There are two types of FFT algorithms. They are Decimation in time (DIT) and Decimation in Frequency (DIF). In DIT algorithm, we divide $x(n)$ into 2 sub-sequences each of length N/2 by grouping the **even**-indexed samples and **odd** indexed samples together, where

$$X(k) = \sum_{Even} x[n] \quad W_N^{kn} + \sum_{Odd} x[n] \quad W_N^{kn}$$

Let $n=2r$ for the even sum and $k=2r+1$ for the odd sum for $r=0,1,2,\ldots,N2/-1$, we get:

$$X(k) = \sum_{r=0}^{N/2-1} x[2r] \quad W_N^{2kr} + \sum_{r=0}^{N/2-1} x[2r+1] \quad W_N^{k(2r+1)} = \sum_{r=0}^{N/2-1} g[r] \quad W_N^{2kr} + W_N^{k} \sum_{r=0}^{N/2-1} h[r] \; W_N^{2kr}$$

since $W_N^2 = W_{\frac{N}{2}}$ then

$$X(k) = \sum_{r=0}^{N/2-1} g[r] \quad W_{N/2}^{rk} + W_N^{k} \sum_{r=0}^{N/2-1} h[r] \; W_{N/2}^{rk}$$

Thus $X(k) = G(k) + W_N^k H(k) \qquad k = 0, 1, 2, 3\ldots\ldots N/2\text{-}1$

Where $G(k)$ and $H(k)$ denote the N/2 point DFTs of the sequences $g(r)$ and $h(r)$, respectively. Since $G(k)$ and $H(k)$ are periodic with period N/2, we can write last equation as:

$$X(k) = G(k) + W_N^k H(k)$$

$$X\left(k + \frac{N}{2}\right) = G(k) + W_N^{k+\frac{N}{2}} H(k) = G(k) - W_N^k H(k)$$

Since $W_N^{k+\frac{N}{2}} = -W_N^k$ *summerty property*

Comparison of DFT and FFFT with Reduced Number of Multiplication and Addition Operations can be given by

Number of points N	Direct computation		Radix – 2 FFT	
	Complex additions $N(N-1)$	Complex Multiplication N^2	Complex additions $N\log_2 N$	Complex Multiplication $(N/2)\log_2 N$
4	12	16	8	4
8	56	64	24	12
16	240	256	64	32
32	992	1,024	160	80
64	4032	4,096	384	192
128	16,256	16,384	896	448

Example: Find the speed improvement factor between DFT and FFFT with Number of Multiplication Operations for N=64?

Solution: The complex multiplications required by DFT is $N^2=64^2= 4096$.

The complex multiplications required by FFT $N/2 \log_2 N=64/2 \log_2 64== 192$.

The speed improvement factor = 4094/192= 21.33

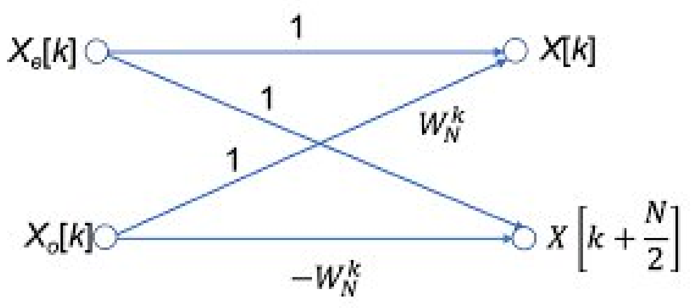
Example : Let us illustrate the ideas behind FFT algorithm by using a simple example having $N = 4$.

Solution:

$$G(k) = \sum_{r=0}^{N/2-1} g[r] \; W_{N/2}^{rk} = \sum_{r=0}^{1} x[2r] \; W_{N/2}^{rk} = x(0) \; W_2^0 + x(2) \; W_2^k$$

$$G(0) = x(0) \; W_2^0 + x(1) \; W_2^0 = x(0) + x(2)$$

$$G(1) = x(0) \; W_2^0 + x(1) \; W_2^1 = x(0) - x(2)$$



Butterfly Diagram

$$H(k) = \sum_{r=0}^{N/2-1} h[r] \; W_{N/2}^{rk} = \sum_{r=0}^{1} x[2r+1] \; W_{N/2}^{rk} = x(1) \; W_2^0 + x(3) \; W_2^k$$

$$H(0) = x(1) \; W_2^0 + x(3) \; W_2^0 = x(1) + x(3)$$

$$H(1) = x(1) \; W_2^0 + x(3) \; W_2^1 = x(1) - x(3)$$

$$X(k) = G(k) + \; W_N^k \; H(k)$$

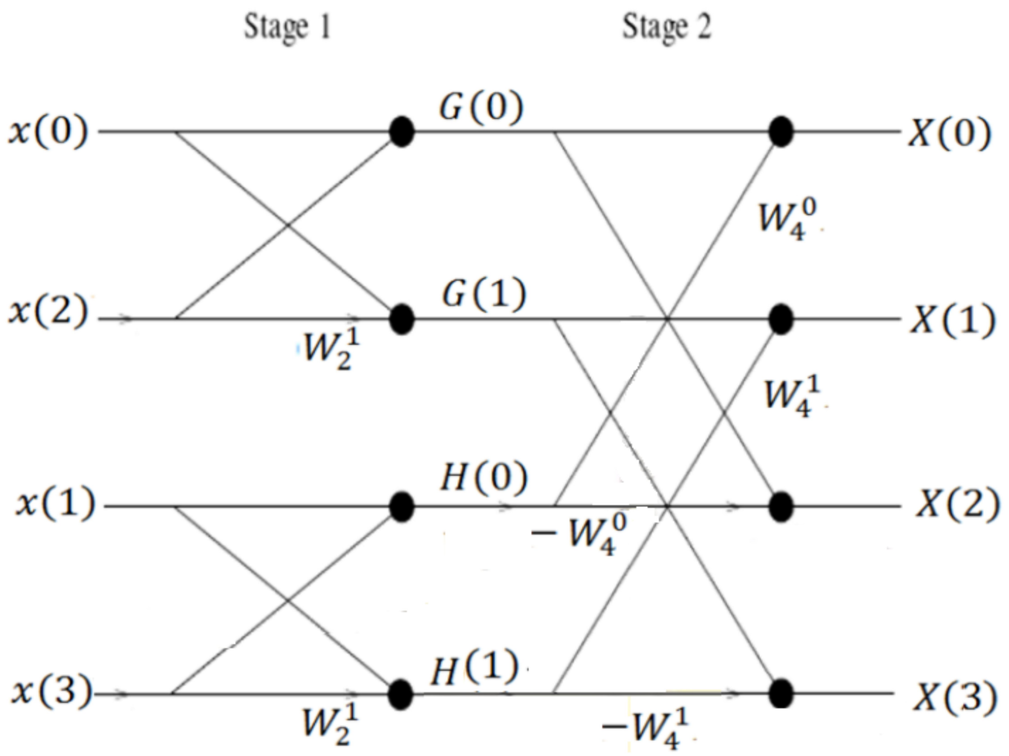
$$X(0) = G(0) + \; W_4^0 \; H(0)$$

$$X(1) = G(1) + \; W_4^1 \; H(1)$$

$$X\left(k + \frac{N}{2}\right) = G(k) - \; W_N^k \; H(k)$$

$$X(2) = G(0) - \; W_4^0 \; H(0)$$

$$X(3) = G(1) - \; W_4^1 \; H(1)$$



Signal flow graph for 4-point DIT FFT algorithm or **FFT butterfly signal flow diagram**

