Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class- Semester-2

Control Engineering

Chapter 11

Lecture 7

Design of Control Systems

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EXAMPLE 11-2

Consider the system shown in Figure 11–12(a). with dominant closed-loop poles s=-0.3307± j0.5846. It is desired to increase the static velocity error constant *K*v to about 5 sec^{-1} without appreciably changing the location of the dominant closed-loop poles. s=-0.3± j0.55

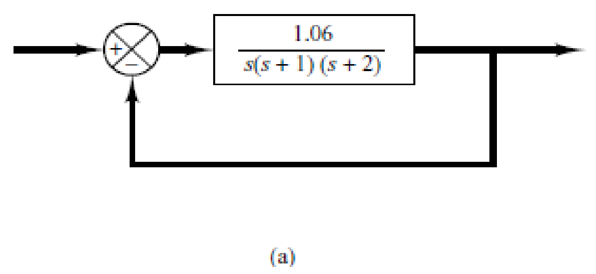


Figure 11-12(a) Control system;

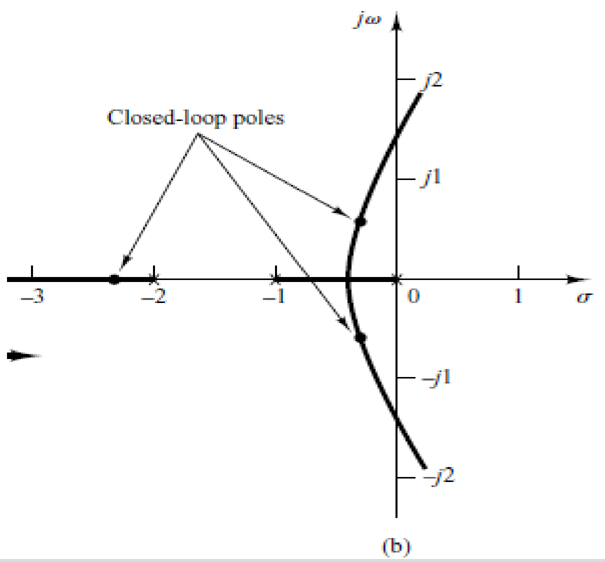
Solution

The feedforward transfer function is $G(s) = \frac{1.06}{s(s+1)(s+2)}$. the static velocity error constant is 0.53 sec^{-1}.The root-locus plot for the system is shown in Figure 11–12(b). The closed-loop transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{1.06}{s(s+1)(s+2)+1.06} = \frac{1.06}{(s+0.3307-j0.5864)(s+0.3307+j0.5864)(s+2.3386)}$$

The dominant closed-loop poles are $s = -0.3307 \pm j0.5864$

The damping ratio of the dominant closed-loop poles is ζ = 0.491. The undamped natural frequency of the dominant closed-loop poles is 0.673 rad_sec. The static velocity error constant is 0.53 sec^{-1}. It is desired to increase the static velocity error constant *K*v to about 5 sec–1 without appreciably changing the location of the dominant closed-loop poles. To meet this specification, let us insert a lag compensator as given by Equation (11–2) in cascade with the given feedforward transfer function. To increase the static velocity error constant by a factor of about 10.



(b) root-locus plot.

let us choose B=10 and place the zero and pole of the lag compensator at s=–0.05 and s=–0.005, respectively. The transfer function of the lag compensator become $G_c(s) = \hat{K}_c \frac{s+0.05}{s+0.005}$

$$G_c(s)G(s) = \hat{K}_c \frac{s+0.05}{s+0.005} \frac{1.06}{s(s+1)(s+2)}$$

$$= \frac{K(s+0.05)}{s(s+0.005)(s+1)(s+2)} \qquad K = 1.06\hat{K}_c$$

The root-locus plot for the compensated system near the dominant closed-loop poles is shown in Figure 11–13(a), together with the original root-locus plot. Figure 11–13(b) shows the root-locus plot of the compensated system

