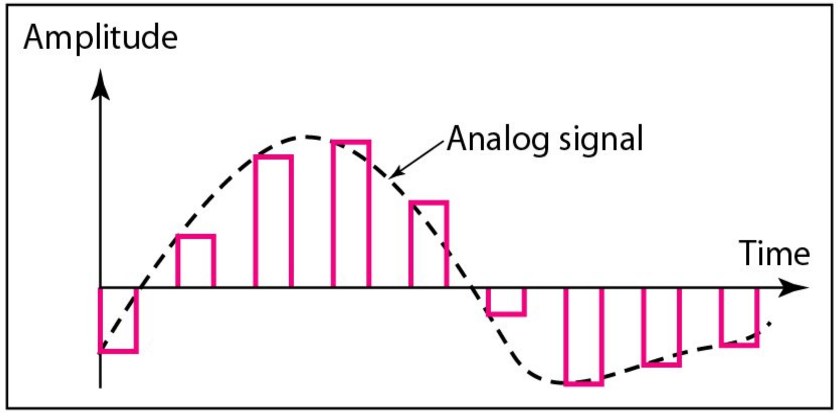
- **Ideal** - an impulse at each sampling instant
- **Natural** - a pulse of short width with varying amplitude
- **Flat top** - sample and hold, like natural but with single amplitude value



a. Ideal sampling



b. Natural sampling



c. Flat-top sampling

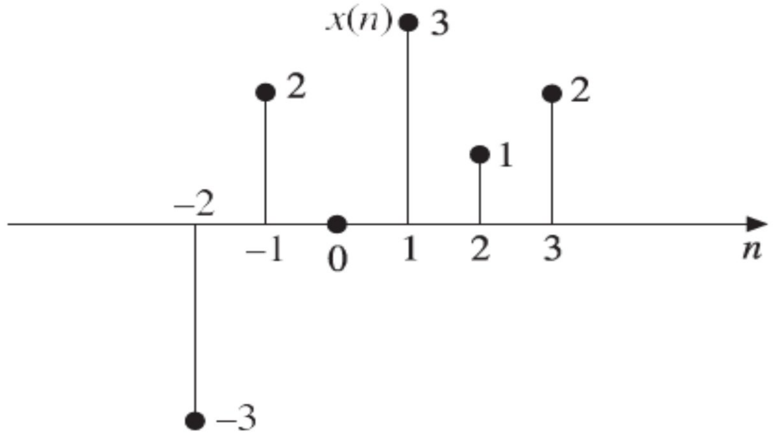
REPRESENTATION OF DISCRETE-TIME SIGNALS

Discrete-time signals are signals which are defined only at discrete instants of time. For those signals, the amplitude between the two time instants is just not defined. For discrete- time signal the independent variable is time ***n***, and it is represented by **x(n).**
There are following four ways of representing discrete-time signals:

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

Graphical Representation

Consider a single **x (n)** with values X (-2) = -3, x(-1) = 2, x(0) = 0, x(1) = 3, x(2) = 1 and x(3) = 2
This discrete-time single can be represented graphically as shown in Figure below



Graphical representation of discrete-time signal

Functional Representation

In this, the amplitude of the signal is written against the values of *n*. The signal given in previous section can be represented using the functional representation as follows:

$$x(n) = \begin{cases} -3 & \text{for } n = -2 \\ 2 & \text{for } n = -1 \\ 0 & \text{for } n = 0 \\ 3 & \text{for } n = 1 \\ 1 & \text{for } n = 2 \\ 2 & \text{for } n = 3 \end{cases}$$

Another example is: $X(n) = 2^n u(n)$

Or $x(n) = 2^n \; for\; n \geq 0$

$\quad 0 \; for\; n < 0$

Tabular Representation

In this, the sampling instant *n* and the magnitude of the signal at the sampling instant are represented in the tabular form. The same signal given above can be represented in tabular form as follows:

n	−2	−1	0	1	2	3
$x(n)$	−3	2	0	3	1	2

Sequence Representation

A finite duration sequence given before can be represented as follows:

$$x(n) = \{-3, 2, \underset{\uparrow}{0}, 3, 1, 2\}$$

The arrow mark ↑ denotes the $n = 0$ term. When no arrow is indicated, the first term corresponds to $n = 0$.

