Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class- Semester-2

Control Engineering

Chapter 12

Lecture 8

Design of Control Systems

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11-4 DESIGN WITH THE PD CONTROLLER

In most examples of control systems, we have discussed thus far, the controller has been typically a simple amplifier with a constant gain *K*. This type of control action is formally known as **proportional control** because the control signal at the output of the controller is simply related to the input of the controller by a proportional constant. Intuitively, one should also be able to use the derivative or integral of the input signal, in addition to the proportional operation. Therefore, we can consider a more general continuous-data controller to be one that contains such components as adders (addition or subtraction), amplifiers, attenuators, differentiators, and integrators.

The designer's task is to determine which of these components should be used, in what proportion, and how they are connected. For example, one of the best-known controllers used in practice is the PID controller, where the letters stand for proportional, integral, and derivative. The integral and derivative components of the PID controller have individual performance implications, and their applications require an understanding of the basics of these elements. To gain an understanding of this controller, we consider just the PD portion of the controller first. Figure 11-17 shows the block diagram of a feedback control system that arbitrarily has a second-order prototype process with the transfer function

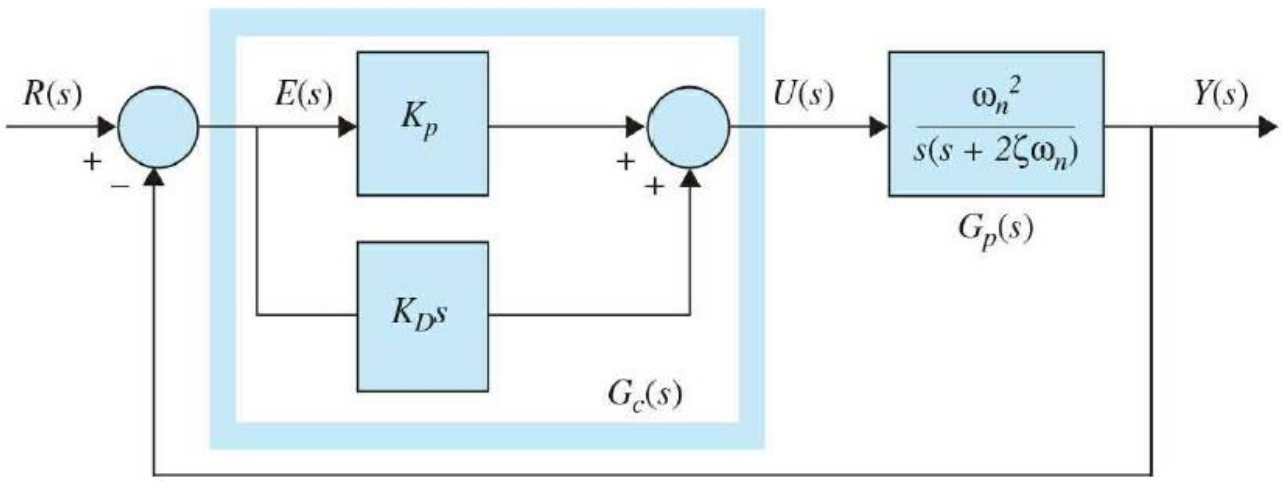

Figure 11-17 Control system with PD controller.

$$G_p(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \qquad 11.7$$

The series controller is a proportional-derivative (PD) type with the transfer function

$$G_c(s) = K_P + K_D s \qquad 11.8$$

Thus, the control signal applied to the process is

$$u(t) = K_P e(t) + K_D \frac{de(t)}{dt} \qquad 11.9$$

where *KP* and *KD* are the proportional and derivative constants, respectively.