Note that the inputs have been shuffled so that the outputs are produced in the correct order. This can be represented as a bit-reversing process. For previous example we have 00 01 10 11 and its bit reversal will be 00 10 01 11 or 0 2 1 3. This is used each time a signal is separated into its even and odd decomposition as shown for the 8-point samples:

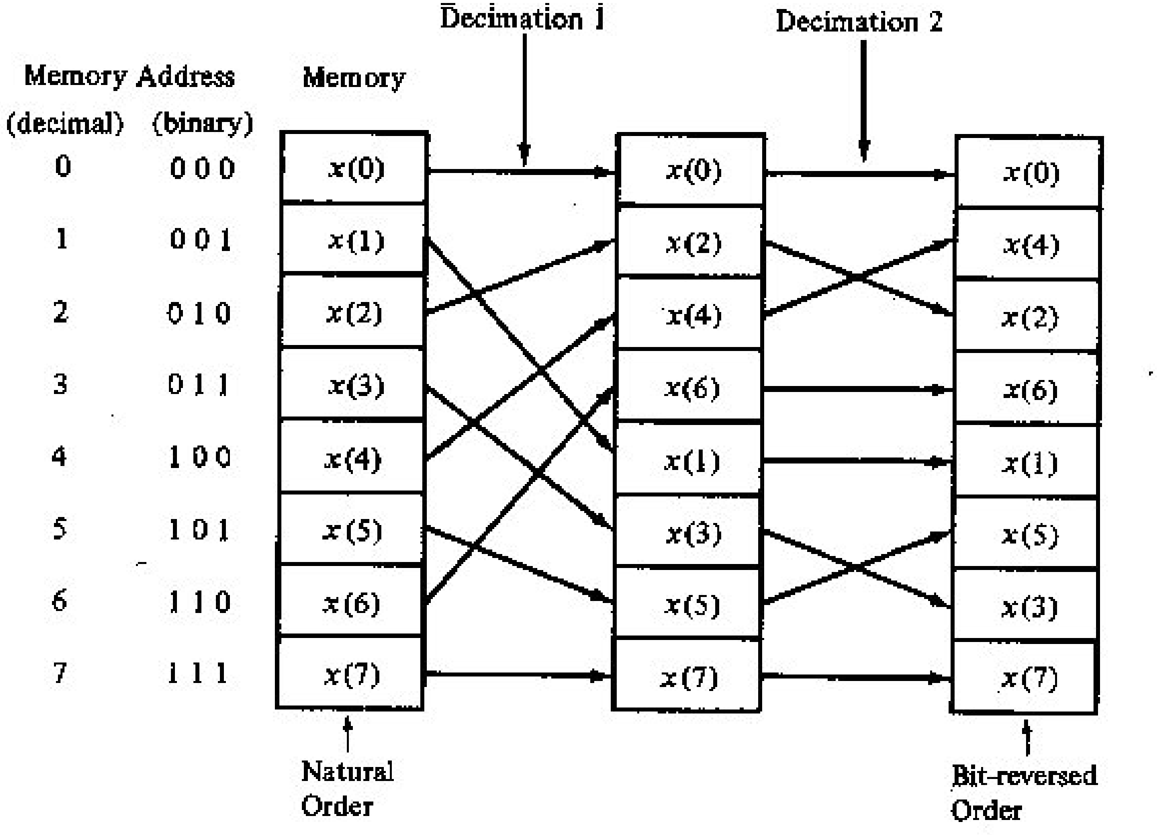
0, 1, 2, 3, 4, 5, 6, 7

0, 2, 4, 6, 1, 3, 5, 7

0, 4, 2, 6, 1, 5, 3, 7

This decomposition is carried out by the bit reversed s

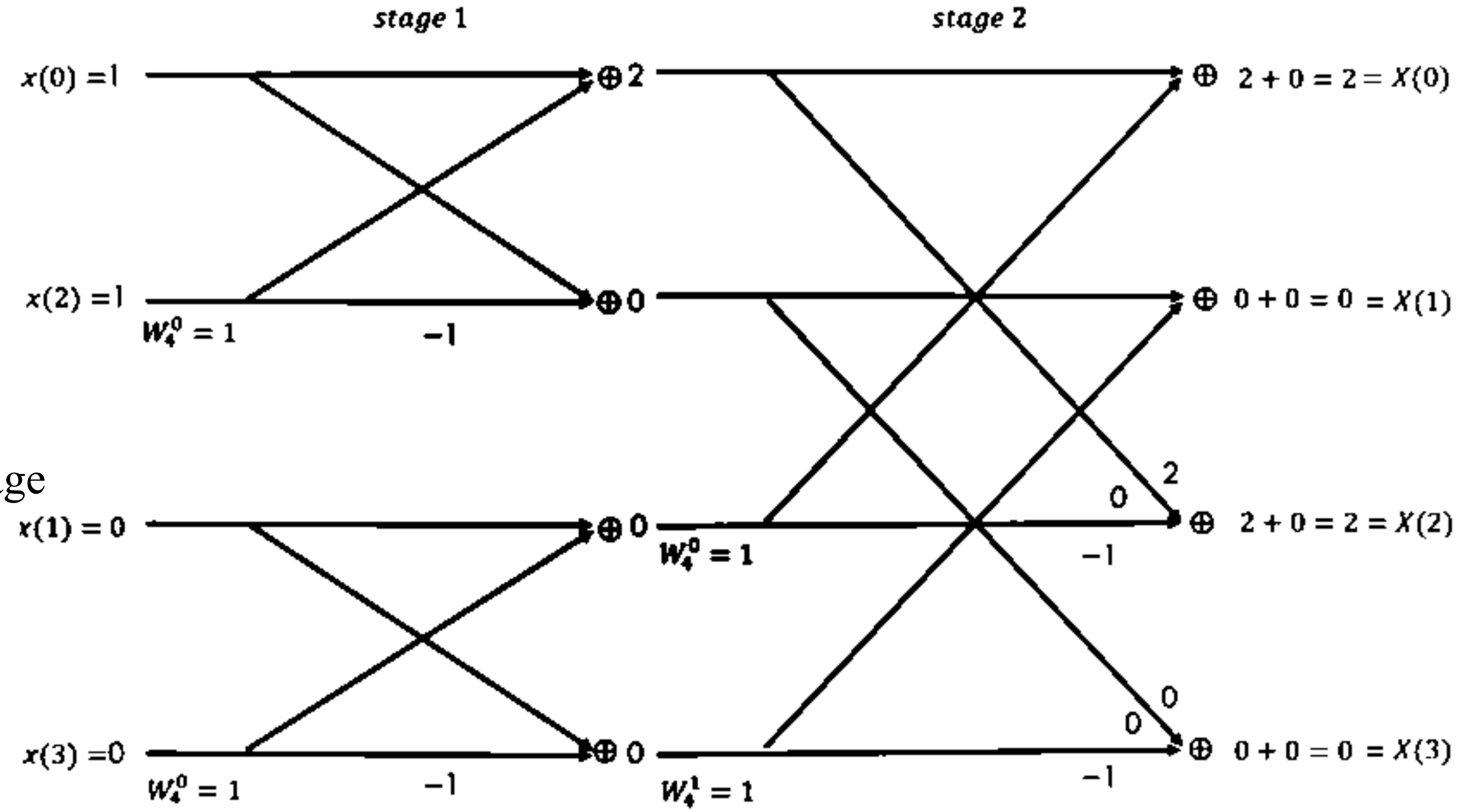
Time Point (n)	Binary Word	Reversed-Bit Word	Order
0	000	000	x[0]
1	001	100	x[4]
2	010	010	x[2]
3	011	110	x[6]
4	100	001	x[1]
5	101	101	x[5]
6	110	011	x[3]
7	111	111	x[7]



Example: Compute the DFT for sequence x[n]={1, 0, 1, 0}? Use 4-point DIT FFT algorithm ?