So a finite duration sequence, that satisfies the condition $x(n) = 0$ for $n < 0$ can be represented as:

$x(n) = \{3, 5, 2, 1, 4, 7\}$

The unit step sequence ***u[n]*** is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

which is shown in Fig. Note that the value of ***u[n]*** at *n* = 0 is defined [unlike the continuous-time step function ***u(t)*** at *t* = 0] and equals unity. Similarly, the shifted unit step sequence ***u[n - k]*** is defined as (see Fig. 1-7(b)).

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases}$$

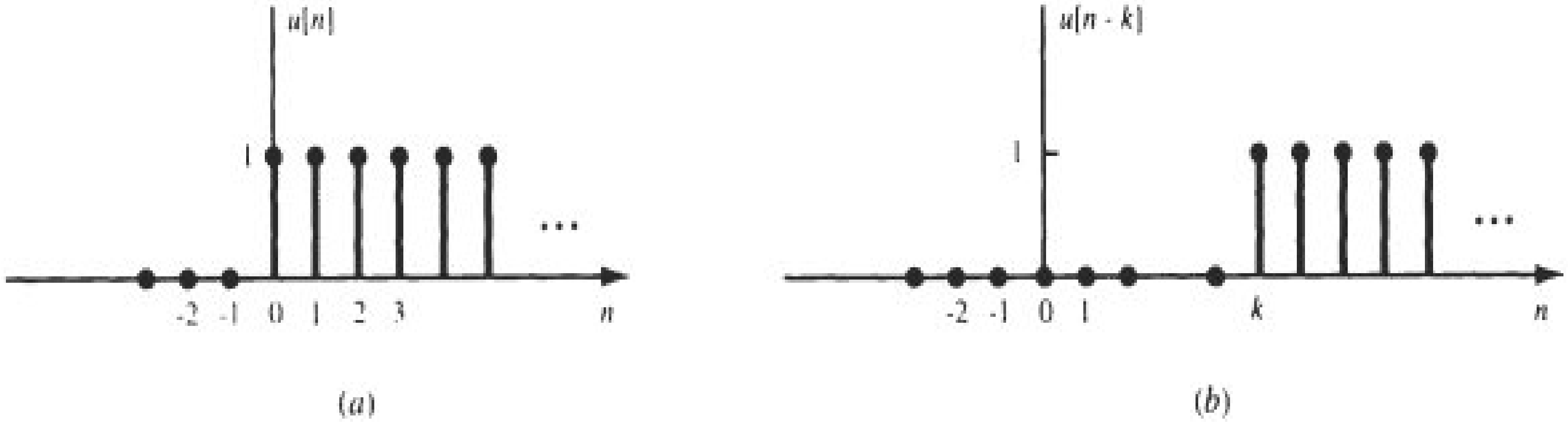


Fig. Unit step sequence; (b) shifted unit step sequence.

Common Digital Sequences- The Unit Impulse Function:

The unit impulse (or unit sample) sequence ***δ[n]*** is defined as

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

which is shown in Fig. 1-9(a). Similarly, the shifted unit impulse (or sample) sequence ***δ [n - k]*** is defined as (see Fig b.)

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

Fig. (a)Unit impulse (sample) sequence; (b) shifted unit impulse sequence.

Unlike the continuous-time unit impulse function ***δ(t), δ[n]*** is defined without mathematical complication or difficulty. ***δ(n), u[n]]***are related by

$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Common Digital Sequences- The Exponential Signals:

The complex exponential sequence (discrete) is of the form:

$$x[n] = e^{j\Omega_0 n} = \cos \Omega_0 n + j \sin \Omega_0 n$$

Thus, ***x[n]*** is a complex sequence whose real part is ***cos*** $\boldsymbol{\Omega_0 n}$ and imaginary part is ***sin*** $\boldsymbol{\Omega_0 n}$.
In order for $e^{j\Omega on}$ to be periodic with period (N > 0), Ω_0 must satisfy the following condition

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} \qquad m = \text{positive integer}$$