Electronic-circuit realizations of the PD controller are shown in Fig. 11-18

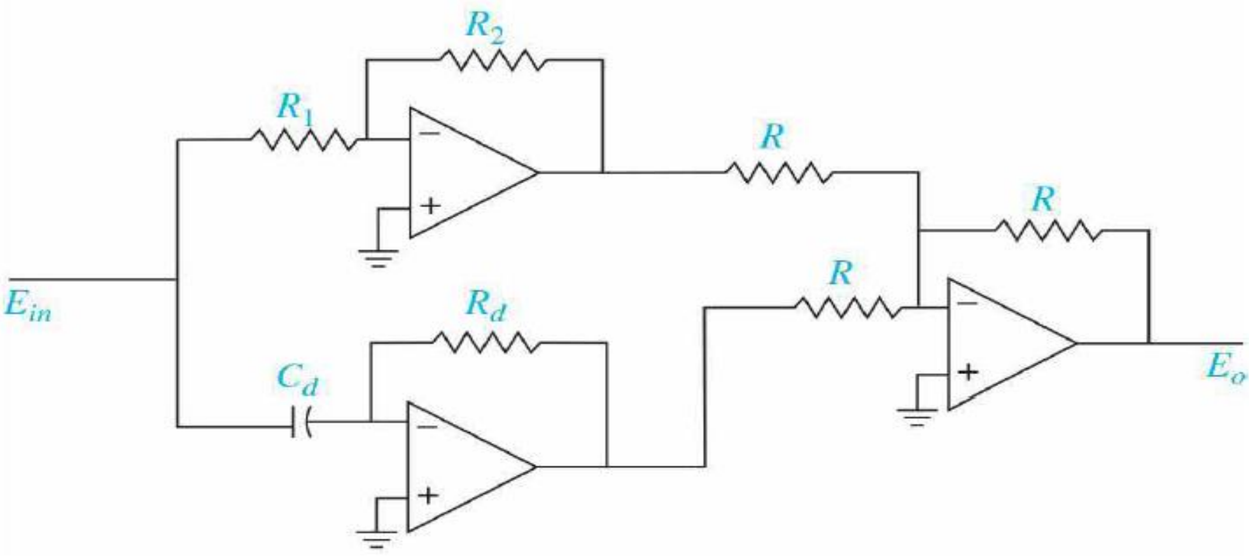

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Figure 11-18 Op-amp circuit realization of the PD controller.

The transfer function of the circuit in Fig. 11-18 is

$$\frac{E_o(s)}{E_{in}(s)} = \frac{R_2}{R_1} + R_d C_d s \qquad 11.10$$

Comparing Eq. (11-8) with Eq. (11-10), we have

$$K_P = R_2/R_1 \qquad K_D = R_d C_d \qquad 11.11$$

The circuit in Fig. 11-4*b* allows *KP* and *KD* to be independently controlled. A large *KD* can be compensated by choosing a large value for *Rd*, thus resulting in a realistic value for *Cd*. The forward-path transfer function of the compensated system is

$$G(s) = \frac{Y(s)}{E(s)} = G_c(s)G_p(s) = \frac{\omega_n^2 (K_P + K_D s)}{s(s + 2\zeta\omega_n)} \qquad 11.12$$

which shows that the PD control is equivalent to adding a simple zero at s = -Kp/KD to the forward-path transfer function.

11-4-1 Time-Domain Interpretation of PD Control

PD control adds a simple zero at s = -*Kp*/*KD* to the forward-path transfer function. The effect of the PD control on the transient response of a control system can be investigated by referring to the time responses shown in Fig. 11-19. Let us assume that the unit-step response of a stable system with only proportional control is as shown in Fig. 11-19*a*, which has a relatively high maximum overshoot and is rather oscillatory. The corresponding error signal, which is the difference between the unit-step input and the output $y(t)$ and its time derivative $de(t)/dt$ are shown in Figs. 11.19*b* and *c*, respectively. The overshoot and oscillation characteristics are also reflected in $e(t)$ and $de(t)/dt$. For the sake of illustration, we assume that the system contains a motor of some kind with its torque proportional to $e(t)$. The performance of the system with proportional control is analyzed as follows:

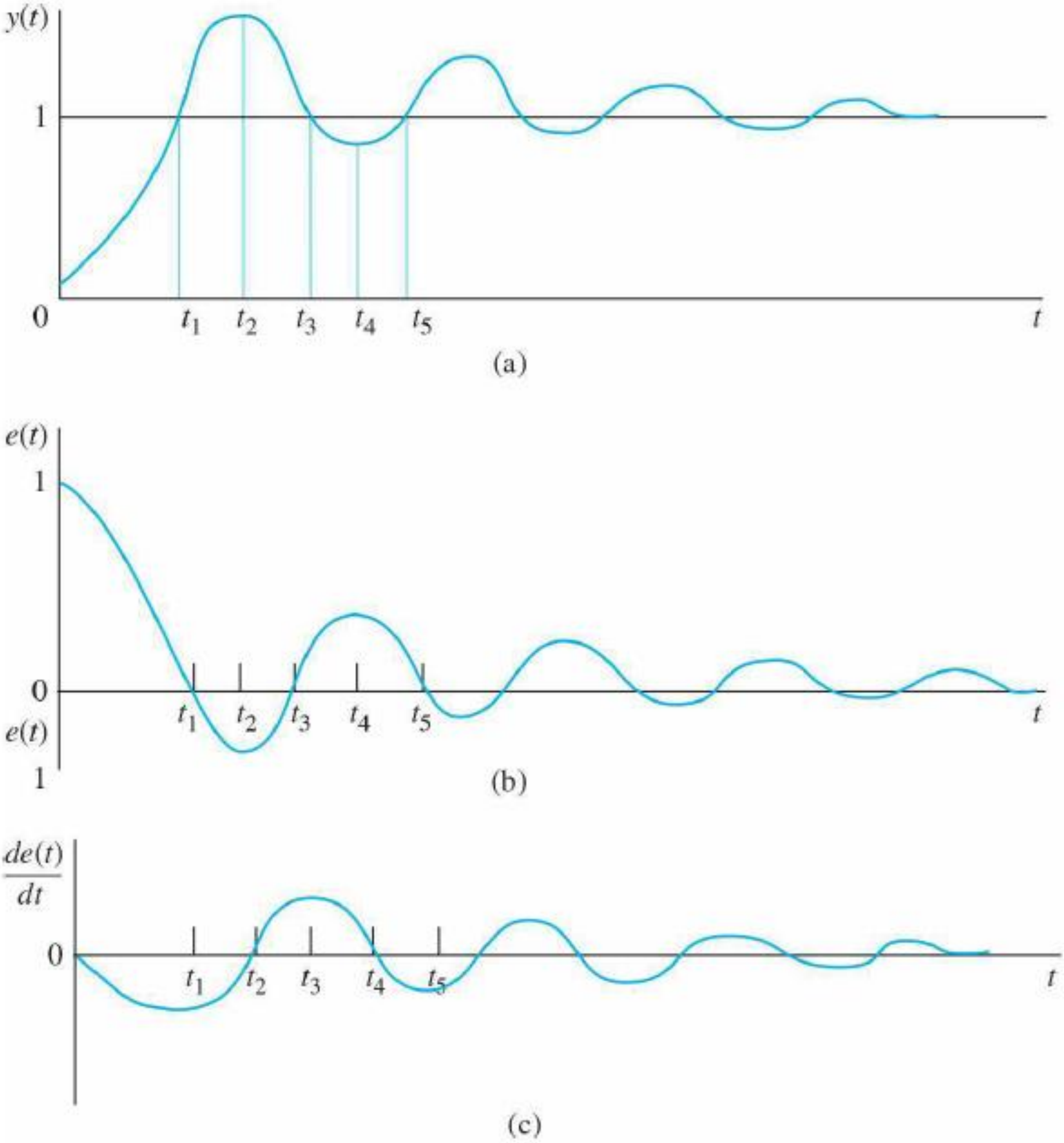

Figure 11-5 Waveforms of y(t), e(t), and de(t)/dt, showing the effect of derivative control. (a) Unit-step response. (b) Error signal. (c) Time rate of change of the error signal.

1. During the time interval 0<t<t1: The error signal e(t) is positive. The motor torque is positive and rising rapidly. The large overshoot and subsequent oscillations in the output y(t) are due to the excessive amount of torque developed by the motor and the lack of damping during this time interval.

2. During the time interval t1<t<t3: The error signal e(t) is negative, and the corresponding motor torque is negative. This negative torque tends to slow down the output acceleration and eventually causes the direction of the output y(t) to reverse and undershoot.

3. During the time interval t3<t<t5: The motor torque is again positive, thus tending to reduce the undershoot in the response caused by the negative torque in the previous time interval. Because the system is assumed to be stable, the error amplitude is reduced with each oscillation, and the output eventually settles to its final value.