1. Determine the length of FFT: N=4
2. Calculate the Twiddle Factors. Two Twiddle factors needed for 4-point FFT $\boldsymbol{W_4^0 = 1}$, and $\boldsymbol{W_4^1 = -j}$
3. Decimate the input sequences (bit reversal process)

Input bit Index	Binary Word	Reverse Bit Word	Bit Reversed Index
0	00	00	0
1	01	10	2
2	10	01	1
3	11	11	3



4. Plotting the Signal Flow Graph for first stage
5. Calculate the Output of the first stage
6. Plotting the Signal Flow Graph for Second stage
7. Calculate the Output of the Second stage
8. Summarizing all FTTs

X[k]={2, 0, 2, 0}

Note that the number of stages = $\log_2 N$

Decimation-in-Frequency (DIF) Algorithm:

The DIF FFT algorithm is obtained by dividing the output sequence $X(k)$ rather than the input sequence $x(n)$ into several sub-sequences. Separate the first N/2 points and the last N/2 points of the sequence $x(n)$ together, we get

$$X(k) = \sum_{n=0}^{N/2-1} x[n] \; W_N^{kn} + \sum_{n=0}^{N/2-1} x\left[n + \frac{N}{2}\right] \; W_N^{k(n+\frac{N}{2})}$$

$$X(k) = \sum_{n=0}^{N/2-1} x[n] \; W_N^{kn} + W_N^{\frac{N}{2}k} \sum_{n=0}^{N/2-1} x\left[n + \frac{N}{2}\right] \; W_N^{kn}$$

Since $W_N^{\frac{N}{2}k} = \begin{cases} -1, & k{:}odd \\ 1, & k{:}even \end{cases} = (-1)^k$ then

$$X(k) = \sum_{n=0}^{N/2-1} x[n] \; W_N^{kn} + (-1)^k \sum_{n=0}^{N/2-1} x\left[n + \frac{N}{2}\right] \; W_N^{kn}$$

For even values of k ($k = 2r$) and by exploiting recursion property: $W_N^{2nr} = W_{N/2}^{nr}$, $r = 0,1, \ldots N/2-1$

$$X(2r) = \sum_{n=0}^{N/2-1} \left[x[n] + x\left[n + \frac{N}{2}\right]\right] W_{\frac{N}{2}}^{nr} = \sum_{n=0}^{N/2-1} g[n] \; W_{\frac{N}{2}}^{nr} = G(k)$$

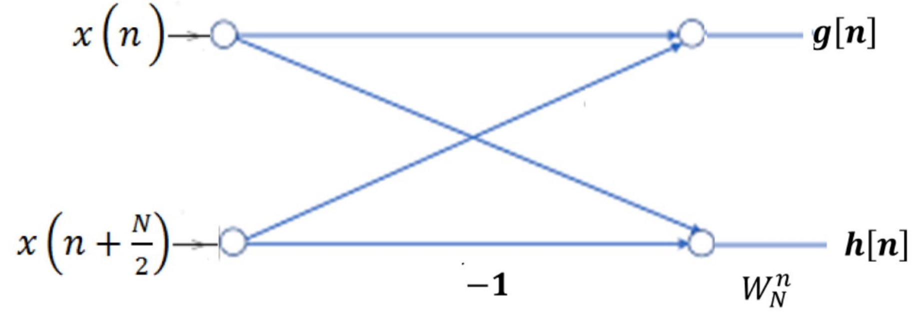
$$where\ g(n) = \left[x[n] + x\left[n + \frac{N}{2}\right]\right]$$

For odd values of k ($k = 2r + 1$) and by exploiting recursion property: $W_N^{(2r+1)n} = W_{N/2}^{nr} W_N^{n}$, $r = 0,1, \ldots N/2-1$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x[n] - x\left[n + \frac{N}{2}\right]\right] W_{N/2}^{nr} W_N^{n} = \sum_{n=0}^{N/2-1} h[n] \; W_{\frac{N}{2}}^{nr} = H(k)$$

$$where\ h(n) = \left[x[n] - x\left[n + \frac{N}{2}\right]\right] W_N^{n}$$

$g(n)$ and $h(n)$ can be recognized by the butterfly.



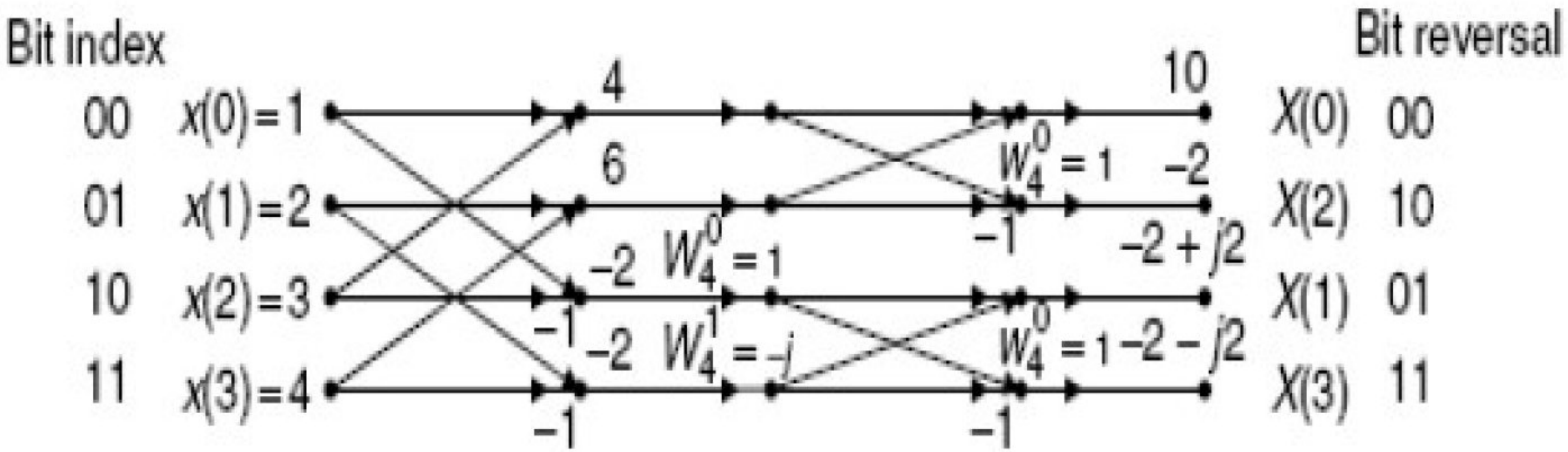
Butterfly of Radix-2 DIF FFT algorithm

Example Given a sequence x(n) for $0 \leq n \leq 3$, where x(0) = 1, x(1) = 2, x(2) = 3, and x(3) = 4
a. Evaluate its DFT X(k) using the DIF FFT method. b. Determine the number of complex multiplications.