Thus, the sequence $e^{j\Omega on}$ is not periodic for any value of Ω_0. It is periodic only if $\Omega_0/2\pi$ is a ***rational number***. Note that this property is quite different from the property that the continuous-time signal $e^{j\omega ot}$ is periodic for any value of ω_o. Thus, if Ω_0 satisfies the periodicity condition, $\Omega_0 \neq 0$ and ***N*** and ***m*** have no factors in common, then the fundamental period of the sequence ***x[n]*** is N_o given by

$$N_0 = m\left(\frac{2\pi}{\Omega_0}\right)$$

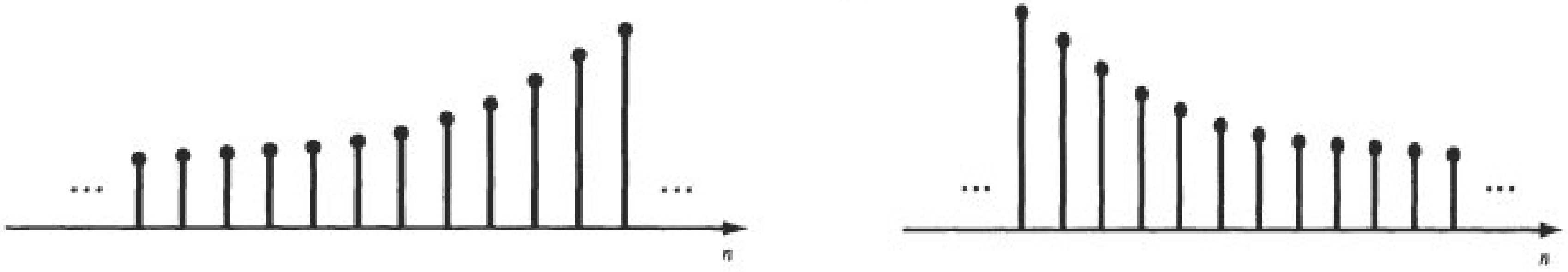


Fig. Exponential sequences

Examples:

Is $x[n] = e^{jn2\pi/3}$ periodic? If it is periodic, what's its fundamental period?

$\Omega_0 = 2\pi/3$

check if $\Omega_0/2\pi$ is a rational number ?

$\Omega_0/2\pi = \dfrac{2\pi/3}{2\pi} = 1/3,$ is a rational number so $x[n]$ is periodic

$$N_0 = m\left(\frac{2\pi}{\Omega_0}\right) = \frac{2\pi}{2\pi/3} = 3 \qquad \text{where } m = 1, 2, 3, \ldots$$

Is $x[n] = e^{jn3\pi/4}$, periodic? If it is periodic, what's its fundamental period?

For $e^{jn3\pi/4}$, $\Omega_0/(2\pi) = 3/8$, so $e^{jn3\pi/4}$ is periodic with fundamental period 8.

Note if $x[n]$ is the sum of above two signals,

$x[n]$ is periodic with fundamental period $24 = lcm(3, 8)$.

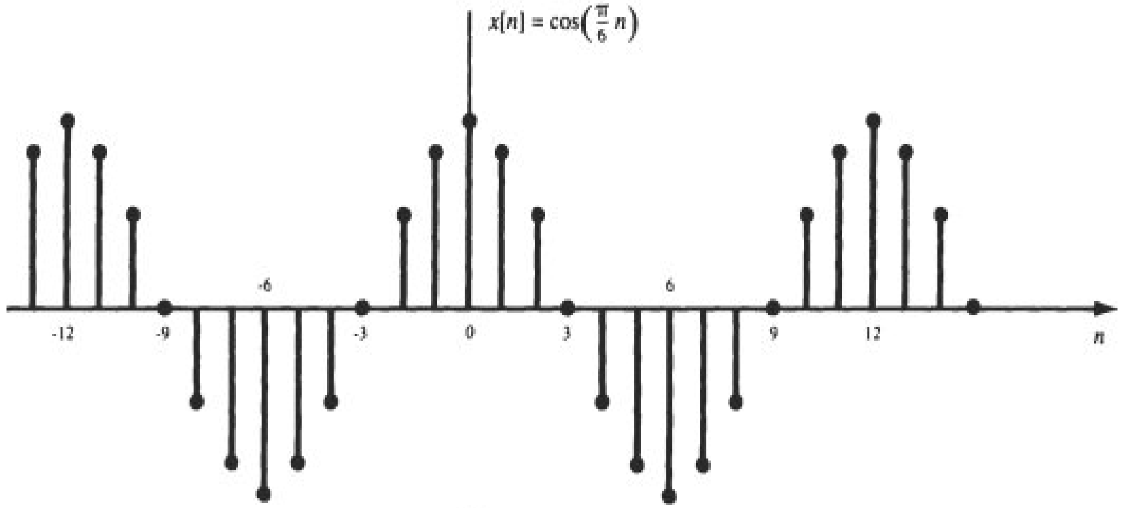
Lcm=Least common multiple

Common Digital Sequences- Sinusoidal Signals:

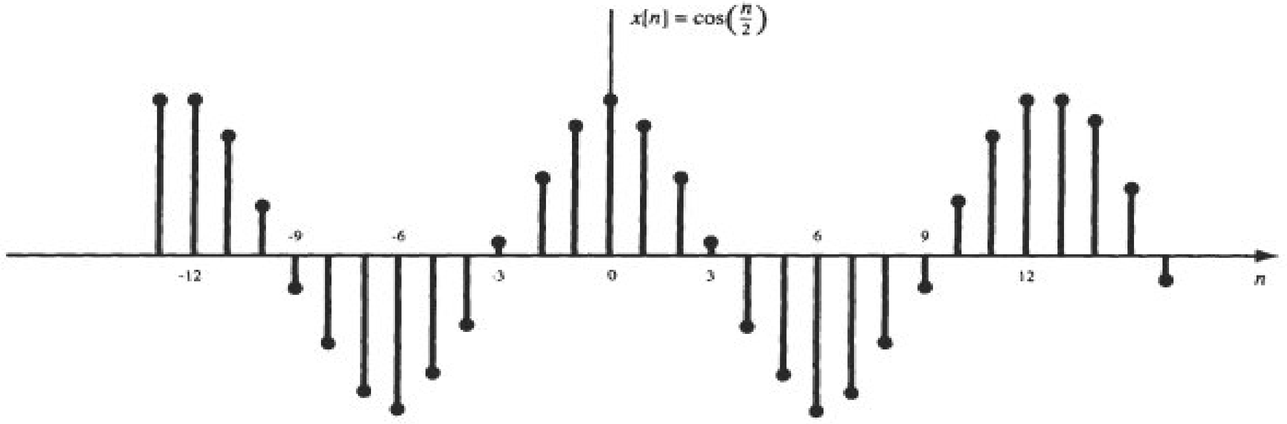
A sinusoidal sequence can be expressed as $x[n] = A\cos(\Omega_0 n + \theta)$

If ***n*** is dimensionless, then both $\boldsymbol{\Omega_0}$, and **θ** have units of radians. Two examples of sinusoidal sequences are shown in Fig. As before, the sinusoidal sequence can be expressed as $A\cos(\Omega_0 n + \theta) = A\,\mathrm{Re}\{e^{j(\Omega_0 n + \theta)}\}$

As we observed in the case of the complex exponential sequence, the same observations also hold for sinusoidal sequences for the periods. The sequence in Fig. (a) is periodic with fundamental period 12, but the sequence in Fig. (b) is not periodic.



(a)



(b)

Examples : Is $x[n] = \sin(8\pi n/31)$ periodic? If it is periodic, what's its fundamental period?

Since $\Omega_0/(2\pi) = 4/31$, $x[n]$ is periodic with fundamental period ;

$$N_0 = m\left(\frac{2\pi}{\Omega_0}\right) = \frac{2\pi}{8\pi/31} = 31 \qquad m=1,2 \ldots.$$

Is $x[n] = \sin(3n/4)$ periodic? If it is periodic, what's its fundamental period?

Since $\frac{\Omega_0}{2\pi} = \frac{3}{8\pi}$ is irrational, $x[n]$ is not periodic;

Example : Find the period of the signal $x[n] = \cos[\pi n/12] + \sin[\pi n/18]$

$$x[n] = \underbrace{\cos[\pi n/12]}_{x_1} + \underbrace{\sin[\pi n/18]}_{x_2}$$

Solution :

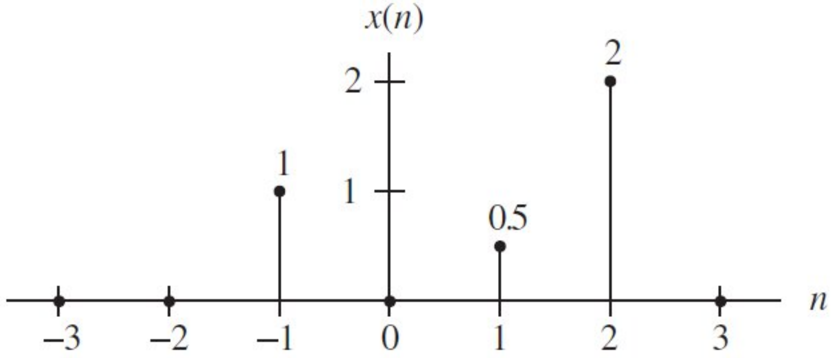
First we have to check if these two signals are periodic or not. If one of them is not periodic then the sum of them also not periodic. If both of them are periodic with N1 and N2 then the period of their sum is given by