Considering the above analysis of the system time response, we can say that the contributing factors to the high overshoot are as follows:

1. The positive correcting torque in the interval 0<t<t1 is too large.

2. The retarding torque in the time interval t1<t<t2 is inadequate.

Therefore, to reduce the overshoot in the step response, without significantly increasing the rise time, a logical approach would be to

1. Decrease the amount of positive correcting torque during 0<*t*<*t1*.

2. Increase the retarding torque during t1<*t*<*t2*.

Similarly, during the time interval, t2<*t*<*t4*, the negative corrective torque in t2<*t*<*t3* should be reduced, and the retarding torque during t3<*t*<*t4*, which is now in the positive direction, should be increased to improve the undershoot of *y*(*t*). The PD control described by Eq. (11-7) gives precisely the compensation effect required. Because the control signal of the PD control is given by Eq. (11-8), Fig. 11-19*c* shows the following effects provided by the PD controller:

1. For 0<*t*<*t1*, *de*(*t*)/*dt* is negative; this will reduce the original torque developed due to *e*(*t*) alone.

2. For t1<*t*<*t2*, both *e*(*t*) and *de*(*t*)/*dt* are negative, which means that the negative retarding torque developed will be greater than that with only proportional control.

3. For $t2<t<t3$, $e(t)$ and $de(t)/dt$ have opposite signs. Thus, the negative torque that originally contributes to the undershoot is reduced also.

Therefore, all these effects will result in smaller overshoots and undershoots in $y(t)$.

PD is essentially an anticipatory control. Another way of looking at the derivative control is that since $de(t)/dt$ represents the slope of $e(t)$, the PD control is essentially an *anticipatory* control. That is, by knowing the slope, the controller can anticipate direction of the error and use it to better control the process.

11-4-2 Frequency-Domain Interpretation of PD Control

For frequency-domain design, the transfer function of the PD controller is written as

$$G_c(s) = K_P + K_D s = K_P\left(1 + \frac{K_D}{K_P}s\right) \qquad 11.13$$

so that it is more easily interpreted on the Bode plot. The Bode plot of Eq. (11-13) is shown in Fig. 11-6 with $Kp=1$. In general, the proportional-control gain KP can be combined with a series gain of the system, so that the zero-frequency gain of the PD controller can be regarded as unity. The high-pass filter characteristics of the PD controller are clearly shown by the Bode plot in Fig. 11-20. The phase-lead property may be utilized to improve the phase margin of a control system. Unfortunately, the magnitude characteristics of the PD controller push the gain-crossover frequency to a higher value. Thus, *the design principle of the PD controller involves the placing of the corner frequency of the controller*, $\omega=Kp/KD$, *such that an effective improvement of the phase margin is realized at the new gain-crossover frequency*. For a given system, there is a range of values of KP/KD that is optimal for improving the damping of the system. Another practical consideration in selecting the values of KP and KD is in the physical implementation of the PD controller. Other apparent effects of the PD control in the frequency domain are that, due to its high-pass characteristics, in most cases it will increase the BW of the system and reduce the rise time of the step response. The practical disadvantage of the PD controller is that the differentiator portion is a high pass filter, which usually accentuates any high-frequency noise that enters at the input.

The PD controller is a high-pass filter. The PD controller has the disadvantage that it accentuates high frequency noise. The PD controller will generally increase the BW and reduce the rise time of the step response