Solution: $\boldsymbol{W_4^0 = e^{-j\frac{2\pi}{4}\times 0} = 1}$, and $\boldsymbol{W_4^1 = e^{-j\frac{\pi}{2}\times 1} = cos\left(-\frac{\pi}{2}\right) + jsin\left(-\frac{\pi}{2}\right) = -j}$,

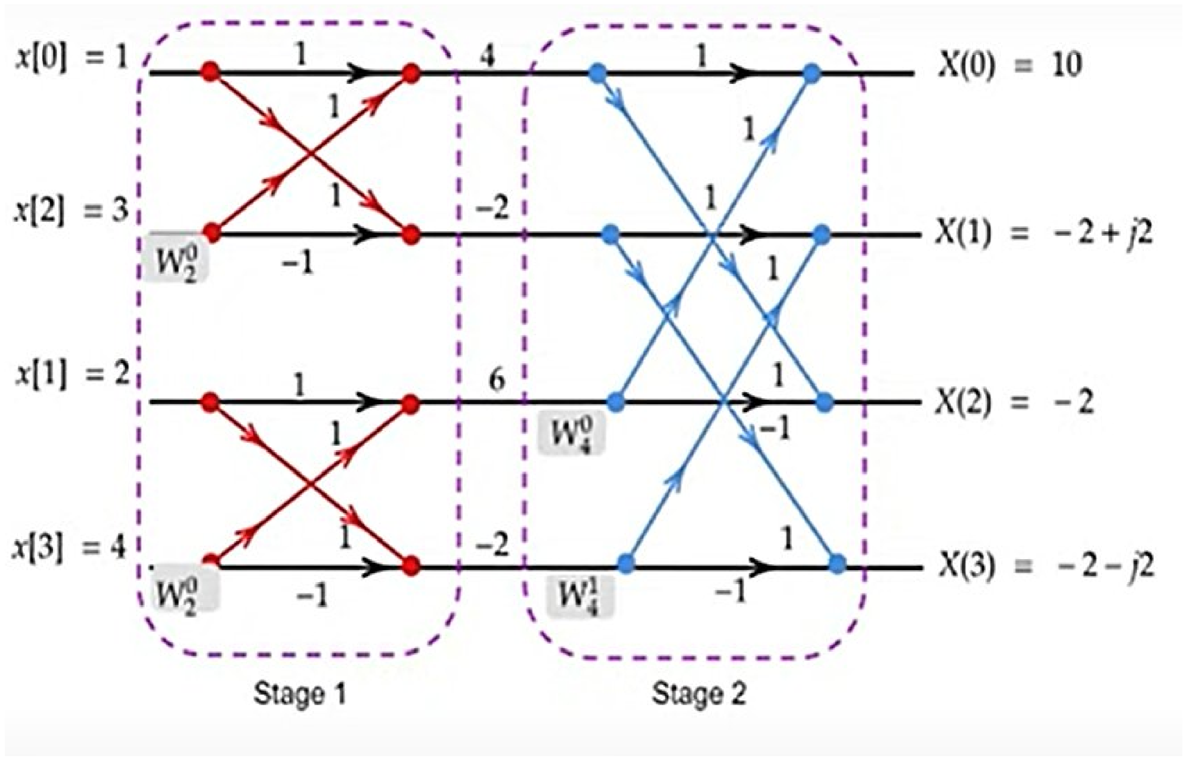
X(k)={10, -2+j2, -2, -2-j2}

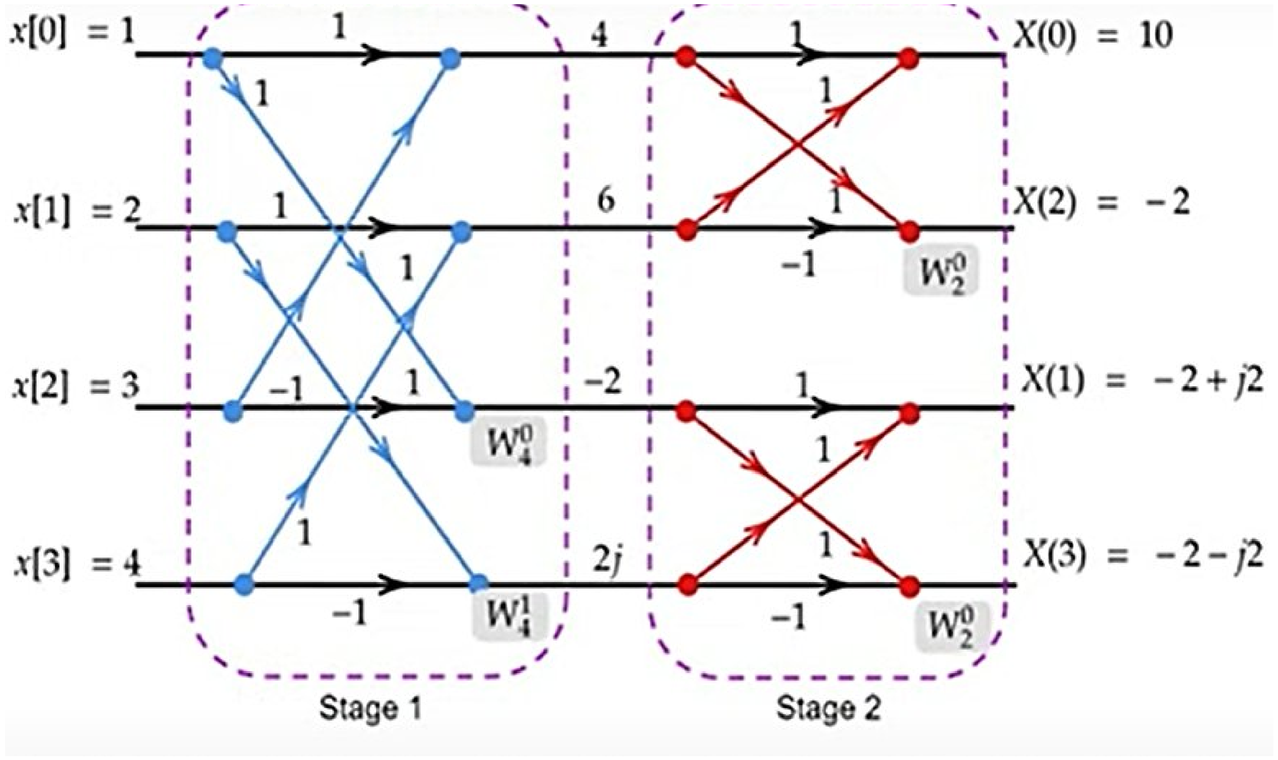
The complex multiplications is
N/2 $\log_2$N=4/2 $\log_2$ 4=4



Example: Compute the 4-point FFT of the aperiodic sequence x[n]={1, 2, 3, 4}? Use DIT and DIF algorithms ?