N=*lcm*(N1,N2)

Since Ω1 /2π= π/12/2 π=1/24 and Ω2/2π= π/18/2 π=1/36 are rational number then both x_1 and x_2 are periodic with N1=24 and N2=36 then N=*LCM*(24,36)=72

Example: Sketch the following sequence ***x(n)=δ(n+1)+0.5 δ(n-1)+ 2 δ(n-2)***

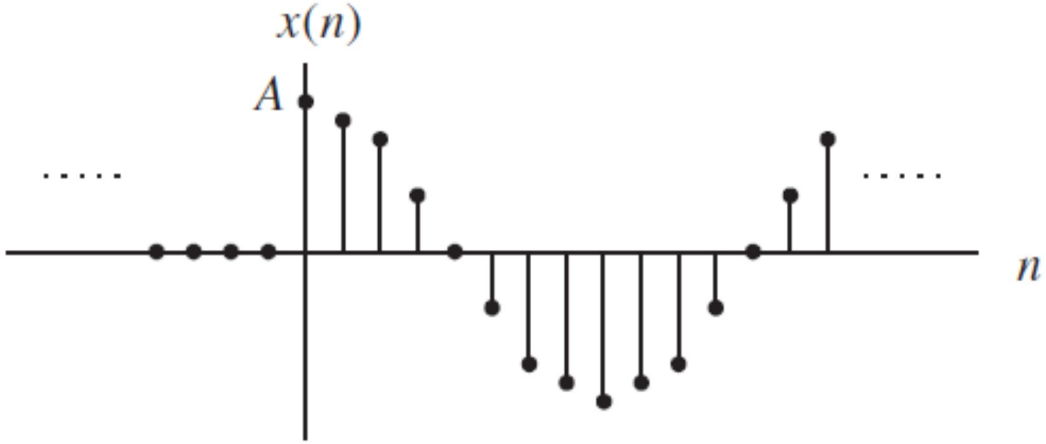
Solution: According to the shift operation, δ(n+1)is obtained by shifting δ(n) to the left by one sample, δ(n-1) and δ(n-2) are yielded by shifting δ(n) to right by one sample and two samples, respectively. Using the amplitude of each impulse function, we obtain the sketch in



Example: Sketch the following sequence ***x(n)= A cos 0.125πn)u(n) with A=10.***

Solution: we can calculate the digital values for the first eight samples (for example) and list their values in Table

n	$x(n) = 10\cos(0.125\pi n)u(n)$
0	10.0000
1	9.2388
2	7.0711
3	3.8628
4	0.0000
5	–3.8628
6	–7.0711
7	–9.2388



Repeat above for x(n)=A (0.75)^{n} u(n).

Example: Assume we have a DSP system with a sampling time interval of 125 microseconds.

a. Convert each of following analog signals *x*(*t*) to a digital signal *x*(*n*) :

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)\,u(t)$

b. Determine and plot the sample values from each obtained digital function.

Solution: $x(n) = x(t)|_{t=nT} = x(nT)$

Since *T* =0.000125 seconds in above equation, substituting *t*= *nT* = *n* × 0.000125 = 0.000125 *n* into the analog signal *x(t)* expressed in (1) leads to the digital sequence

1. $x(n) = x(nT) = 10e^{-5000 \times 0.000125n}u(nT) = 10e^{-0.625n}\,u(n)$

Similarly, the digital sequence for (2) is achieved as follows:

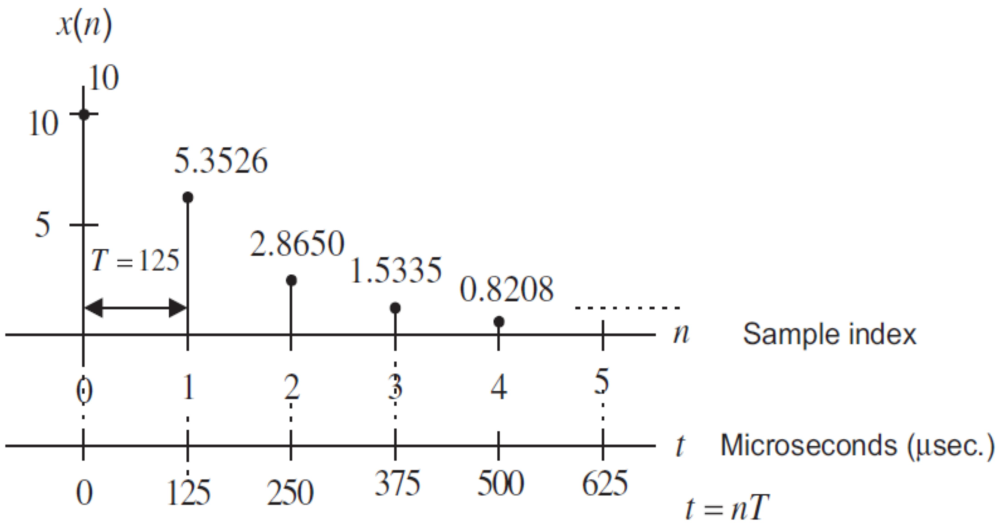
2. $x(n) = x(nT) = 10\sin(2000\pi \times 0.000125n\, u(nT) = 10\,sin(0.25\pi n)\,u(n)$

b. The first five sample values for (1) are calculated and plotted in Fig

$x(0) = 10e^{-0.625\,(0)}\,u(0) = 10$

$x(1) = 10e^{-0.625\,(1)}\,u(1) = 5.3526$

And so on.... Complete the other values and for part 2.



Manipulation of discrete time signals:

- ✓ **Manipulations involving the independent variable : n**
  - **Shifted in time Operations.**
  - **Folding, reflection or time reversal.**
  - **Time Scaling.**
- ✓ **Manipulations involving the signal amplitude (dependent variable).**

➢ Manipulations involving the independent variable n

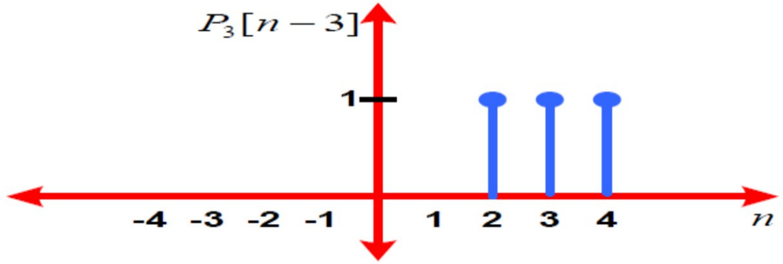
1. Shifted in time Operations:

Given a D-T signal $x[n]$ and a positive integer p , then

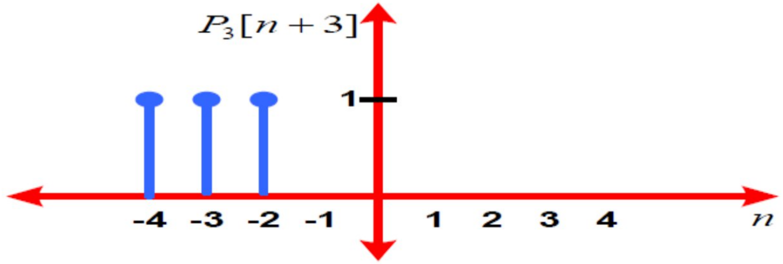
- $y[n] = x[n - p]$ is the p -step right shift of $x[n]$ that results in a **delay** of the signal by p units of time (replacing n by $n - p$).
- $y[n] = x[n + p]$ is the p -step left shift of $x[n]$ that results in an **advance** of the signal by p units of time (replacing n by $n + p$).