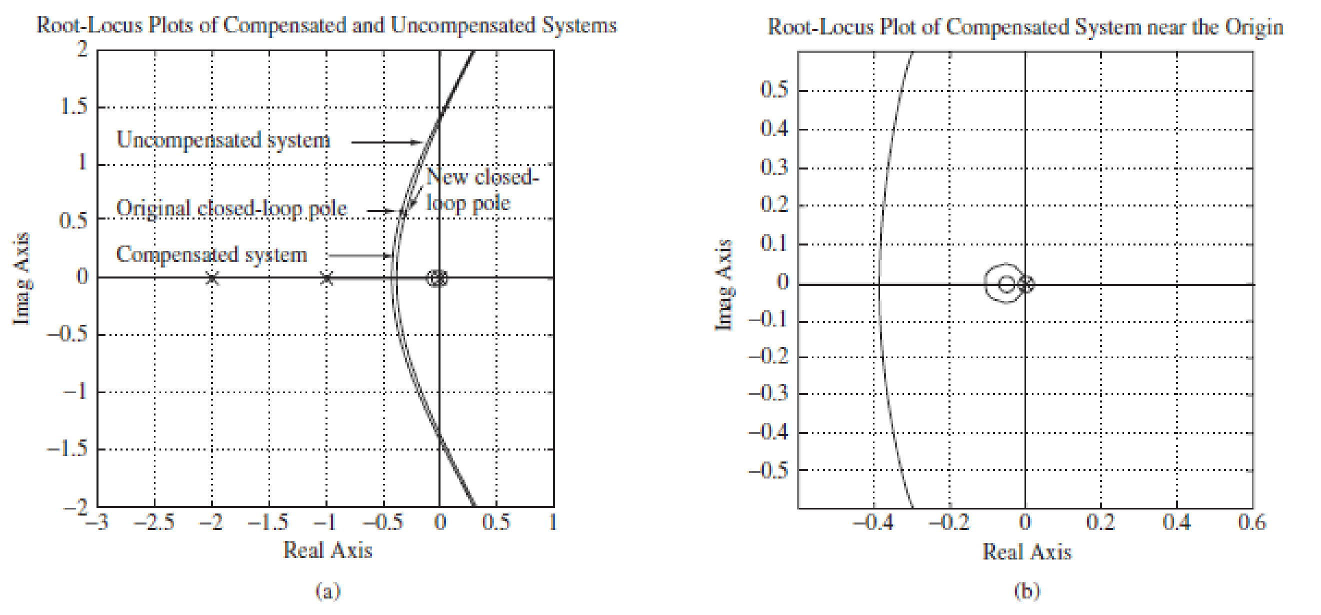


Figure 11–13 (a) Root-locus plots of the compensated system and uncompensated system; (b) root-locus plot of compensated system near the origin.

The open-loop gain K is determined from the magnitude condition as follows:

$$K = \left|\frac{s(s+0.005)(s+1)(s+2)}{s+0.05}\right|_{s=-0.31+j0.55}$$

$$= 1.0235$$

Then the lag compensator gain $\hat{K}_c$ is determined as

$$\hat{K}_c = \frac{K}{1.06} = \frac{1.0235}{1.06} = 0.9656$$

Thus the transfer function of the lag compensator designed is

$$G_c(s) = 0.9656\frac{s+0.05}{s+0.005} = 9.656\frac{20s+1}{200s+1}$$

Then the compensated system has the following open-loop transfer function:

$$G_1(s) = \frac{1.0235(s+0.05)}{s(s+0.005)(s+1)(s+2)}$$

$$= \frac{5.12(20s+1)}{s(200s+1)(s+1)(0.5s+1)}$$

The static velocity error constant K_v is

$$K_v = \lim_{s\to 0} sG_1(s) = 5.12 \text{ sec}^{-1}$$

11–4 Lag–Lead Compensation

Lead compensation basically speeds up the response and increases the stability of the system. Lag compensation improves the steady-state accuracy of the system, but reduces the speed of the response. If improvements in both transient response and steady-state response are desired, then both a lead compensator and a lag compensator may be used simultaneously. Rather than introducing both a lead compensator and a lag compensator as separate units, however, it is economical to use a single lag–lead compensator.

Lag–lead compensation combines the advantages of lag and lead compensations. Since the lag–lead compensator possesses two poles and two zeros, such a compensation increases the order of the system by 2, unless cancellation of pole(s) and zero(s) occurs in the compensated system.

11-4-1 Electronic Lag–Lead Compensator Using Operational Amplifiers.