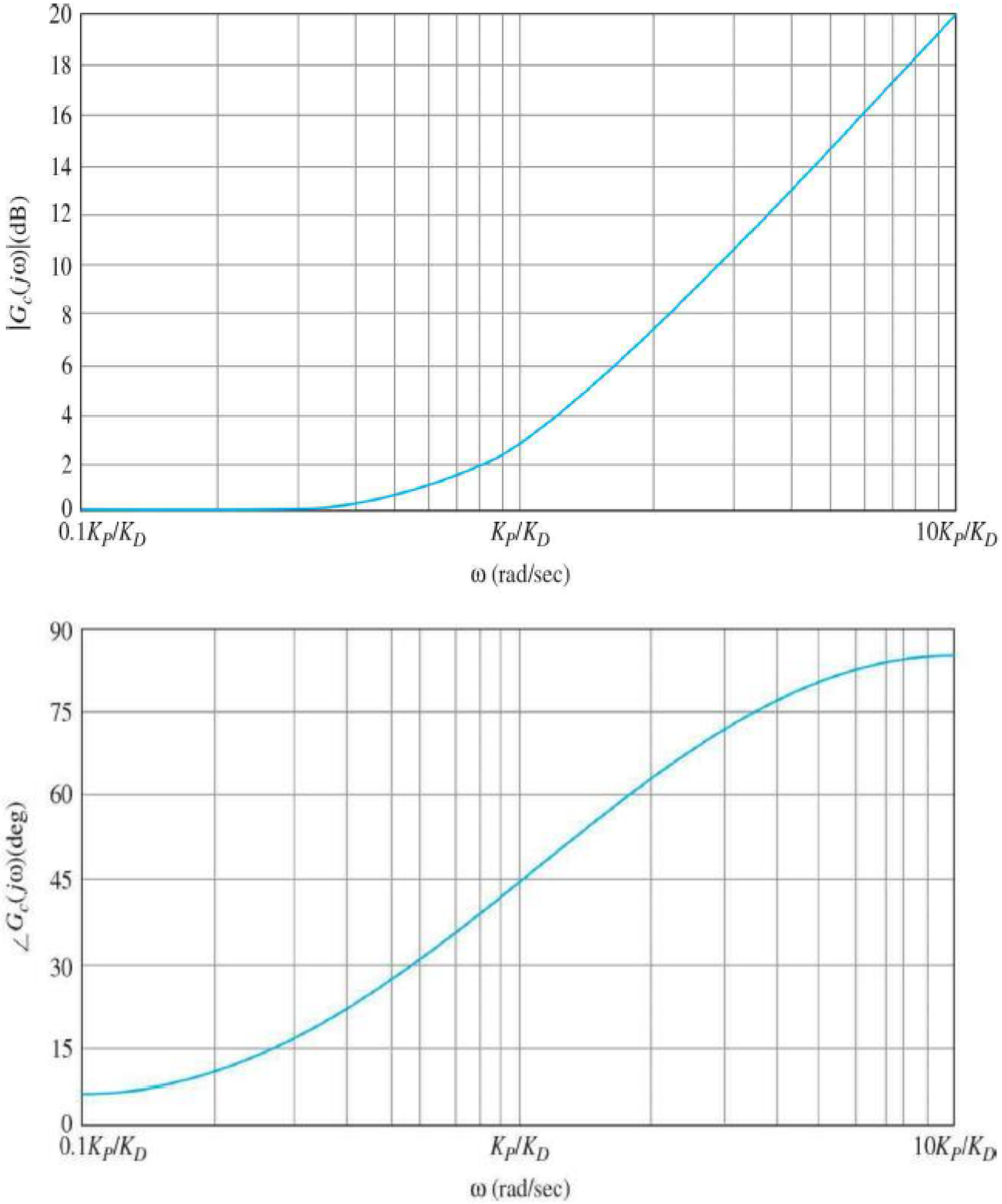

Figure 11-20 Bode diagram

11-4-3 Summary of Effects of PD Control

Though it is not effective with lightly damped or initially unstable systems, a properly designed PD controller can affect the performance of a control system in the following ways:

1. Improving damping and reducing maximum overshoot.

2. Reducing rise time and settling time.

3. Increasing BW.

4. Improving GM, PM, and *Mr*.

5. Possibly accentuating noise at higher frequencies.

6. Possibly requiring a relatively large capacitor in circuit

11-5 DESIGN WITH THE PI CONTROLLER

We see from Sec. 11-4 that the PD controller can improve the damping and rise time of a control system at the expense of higher bandwidth and resonant frequency, and the steady-state error is not affected unless it varies with time, which is typically not the case for step-function inputs. Thus, the PD controller may not fulfil the compensation objectives in many situations. The integral part of the PID controller produces a signal that is proportional to the time integral of the input of the controller. Figure 11-21 illustrates the block diagram of a prototype second-order system with a series PI controller. The transfer function of the PI controller is