Solution $W_4^0 = e^{-j\frac{2\pi}{4}\times 0} = \mathbf{1}$, and $W_4^1 = e^{-j\frac{\pi}{2}\times 1} = cos\left(-\frac{\pi}{2}\right) + jsin\left(-\frac{\pi}{2}\right) = -j$





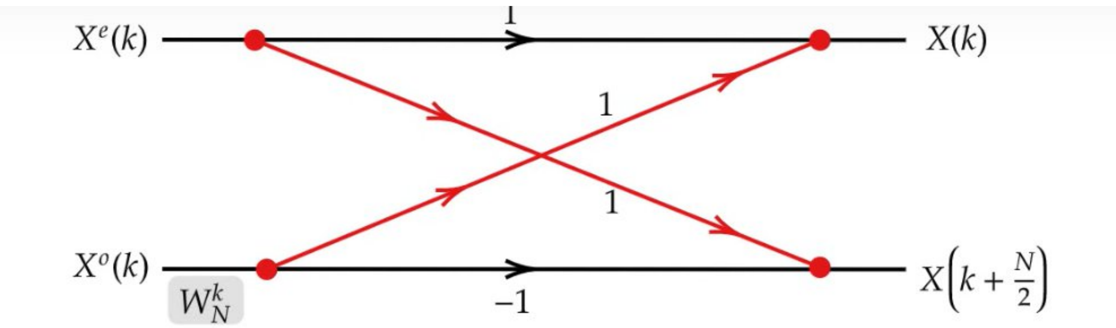
Difference between DIT and DIF:

- In DIT, the input is bit–reversed while the output is in natural order. For DIF, the input is normal order, while output is bit–reversed.
- Considering the butterfly diagram, in DIF, the complex multiplication takes place after the add–subtract operation.

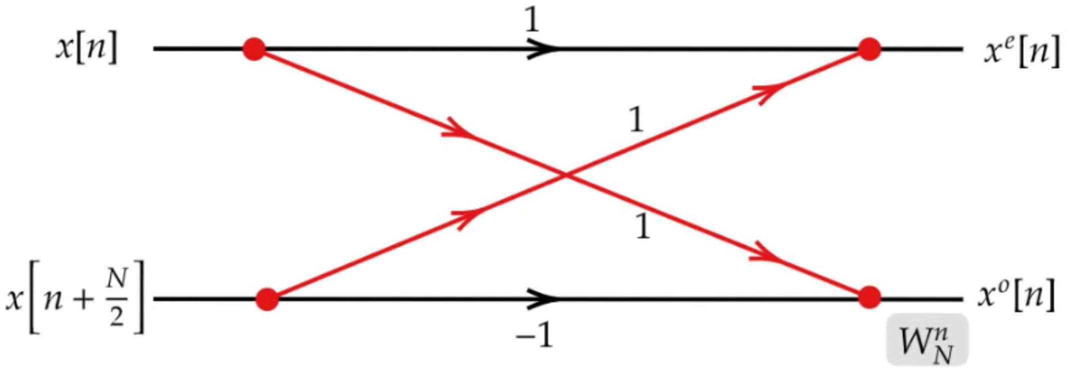
Similarities:

- Both algorithm requires same number of operations to compute DFT.
- Both algorithms require bit–reversed at same place during computation.