Figure 11–14 shows an electronic lag–lead compensator using operational amplifiers. The transfer

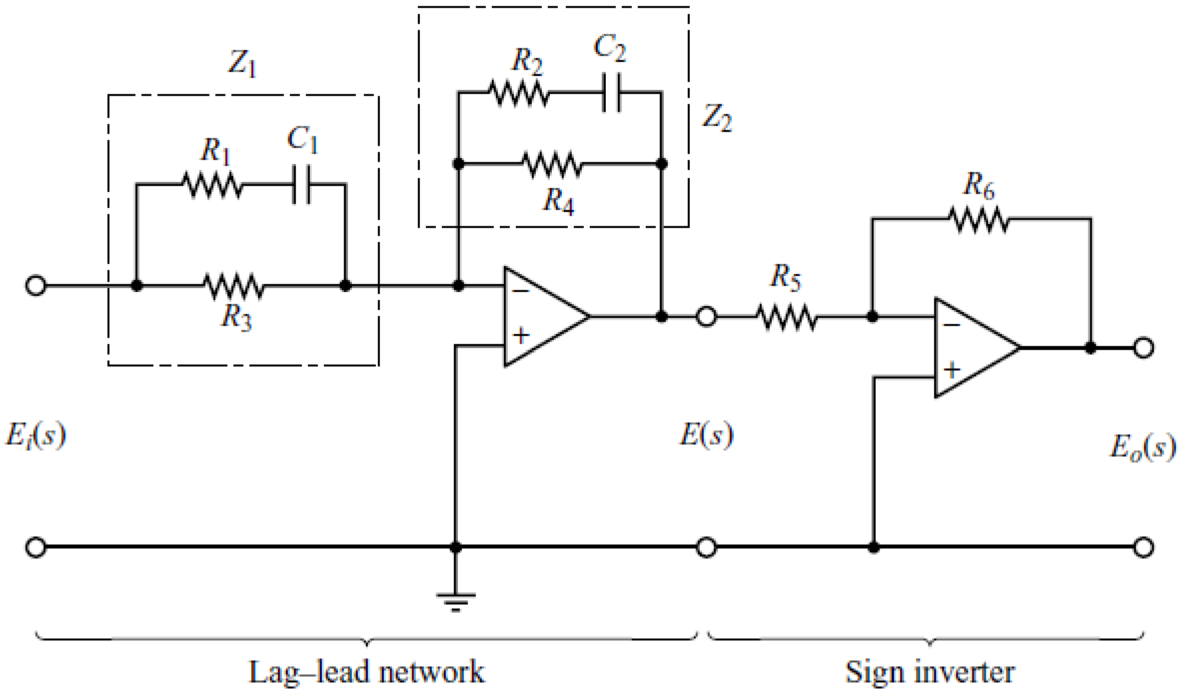


Figure 11-14 Lag–lead compensator.

The transfer function of the compensator shown in Figure 11–14 is

$$\frac{E_o(s)}{E_i(s)} = \frac{E_o(s)}{E(s)}\frac{E(s)}{E_i(s)} = \frac{R_4R_6}{R_3R_5}\left[\frac{(R_1 + R_3)C_1 s + 1}{R_1C_1 s + 1}\right]\left[\frac{R_2C_2 s + 1}{(R_2 + R_4)C_2 s + 1}\right] \quad 11.3$$

Let us define

$$T_1 = (R_1 + R_3)C_1, \qquad \frac{T_1}{\gamma} = R_1C_1, \qquad T_2 = R_2C_2, \qquad \beta T_2 = (R_2 + R_4)C_2$$

Then Equation (11-3) becomes

$$\frac{E_o(s)}{E_i(s)} = K_c\frac{\beta}{\gamma}\left(\frac{T_1s + 1}{\frac{T_1}{\gamma}s + 1}\right)\left(\frac{T_2s + 1}{\beta T_2s + 1}\right) = K_c\frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\gamma}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)} \qquad 11.4$$

Where

$$\gamma = \frac{R_1 + R_3}{R_1} > 1, \qquad \beta = \frac{R_2 + R_4}{R_2} > 1, \qquad K_c = \frac{R_2R_4R_6}{R_1R_3R_5}\frac{R_1 + R_3}{R_2 + R_4}$$

Note that γ is often chosen to be equal to β.

11-4-2 Lag–lead Compensation Techniques Based on the Root-Locus Approach.

Consider the system shown in Figure 11-14. Assume that we use the lag–lead compensator

$$G_c(s) = K_c\frac{\beta}{\gamma}\frac{(T_1s + 1)(T_2s + 1)}{\left(\frac{T_1}{\gamma}s + 1\right)(\beta T_2s + 1)} = K_c\left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}}\right)\left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}}\right) \qquad 11.5$$

where β >1 and γ >1 (Consider *K*c to belong to the lead portion of the lag–lead compensator.)