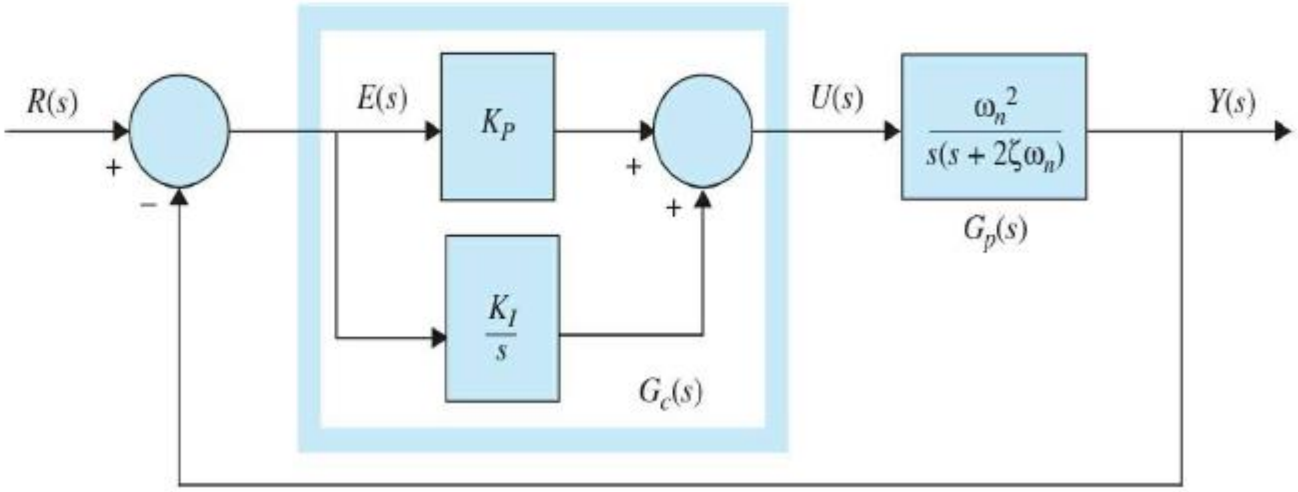

Figure 11-21 Control system with PI controller.

$$G_c(s) = K_P + \frac{K_I}{s} \qquad 11.14$$

op-amp-circuit realizations of Eq. (11-34) are shown in Fig. 11-22. The transfer function of the three-op-amp circuit in Fig. 11-22 is

$$G_c(s) = \frac{E_o(s)}{E_{in}(s)} = \frac{R_2}{R_1} + \frac{1}{R_i C_i s} \qquad 11.15$$

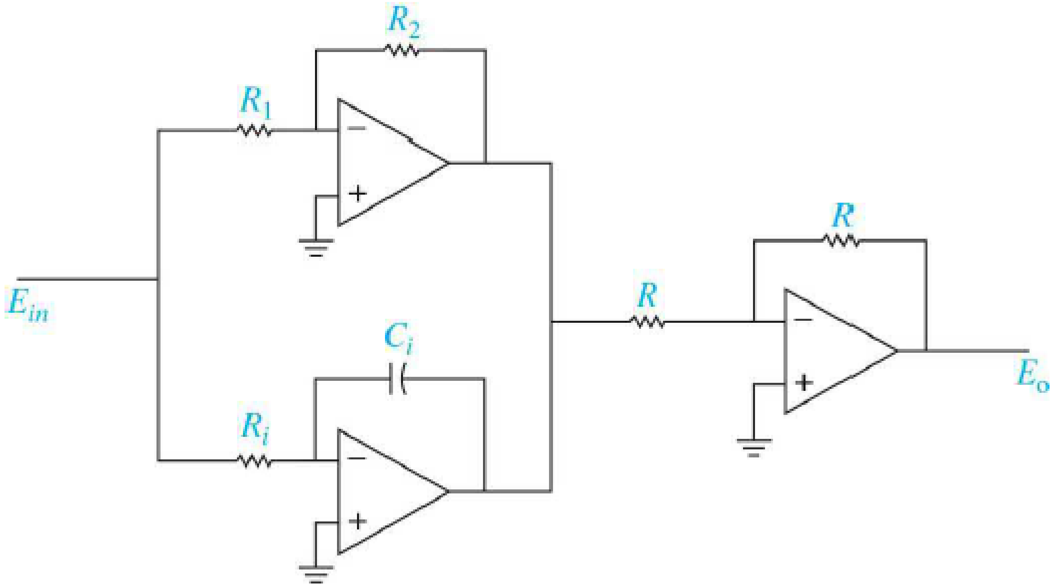

Figure 11-22 Op-amp-circuit realization of the PI controller

Thus, the parameters of the PI controller are related to the circuit parameters as

$$K_P = \frac{R_2}{R_1} \quad K_I = \frac{1}{R_i C_i} \qquad 11.16$$

The advantage with the circuit in Fig. 11-22 is that the values of *KP* and *KI* are independently related to the circuit parameters.

