Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class- Semester-2

Control Engineering

Chapter 10

Lecture 4

Nyquist Plot

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10. 1ntroduction

Using Routh-Hurwitz criterion, only the absolute stability of a system can be found out. On the other hand, the Nyquist criterion uses a different approach for finding the stability of a system. It focuses also on the relative stability of the system. It is possible to determine the stability of closed-loop pole from open-loop pole without knowing the roots of the closed-loop system. Nyquist plot is based on polar plot. The aim of this chapter is to introduce the Nyquist plot as well as the stability analysis of the system.

10.1 Basic Definitions

Encircled: If point is found to be inside the path, the point is said to be encircled by the closed path. Figure 10.1 shows that point X is encircled by the closed path where the point Y is not encircled by the closed path.

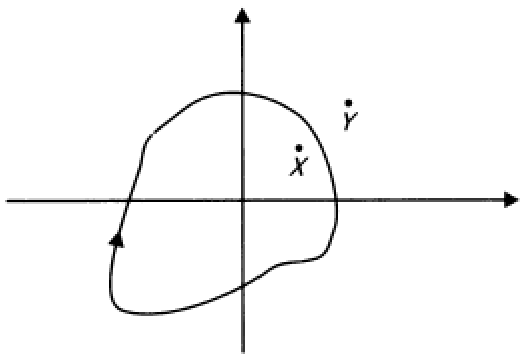

Fig 10.1

Enclosed: If a path is traversed in the clockwise direction and the point is found to lie to the right of the path, the point is said to be enclosed by the path. Figure 10.2(a) shows that point X is enclosed by the path, whereas point Y is enclosed by the path in Figure 10.2(b).

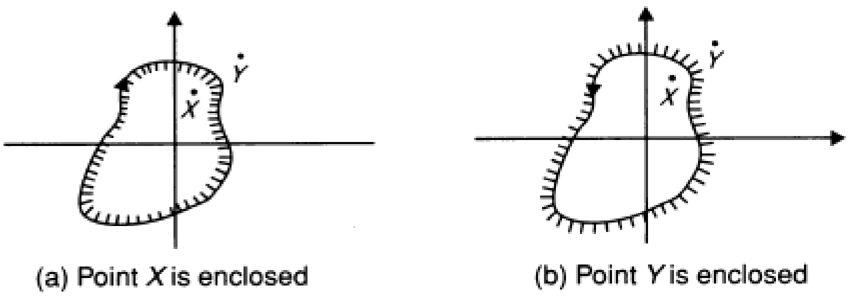

Fig. 10.2 Definition of enclosed

10.2 Nyquist Analysis

10.2.1 Mapping Theorem and the Principle of Argument

Let us consider a function as follows

$$GH(s) = \frac{(s-a)(s-b)\cdots}{(s-a_1)(s-b_1)\cdots}$$

Let us take a path of s in the s-plane for the following cases:

(1) No poles or zeros of GH(s) are covered.

(2) Only zero is covered.

(3) Only pole is covered.

(4) One pole and one zero are covered.

(5) Two, three, etc. zeros are covered

(6) Two, three, etc. poles are covered.

(7) Entire s-plane.

Let us discuss the above cases one by one.

Case 1: No poles and zeros covered

For the right-hand side of the travel direction, the region enclosed by *ABCDEF* and the corresponding region may be *ABCDEF* in the *G(jw)* plane. The origin covered by *ABCDEF* is important rather than its shape.

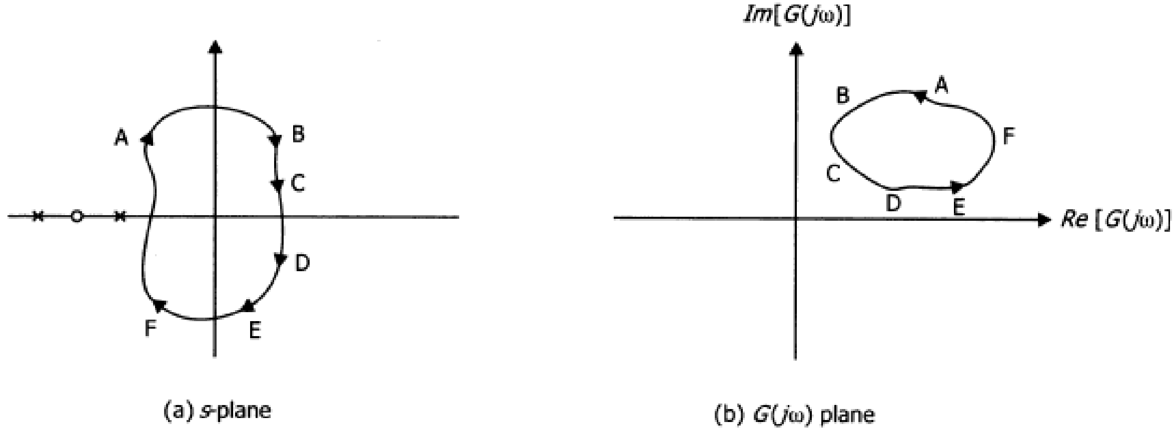

Fig. 10.3 Case l

Case 2: One zero only is covered

The origin is enclosed here only once in the clockwise (CW) direction.