In designing lag–lead compensators, we consider two cases where γ ≠ β and γ=β

Case 1 γ ≠ β. In this case, the design process is a combination of the design of the lead compensator and that of the lag compensator. The design procedure for the lag–lead compensator follows:

1. From the given performance specifications, determine the desired location for the dominant closed-loop poles.

2. Using the uncompensated open-loop transfer function G(s), determine the angle deficiency ϕ if the dominant closed-loop poles are to be at the desired location. The phase-lead portion of the lag–lead compensator must contribute this angle ϕ.

3. Assuming that we later choose T2 sufficiently large so that the magnitude of the lag portion

$$\left|\frac{s_1+\dfrac{1}{T_2}}{s_1+\dfrac{1}{\beta T_2}}\right|$$

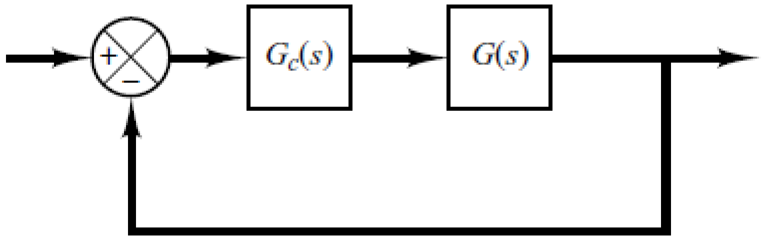


Figure 11–15 Control system.

is approximately unity, $S=S_1$ where is one of the dominant closed-loop poles, choose the values of T1 and γ from the requirement that

$$\angle\frac{s_1+\dfrac{1}{T_1}}{s_1+\dfrac{\gamma}{T_1}}=\phi$$

The choice of T1 and γ is not unique. (Infinitely many sets of T1 and γ are possible.)

Then determine the value of Kc from the magnitude condition:

$$\left|K_c\frac{s_1+\dfrac{1}{T_1}}{s_1+\dfrac{\gamma}{T_1}}G(s_1)\right|=1$$

4. If the static velocity error constant Kv is specified, determine the value of β to satisfy the requirement for Kv .The static velocity error constant Kv is given by

$$\begin{aligned}K_v&=\lim_{s\to 0}sG_c(s)G(s)\\&=\lim_{s\to 0}sK_c\left(\frac{s+\dfrac{1}{T_1}}{s+\dfrac{\gamma}{T_1}}\right)\left(\frac{s+\dfrac{1}{T_2}}{s+\dfrac{1}{\beta T_2}}\right)G(s)\\&=\lim_{s\to 0}sK_c\frac{\beta}{\gamma}G(s)\end{aligned}$$

where Kc and γ are already determined in step 3. Hence, given the value of Kv, the value of β can be determined from this last equation. Then, using the value of β thus determined, choose the value of T2 such that

$$\left| \frac{s_1 + \dfrac{1}{T_2}}{s_1 + \dfrac{1}{\beta T_2}} \right| \doteq 1$$

$$-5^\circ < \angle \frac{s_1 + \dfrac{1}{T_2}}{s_1 + \dfrac{1}{\beta T_2}} < 0^\circ$$

(The preceding design procedure is illustrated in Example 11–3.)

Case 2 β=γ. If β=γ is required in Equation (11–5), then the preceding design procedure for the lag–lead compensator may be modified as follows:

1. From the given performance specifications, determine the desired location for the dominant closed-loop poles

2. The lag–lead compensator given by Equation (11–5) is modified to

$$G_c(s) = K_c \frac{(T_1 s + 1)(T_2 s + 1)}{\left(\dfrac{T_1}{\beta} s + 1\right)(\beta T_2 s + 1)} = K_c \frac{\left(s + \dfrac{1}{T_1}\right)\left(s + \dfrac{1}{T_2}\right)}{\left(s + \dfrac{\beta}{T_1}\right)\left(s + \dfrac{1}{\beta T_2}\right)} \qquad 11.6$$

where β>1. The open-loop transfer function of the compensated system is Gc(s)G(s). If the static velocity error constant Kv is specified, determine the value of constant Kc from the following equation:

$$\begin{aligned} K_v &= \lim_{s \to 0} s G_c(s) G(s) \\ &= \lim_{s \to 0} s K_c G(s) \end{aligned}$$

3. To have the dominant closed-loop poles at the desired location, calculate the angle contribution ϕ needed from the phase-lead portion of the lag–lead compensator.