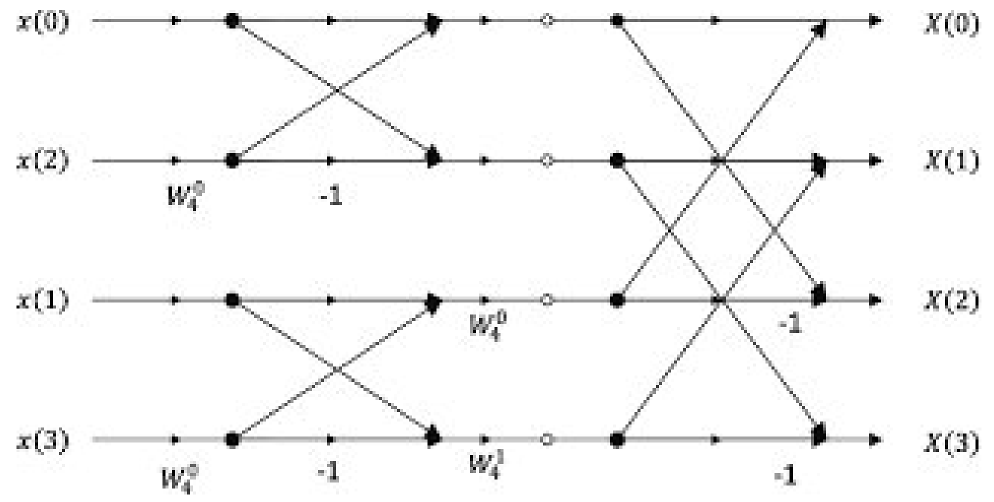
DIT



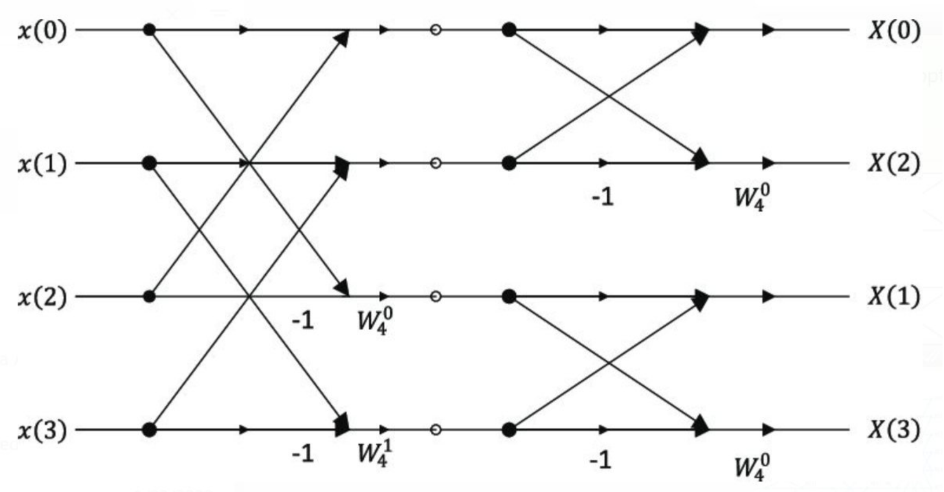
DIF



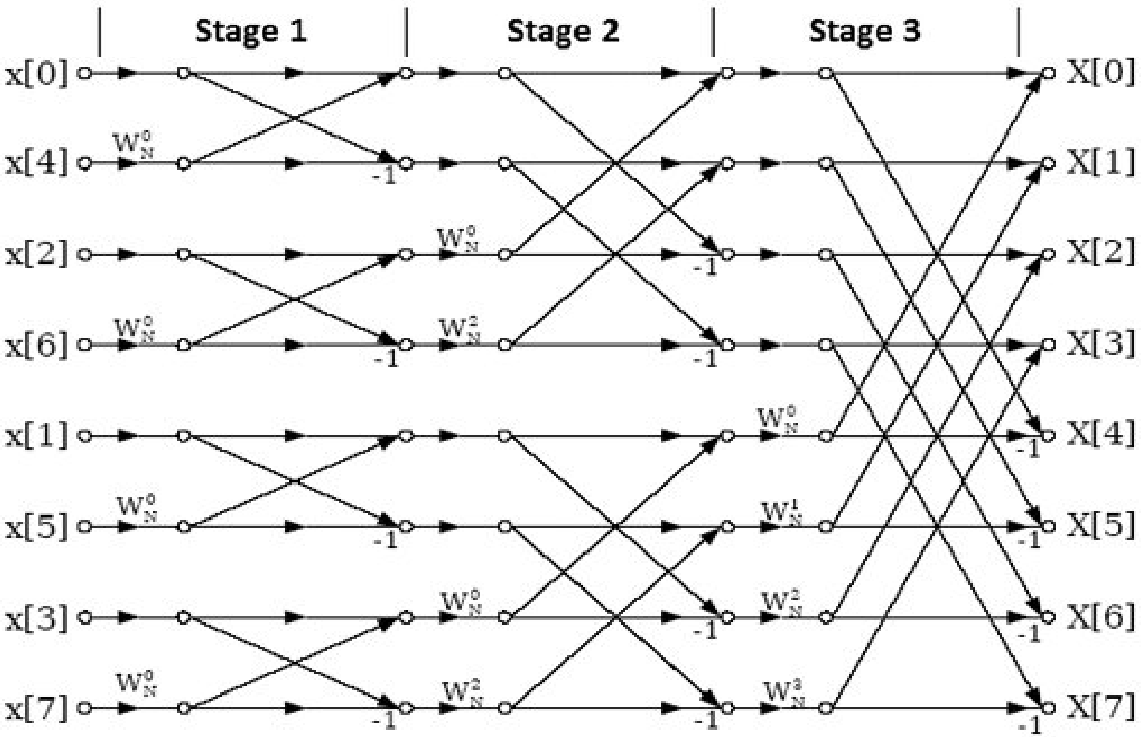
4 point DIT FFT structure



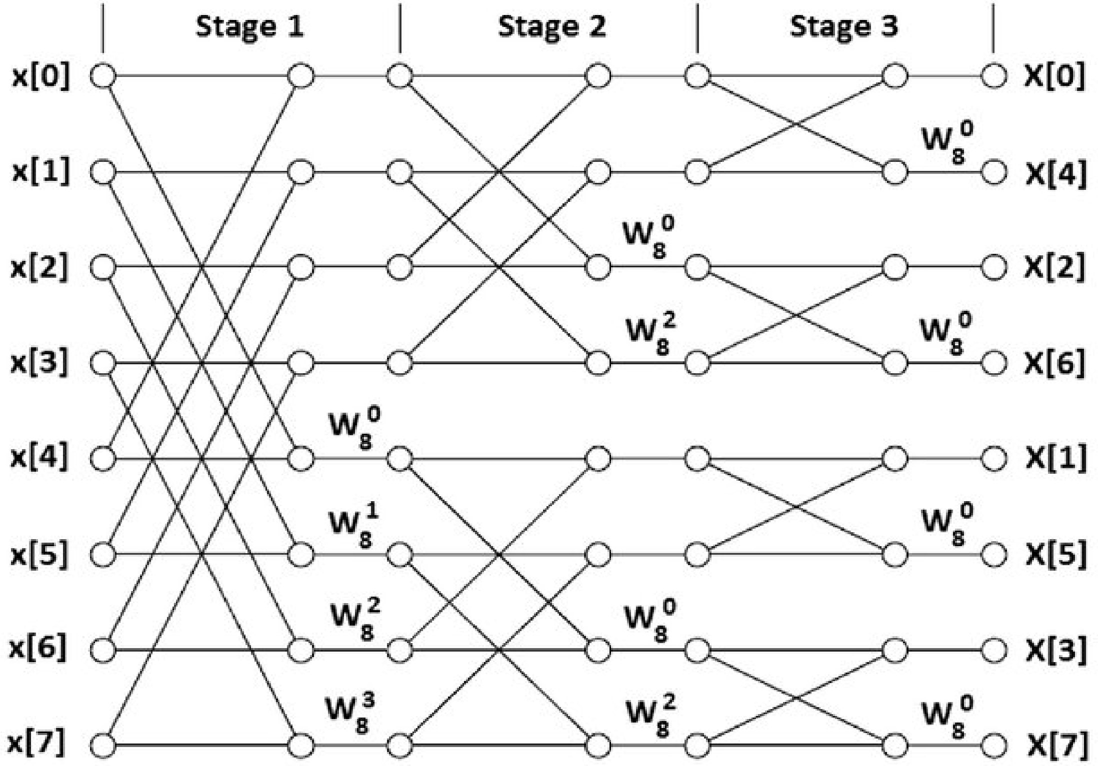
4 point DIF FFT structure



8 point DIT FFT structure



8 point DIF FFT structure



Example Compute the 8 point DFT of the sequence x(n)= [1, -1, 2, 0, 1, 3, 1, -1] using

1-Direct computation DFT

2- Linear Transformation

3- FFT- DIT algorithm

Solution:

(1) Direct computation DFT

$$X(k) = \sum_{n=0}^{N-1} x[n] \quad W_N^{kn}$$

$$k = 0, 1, 2, 3 \ldots\ldots N\text{-}1$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$X(0) = \sum_{n=0}^{7} x(n)W_8^0 = x(0)W_8^0 + x(1)W_8^0 + x(2)W_8^0 + x(3)W_8^0 + x(4)W_8^0 + x(5)W_8^0 + x(6)W_8^0 + x(7)W_8^0 = 1 - 1 + 2 + 0 + 1 + 3 + 1 - 1 = 6$$

$$X(1) = \sum_{n=0}^{7} x(n)W_8^n = x(0)W_8^0 + x(1)W_8^1 + x(2)W_8^2 + x(3)W_8^3 + x(4)W_8^4 + x(5)W_8^5 + x(6)W_8^6 + x(7)W_8^7$$

$$X(1) = 1(1) - 1\left[\frac{1}{\sqrt{2}}(1-j)\right] - 2j + 0\left[\frac{1}{\sqrt{2}}(-1-j)\right] - 1 + 3\left[\frac{1}{\sqrt{2}}(-1+j)\right] + 1j - 1\left[\frac{1}{\sqrt{2}}(1+j)\right] = -\frac{5}{\sqrt{2}} + (\frac{3}{\sqrt{2}} - 1)j$$

$$X(2) = \sum_{n=0}^{7} x(n)W_8^{2n} = x(0)W_8^0 + x(1)W_8^2 + x(2)W_8^4 + x(3)W_8^6 + x(4)W_8^8 + x(5)W_8^{10} + x(6)W_8^{12} + x(7)W_8^{14}$$

$$X(2) = x(0)W_8^0 + x(1)W_8^2 + x(2)W_8^4 + x(3)W_8^6 + x(4)W_8^0 + x(5)W_8^2 + x(6)W_8^4 + x(7)W_8^6 = 1 - 1(-j) + 2(-1) + 0(j) + 1 + 3(-j) + 1(-1) - 1(j)$$