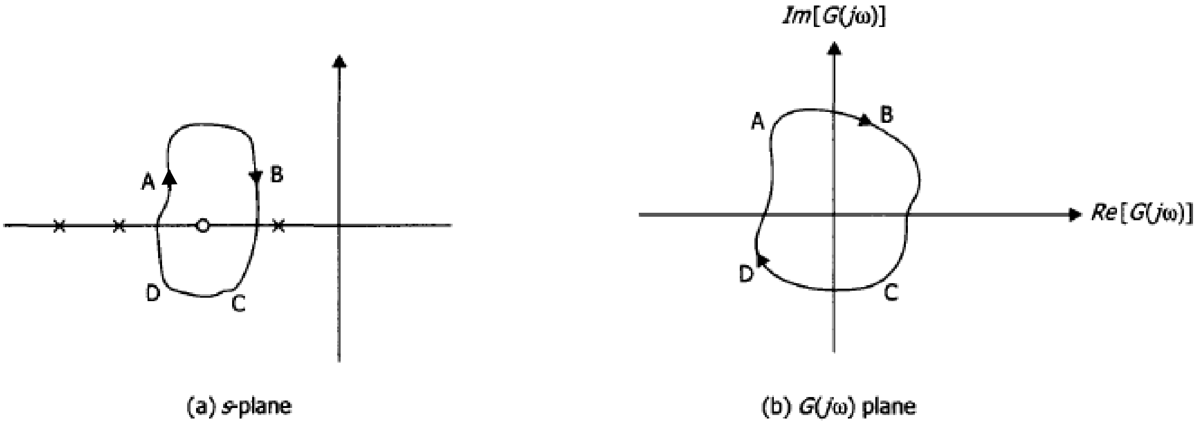

Fig. 10.4 Case 2

Case 3: One pole only is covered

The origin is enclosed here only once in the counter-clockwise (CCW) direction.