4. For the lag–lead compensator, we later choose T2 sufficiently large so that

$$\left| \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right|$$

is approximately unity, where $S=S_1$ is one of the dominant closed-loop poles. Determine the values of and β from the magnitude and angle conditions:

$$\left| K_c \left(\frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\beta}{T_1}} \right) G(s_1) \right| = 1$$

$$\angle \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\beta}{T_1}} = \phi$$

5. Using the value of β just determined, choose T2 so that

The value of βT_2 the largest time constant of the lag–lead compensator, should not be too large to be physically realized

EXAMPLE 11-4 Consider the control system shown in Figure 11–16.

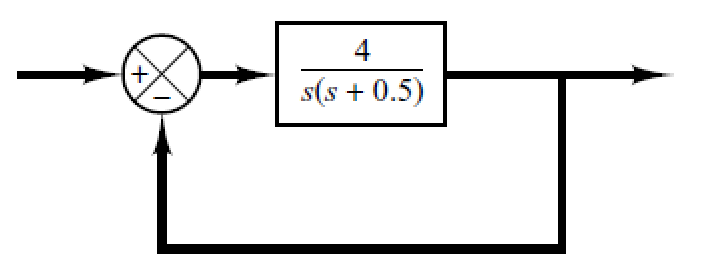


Figure 11-16 Control system.

It is desired to make the damping ratio of the dominant closed-loop poles equal to 0.5 and to increase the undamped natural frequency to 5 rad/sec and the static velocity error constant to 80 sec–1. Design an appropriate compensator to meet all the performance specifications.

Solution

The feedforward transfer function is

$$G(s) = \frac{4}{s(s + 0.5)}$$

This system has closed-loop poles at $s = -0.2500 \pm j1.9843$

The damping ratio is 0.125, the undamped natural frequency is 2 rad/sec, and the static velocity error constant is 8 sec–1.

Let us assume that we use a lag–lead compensator having the transfer function

$$G_c(s) = K_c\left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}}\right)\left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}}\right) \qquad (\gamma > 1, \beta > 1)$$

where γ is not equal to β. Then the compensated system will have the open-loop transfer function

$$G_c(s)G(s) = K_c\left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}}\right)\left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}}\right)G(s)$$

From the performance specifications, the dominant closed-loop poles must be at

$$s = -2.50 \pm j4.33$$

Since

$$\left/\frac{4}{s(s + 0.5)}\right|_{s=-2.50+j4.33} = -235°$$

the phase-lead portion of the lag–lead compensator must contribute 55° so that the root locus passes through the desired location of the dominant closed-loop poles.

To design the phase-lead portion of the compensator, we first determine the location of the zero and pole that will give 55° contribution. There are many possible choices, but we shall here choose the zero at s=–0.5 so that this zero will cancel the pole at s=–0.5 of the plant. Once the zero is chosen, the pole can be located such that the angle contribution is 55°. By simple calculation or graphical analysis, the pole must be located at s=–5.02. Thus, the phase-lead portion of the lag–lead compensator becomes

$$K_c\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} = K_c\frac{s + 0.5}{s + 5.02}$$

Thus

$$T_1 = 2, \qquad \gamma = \frac{5.02}{0.5} = 10.04$$

Next, we determine the value of Kc from the magnitude condition:

$$\left| K_c \frac{s + 0.5}{s + 5.02} \frac{4}{s(s + 0.5)} \right|_{s=-2.5+j4.33} = 1$$

Hence,

$$K_c = \left| \frac{(s + 5.02)s}{4} \right|_{s=-2.5+j4.33} = 6.26$$

The phase-lag portion of the compensator can be designed as follows: First the value of β is determined to satisfy the requirement on the static velocity error constant:

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} sK_c \frac{\beta}{\gamma} G(s)$$

$$= \lim_{s\to 0} s(6.26) \frac{\beta}{10.04} \frac{4}{s(s + 0.5)} = 4.988\beta = 80$$