Examples:

a) $P_3[n-3]$: Three-step right shift of DT rectangular pulse $P_3[n]$ (see figure).



b) $P_3[n+3]$: Three-step left shift of DT rectangular pulse $P_3[n]$ (see figure)



Example: The DT signal

$$x[n]=\begin{cases}3 & n=1,2\\ -2 & n=-1,-2\\ 0 & n=0\ and\ |n|>2\end{cases}$$

Find the time-shifted signal] y[n]=x[n+4]

Solution

$$x[n]=\begin{cases}3 & n=-2,-3\\ -2 & n=-5,-6\\ 0 & n=-4\ ,n<-6\quad and\ n>-2\end{cases}$$

2. Folding, reflection or time reversal:

Let x[n] be the original sequence, and y[n] be reflected sequence, then y[n] is defined by y[n]=x[-n] , this means that we replace the independent variable n by –n ; the result of this operation is a **folding or reflection** of the signal about the time origin n=0. It is important to note that the operation of folding and time delaying (or advancing) a signal are **not commutative**

if SO (shifted operation, for example Time-Delay) and FO (folding operation), we can write:

$SO_k\{x[n]\} = x[n-k], k>0.$

$FO\{x[n]\}=x[-n].$

Now:

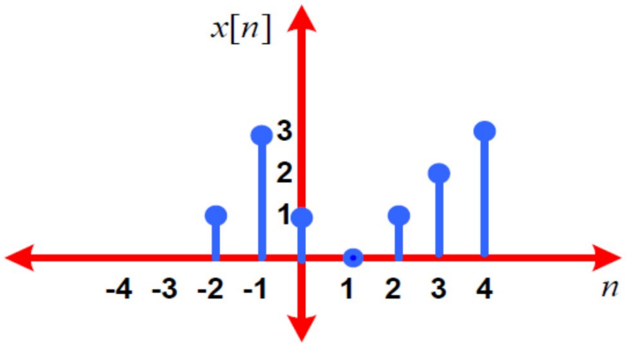
$SO_k\{FO\{x[n]\}\}= SO_k\{ x[-n]\}= x[-n+k]$ where as

$FO\{SO_k\{x[n]\}\}=FO\{x[n-k\}= x[-n-k]$

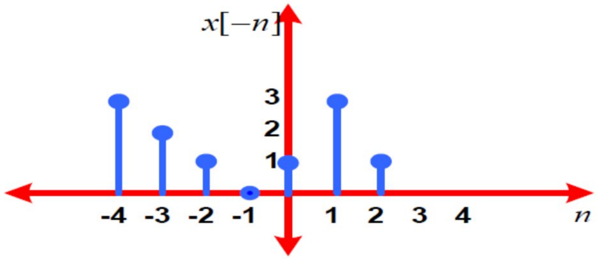
so :

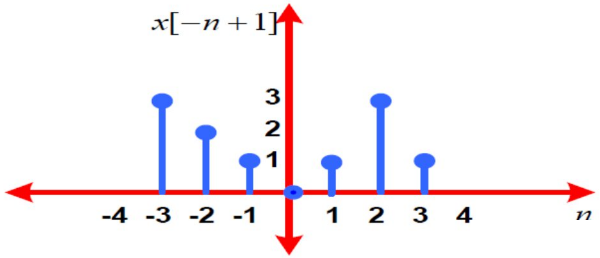
$$SO_k\{FO\{x[n]\}\} \neq FO\{SO_k\{x[n]\}\}$$

Example: Show the graphical representation of the signals x[-n] and x[-n+1], where is the signal x[n] illustrated in figure below



Solution: A simple way to verify that the result is correct is to compute samples,





$$\begin{aligned}
y[-3] &= x[4] = 3,\\
y[-2] &= x[3] = 2,\\
y[-1] &= x[2] = 1,\\
y[0] &= x[1] = 0,\\
y[1] &= x[0] = 1,\\
y[2] &= x[-1] = 3,\\
y[3] &= x[-2] = 1
\end{aligned}$$

Example: The DT signal

$$x[n] = \begin{cases} +1 & n = 1 \\ -1 & n = -1 \\ 0 & otherwise \end{cases}$$

Find the composite signal y[n]=x[n]+x[-n]

Solution:

y[n]=0 for all integer values of n

Precedence rules for time shifting and time folding	
Order of shifting and folding operations	**Output signal**
1. Folding →Shift to the right	$x[n] \xrightarrow{FO} x[-n] \xrightarrow{SO_R} x[-(n-p)] = x[-n+p]$
2. Shift to the left →Folding	$x[n] \xrightarrow{SO_L} x[n+p] \xrightarrow{FO} x[-n+p]$
3. Folding →Shift to the left	$x[n] \xrightarrow{FO} x[-n] \xrightarrow{SO_L} x[-(n+p)] = x[-n-p]$
4. Shift to the right→Folding	$x[n] \xrightarrow{SO_R} x[n-p] \xrightarrow{FO} x[-n-p]$