$$X(2) = -1 - 3j$$

$$X(3) = \sum_{n=0}^{7} x(n)W_8^{3n} = x(0)W_8^0 + x(1)W_8^3 + x(2)W_8^6 + x(3)W_8^9 + x(4)W_8^{12} + x(5)W_8^{15} + x(6)W_8^{18} + x(7)W_8^{21}$$

$$X(3) = x(0)W_8^0 + x(1)W_8^3 + x(2)W_8^6 + x(3)W_8^1 + x(4)W_8^4 + x(5)W_8^7 + x(6)W_8^2 + x(7)W_8^5$$

$$X(3) = 1(1) - 1\left[\frac{1}{\sqrt{2}}(-1-j)\right] + 2j + 0\left[\frac{1}{\sqrt{2}}(1-j)\right] - 1 + 3\left[\frac{1}{\sqrt{2}}(1+j)\right] - 1j - 1\left[\frac{1}{\sqrt{2}}(-1+j)\right] = \frac{5}{\sqrt{2}} + (\frac{3}{\sqrt{2}} + 1)j$$

$$X(4) = \sum_{n=0}^{7} x(n)W_8^{4n} = x(0)W_8^0 + x(1)W_8^4 + x(2)W_8^8 + x(3)W_8^{12} + x(4)W_8^{16} + x(5)W_8^{20} + x(6)W_8^{24} + x(7)W_8^{28}$$

$$X(4) = x(0)W_8^0 + x(1)W_8^4 + x(2)W_8^0 + x(3)W_8^4 + x(4)W_8^0 + x(5)W_8^4 + x(6)W_8^0 + x(7)W_8^4 = 1 - 1(-1) + 2 + 0(-1) + 1 + 3(-1) + 1 - 1(-1) = 4$$

$$X(5) = \sum_{n=0}^{7} x(n)W_8^{5n} = x(0)W_8^0 + x(1)W_8^5 + x(2)W_8^{10} + x(3)W_8^{15} + x(4)W_8^{20} + x(5)W_8^{25} + x(6)W_8^{30} + x(7)W_8^{35}$$

$$X(5) = x(0)W_8^0 + x(1)W_8^5 + x(2)W_8^2 + x(3)W_8^7 + x(4)W_8^4 + x(5)W_8^1 + x(6)W_8^6 + x(7)W_8^3$$

$$X(5) = 1(1) - 1\left[\frac{1}{\sqrt{2}}(-1+j)\right] + 2(-j) + 0\left[\frac{1}{\sqrt{2}}(1+j)\right] + 1(-1) + 3\left[\frac{1}{\sqrt{2}}(1-j)\right] + 1j - 1\left[\frac{1}{\sqrt{2}}(-1-j)\right] = \frac{5}{\sqrt{2}} - \left(\frac{3}{\sqrt{2}} + 1\right)j$$

$$X(6) = \sum_{n=0}^{7} x(n)W_8^{6n} = x(0)W_8^0 + x(1)W_8^6 + x(2)W_8^{12} + x(3)W_8^{18} + x(4)W_8^{24} + x(5)W_8^{30} + x(6)W_8^{36} + x(7)W_8^{42}$$

$$X(6) = x(0)W_8^0 + x(1)W_8^6 + x(2)W_8^4 + x(3)W_8^2 + x(4)W_8^0 + x(5)W_8^6 + x(6)W_8^4 + x(7)W_8^2 = 1 - 1(j) + 2(-1) + 0(-j) + 1 + 3(j) + 1(-1) - 1(-j)$$

$$X(6) = -1 + 3j$$

$$X(7) = \sum_{n=0}^{7} x(n)W_8^{7n} = x(0)W_8^0 + x(1)W_8^7 + x(2)W_8^{14} + x(3)W_8^{21} + x(4)W_8^{28} + x(5)W_8^{35} + x(6)W_8^{42} + x(7)W_8^{49}$$

$$X(7) = x(0)W_8^0 + x(1)W_8^7 + x(2)W_8^6 + x(3)W_8^5 + x(4)W_8^4 + x(5)W_8^3 + x(6)W_8^2 + x(7)W_8^1$$

$$X(7) = 1(1) - 1\left[\frac{1}{\sqrt{2}}(+1+j)\right] + 2j + 0\left[\frac{1}{\sqrt{2}}(-1+j)\right] + 1(-1) + 3\left[\frac{1}{\sqrt{2}}(-1-j)\right] + 1(-j) - 1\left[\frac{1}{\sqrt{2}}(1-j)\right] == -\frac{5}{\sqrt{2}} - (\frac{3}{\sqrt{2}} - 1)j$$

Example Compute the DFT of the sequence x(n)= [1, -1, 2, 0, 1, 3, 1, -1] using

Solution:

(2) Linear Transformation

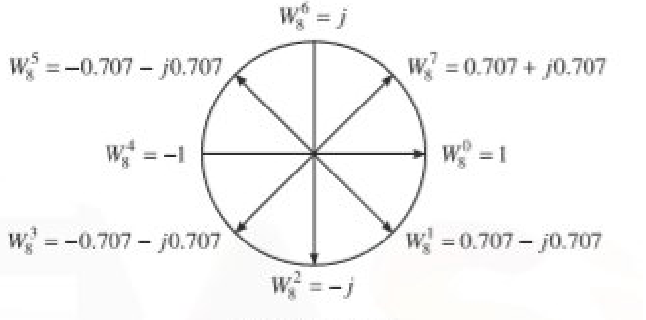
$$X(k) = \sum_{n=0}^{N-1} x[n] \quad W_N^{kn}$$

$k = 0, 1, 2, 3...... N\text{-}1$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$\mathbf{W}_8 = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^0 & W_8^2 & W_8^4 & W_8^6 \\ W_8^0 & W_8^3 & W_8^6 & W_8^1 & W_8^4 & W_8^7 & W_8^2 & W_8^5 \\ W_8^0 & W_8^4 & W_8^0 & W_8^4 & W_8^0 & W_8^4 & W_8^0 & W_8^4 \\ W_8^0 & W_8^5 & W_8^2 & W_8^7 & W_8^4 & W_8^1 & W_8^6 & W_8^3 \\ W_8^0 & W_8^6 & W_8^4 & W_8^2 & W_8^0 & W_8^6 & W_8^4 & W_8^2 \\ W_8^0 & W_8^7 & W_8^6 & W_8^5 & W_8^4 & W_8^3 & W_8^2 & W_8^1 \end{bmatrix}$$

$$X(K) = \mathbf{W}_8 \times x(n)^T$$



(b) 8-point DFT

$W_8^0 = 1$ $\qquad$ $W_8^4 = -1$

$W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $\qquad$ $W_8^5 = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$W_8^2 = -j$ $\qquad$ $W_8^6 = j$