The forward-path transfer function of the compensated system is

$$G(s) = G_c(s)G_p(s) = \frac{\omega_n^2(K_P s + K_I)}{s^2(s + 2\zeta\omega_n)} \qquad 11.17$$

Clearly, the immediate effects of the PI controller are as follows:

1. Adding a zero at s = -*K1*/*KP* to the forward-path transfer function.

2. Adding a pole at *s* = 0 to the forward-path transfer function.

This means that the system type is increased by 1 to a type 2 system. Thus, the steady-state error of the original system is improved by one order; that is, if the steady-state error to a given input is constant, the PI control reduces it to zero (provided that the compensated system remains stable).

The system in Fig. 11-21, with the forward-path transfer function in Eq (11-17), will now have a zero steady-state error when the reference input is a ramp function. However, because the system is now of the third order, it may be less stable than the original second-order system or even become unstable if the parameters KP and KI are not properly chosen. In the case of a type 1 system with a PD control, the value of KP is important because the ramp-error constant Kv is directly proportional to KP, and thus the magnitude of the steady-state error is inversely proportional to KP when the input is a ramp. On the other hand, if KP is too large, the system may become unstable. Similarly, for a type 0 system, the steady-state error due to a step input will be inversely proportional to KP. When a type 1 system is converted to type 2 by the PI controller, KP no longer affects the steady-state error, and the latter is always zero for a stable system with a ramp input. The problem is then to choose the proper combination of KP and KI so that the transient response is satisfactory.

11-5-1 Time-Domain Interpretation and Design of PI Control

The pole-zero configuration of the PI controller in Eq. (11-14) is shown in Fig. 11-23. At first glance, it may seem that PI control will improve the steady-state error at the expense of stability. However, we can show that, if the location of the zero of $Gc(s)$ is selected properly, both the damping and the steady-state error can be improved. Because the PI controller is essentially a low-pass filter, the compensated system usually will have a slower rise time and longer settling time. *A viable method of designing the PI control is to select the zero at s = -K1/KP so that it is relatively close to the origin and away from the most significant poles of the process; the values of KP and KI should be relatively small*.

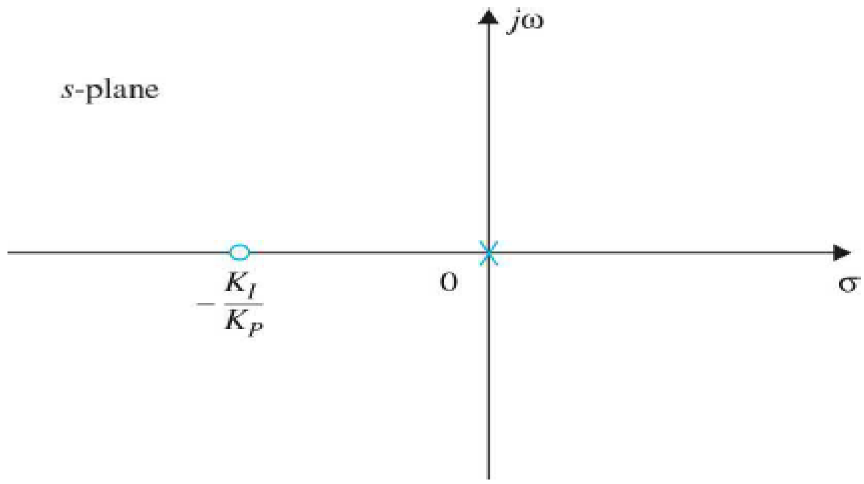

Figure 11-21 Pole-zero configuration of a PI controller.

we can summarize the advantages and

disadvantages of a properly designed PI controller as the following:

1. Improving damping and reducing maximum overshoot.

2. Increasing rise time.

3. Decreasing BW.

4. Improving gain margin, phase margin, and *Mr*.

5. Filtering out high-frequency noise.