β = 16.04

Finally, we choose the value T2 such that the following two conditions are satisfied:

$$\left| \frac{s + \frac{1}{T_2}}{s + \frac{1}{16.04T_2}} \right|_{s=-2.5+j4.33} \doteqdot 1, \qquad -5^\circ < \left/ \frac{s + \frac{1}{T_2}}{s + \frac{1}{16.04T_2}} \right|_{s=-2.5+j4.33} < 0^\circ$$

We may choose several values for T2 and check if the magnitude and angle conditions are satisfied. After simple calculations we find for T2 = 5

$$1 > \text{magnitude} > 0.98, \qquad -2.10^\circ < \text{angle} < 0^\circ$$

Now the transfer function of the designed lag–lead compensator is given by

$$G_c(s) = (6.26)\left(\frac{s + \frac{1}{2}}{s + \frac{10.04}{2}} \right)\left(\frac{s + \frac{1}{5}}{s + \frac{1}{16.04 \times 5}} \right)$$

$$= 6.26\left(\frac{s + 0.5}{s + 5.02} \right)\left(\frac{s + 0.2}{s + 0.01247} \right)$$

The compensated system will have the open-loop transfer function

$$G_c(s)G(s) = \frac{25.04(s + 0.2)}{s(s + 5.02)(s + 0.01247)}$$

The characteristic equation for the compensated system is

$$s(s + 5.02)(s + 0.01247) + 25.04(s + 0.2) = 0$$

or

$$s^3 + 5.0325s^2 + 25.1026s + 5.008$$
$$= (s + 2.4123 + j4.2756)(s + 2.4123 - j4.2756)(s + 0.2078) = 0$$

Hence the new closed-loop poles are located at

$$s = -2.4123 \pm j4.2756$$

The new damping ratio is ζ=0. 491.Therefore the compensated system meets all the required performance specifications.

EXAMPLE 11–4 Consider the control system of Example 11–3 again. Suppose that we use a lag–lead compensator of the form given by Equation (11–6), or

$$G_c(s) = K_c \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)} \qquad (\beta > 1)$$

SOLUTION

The desired locations for the dominant closed-loop poles are at s = -2.50±j4.33

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = K_c \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)} \cdot \frac{4}{s(s + 0.5)}$$

Since the requirement on the static velocity error constant Kv is 80 sec^{-1}, we have

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} K_c \frac{4}{0.5} = 8K_c = 80 \qquad K_c=10$$

The time T1 constant and the value of β are determined from

$$\left|\frac{s + \frac{1}{T_1}}{s + \frac{\beta}{T_1}}\right| \left|\frac{40}{s(s + 0.5)}\right|_{s=-2.5+j4.33} = \left|\frac{s + \frac{1}{T_1}}{s + \frac{\beta}{T_1}}\right| \frac{8}{4.77} = 1$$

$$\left. \angle \frac{s + \frac{1}{T_1}}{s + \frac{\beta}{T_1}} \right|_{s=-2.5+j4.33} = 55^\circ$$

(The angle deficie ncy of 55° was obtained in Example 11–3.)

If $\frac{1}{T1}$ =2.38. → $\frac{\beta}{T1}$ = 8.34

$$T_1 = \frac{1}{2.38} = 0.420, \qquad \beta = 8.34T_1 = 3.503$$

The phase-lead portion of the lag–lead network thus becomes

$$10\left(\frac{s + 2.38}{s + 8.34}\right)$$

For the phase-lag portion, we choose T2 such that it satisfies the conditions

$$\left| \frac{s + \frac{1}{T_2}}{s + \frac{1}{3.503T_2}} \right|_{s=-2.50+j4.33} \doteqdot 1, \qquad -5^\circ < \left. \angle \frac{s + \frac{1}{T_2}}{s + \frac{1}{3.503T_2}} \right|_{s=-2.50+j4.33} < 0^\circ$$

if we choose T2=10, then

1 > magnitude > 0.99, -1° < angle < 0°

Since T2 is one of the time constants of the lag–lead compensator, it should not be too large.

$$\frac{1}{\beta T_2} = \frac{1}{3.503 \times 10} = 0.0285$$

Thus, the lag–lead compensator becomes

$$G_c(s) = (10)\left(\frac{s + 2.38}{s + 8.34}\right)\left(\frac{s + 0.1}{s + 0.0285}\right)$$

The compensated system will have the open-loop transfer function

$$G_c(s)G(s) = \frac{40(s + 2.38)(s + 0.1)}{(s + 8.34)(s + 0.0285)s(s + 0.5)}$$