$W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $\qquad$ $W_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt{2}}(1-j) & -j & \frac{1}{\sqrt{2}}(-1-j) & -1 & \frac{1}{\sqrt{2}}(-1+j) & j & \frac{1}{\sqrt{2}}(1+j) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{1}{\sqrt{2}}(-1+j) & j & \frac{1}{\sqrt{2}}(1-j) & -1 & \frac{1}{\sqrt{2}}(1+j) & -j & \frac{1}{\sqrt{2}}(-1+j) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{1}{\sqrt{2}}(-1+j) & -j & \frac{1}{\sqrt{2}}(1+j) & -1 & \frac{1}{\sqrt{2}}(1-j) & j & \frac{1}{\sqrt{2}}(-1-j) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{1}{\sqrt{2}}(1+j) & j & \frac{1}{\sqrt{2}}(-1+j) & -1 & \frac{1}{\sqrt{2}}(-1-j) & -j & \frac{1}{\sqrt{2}}(1-j) \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \\ 1 \\ 3 \\ 1 \\ -1 \end{bmatrix}$$

$$X(K) = \left[6, -\frac{5}{\sqrt{2}} + \left(\frac{3}{\sqrt{2}} - 1\right)j, -1 - 3j, \frac{5}{\sqrt{2}} + (\frac{3}{\sqrt{2}} - 1)j, 4, \frac{5}{\sqrt{2}} - (\frac{3}{\sqrt{2}} - 1)j, -1 + 3j, -\frac{5}{\sqrt{2}} - (\frac{3}{\sqrt{2}} - 1)j \right]$$

Example Compute the DFT of the sequence x(n)= [1, -1, 2, 0, 1, 3, 1, -1] using

Solution:

(3) FFT- DIT algorithm

$$X(k) = \sum_{n=0}^{N-1} x[n] \quad W_N^{kn}$$

$$k = 0,\ 1,\ 2,\ 3\ldots\ldots\ N\text{-}1$$

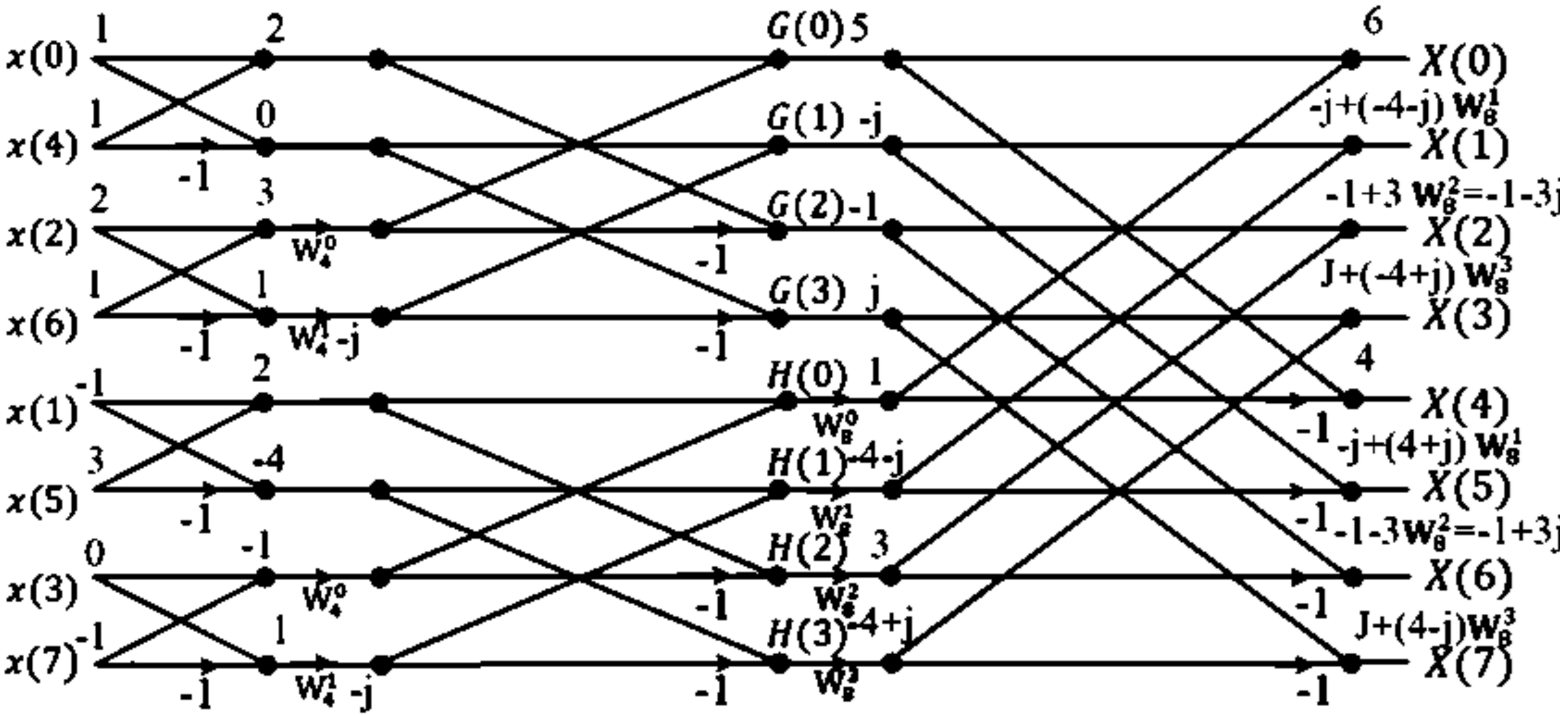
$$W_N = e^{-j\frac{2\pi}{N}}$$

$W_8^0 = 1$

$W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$W_8^2 = -j$

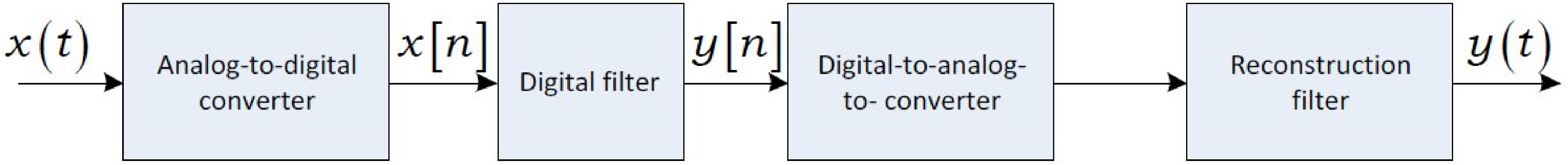
$W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$



$$X(K) = \left[6, -\frac{5}{\sqrt{2}} + \left(\frac{3}{\sqrt{2}} - 1\right)j, -1 - 3j, \frac{5}{\sqrt{2}} + (\frac{3}{\sqrt{2}} - 1)j, 4, \frac{5}{\sqrt{2}} - (\frac{3}{\sqrt{2}} - 1)j, -1 + 3j, -\frac{5}{\sqrt{2}} - (\frac{3}{\sqrt{2}} - 1)j\right]$$

Digital Filter Design

- A filter is essentially a system or network that selectively changes the wave shape, amplitude and or phase-frequency characteristics of a signal in a desired manner.
- Common filtering objectives are to improve the quality of a signal …. (to remove or reduce noise), to extract information from signals or to separate two or more signals previously combined to make, for example, efficient use of available communication channel.
- A digital filter is a mathematical algorithm implemented in **hardware and /or software** that operate on a digital input signal to produce a digital output signal of achieving a filter objective.   A simplified block diagram of a real time digital filter with analogue input and output signals is shown in Figure below. So digital filter is a system that perform the mathematical operation in signal processing on a sampled signal to reduce or enhance certain aspects of that signal



Electronic Filter

Low-Pass Electrical Filter

EMI Filters