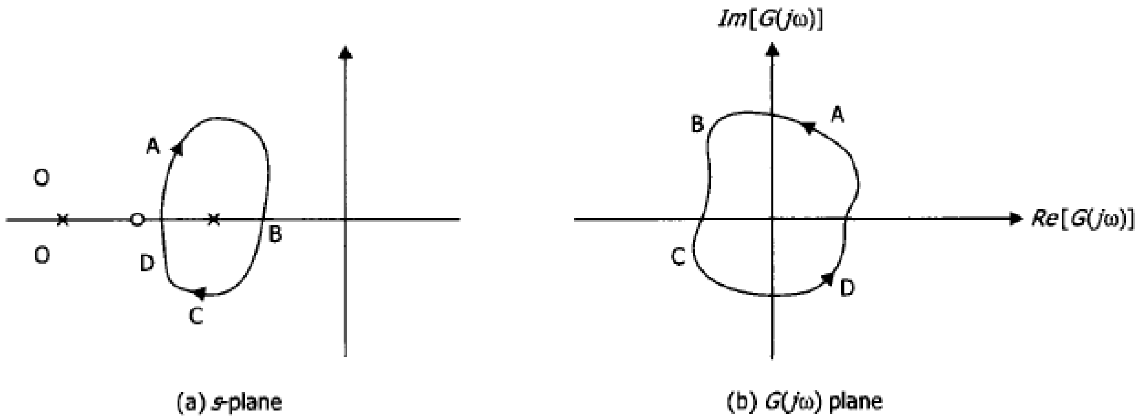

Fig. 15.5 Case 3

Case 4: One pole and one zero is covered

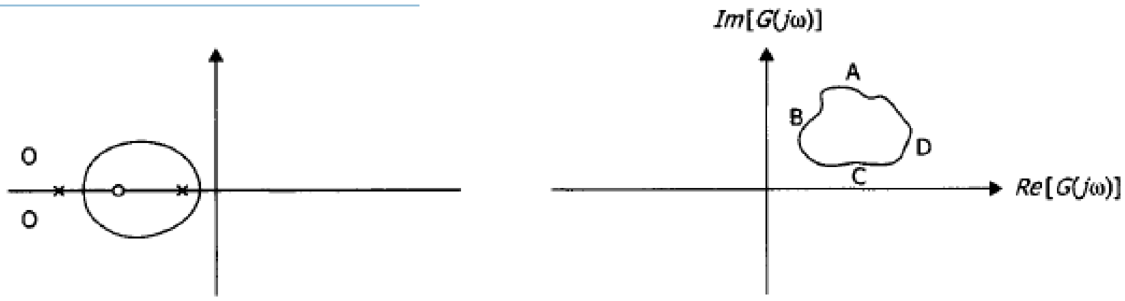

Fig. 10.6 Case 4

Figure 10.6 shows that the origin is not covered here.

Case 5: 2,3, etc. zeros covered

There may be two, three, etc. times the origin is enclosed in the GH(*jw*) plane in the CW direction, i.e., the number of zeros is equal to the number of CW encirclements about the origin.

Case 6: 2,3, etc. poles covered

There may be two, three, etc. times the origin is enclosed in the GH(*jw*) plane in the CCW direction, i.e., the number of poles is equal to the number of CCW encirclements about the origin.

Case 7: Entire s-plane/any region of the s-plane

Suppose we choose any region in the s-plane (or the entire s-plane). If there are Z zeros and P poles, the number of encirclements, N, about the origin is given by

$$N=Z-P \qquad 10.1$$

For Z = 4, P =2, N = 2, 2 CW encirclements

Z = 2, P = 2, N = 0, No encirclements

Z = 2, P = 4, N = - 2 i.e., 2 CCW encirclements

Equation (10.1) is known as the mapping theorem. If F(s) be a ratio of two polynomials of s having P poles and Z zeros in some closed contour in the s-plane, this closed contour must pass through any pole or zero (but it can contain them). This closed contour if mapped into the F(s) plane will be a closed curve so that the total number of clockwise encirclements about the origin is given by

N = Z – P

10.2.2 Application of Mapping Theorem to Stability

In polar plot jw is varied from 0 to ∞, and, in the mapping theorem discussion, we made s to vary in some closed contour. We can apply the above concepts to determine the stability.

$$G(s)H(s) = \frac{N(s)}{D(s)} = \frac{(s-a)(s-b)}{(s-a_1)(s-b_1)}$$

The poles and zeros of the open-loop transfer function are easy to locate. Now the closed-loop transfer function (CLTF) is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

The poles of the CLTF must not lie in the RHP from stability considerations. The closed poles are obtained from

1 + G(s)H(s) = 0

$$q(s) = 1 + G(s)H(s) = \frac{N_1(s)}{D_1(s)} \quad 10.2$$

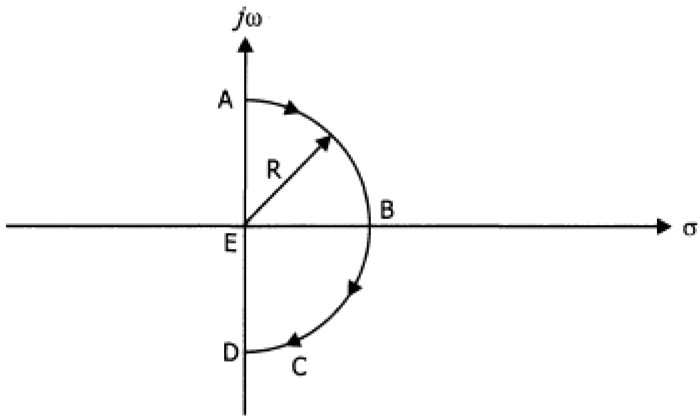

Fig. 10.7 Path covering the RHP of the s-plane (1 + R→∞)

The poles of the closed-loop transfer function are given by q(s) = 0

i.e., $$\frac{N_1(s)}{D_1(s)} = 0$$

i.e., $$N_1(s) = 0 \quad 10.3$$

i.e., zeros o f q(s) are poles o f a CLTF.

Therefore, the zeros of q (s) [i.e., the zeros of 1 + G(s)H(s)] should not lie in the RHP from stability point of view.

For given q(s), let us choose a closed contour such that it covers the entire RHP shown in Fig. 10.7. Plotting the Nyquist plot o f q(s) for such a closed contour, we can conclude whether there is zero in the RHP by examining the number of encirclements about the origin using the mapping theorem. For a given q(s), let us choose the RHP as path ABCDE with limit R—» ∞ so that we can map the RHP.

We can decide the stability after getting the encirclement(s) of the polar plot about the origin. If the contour of q(s) encloses the origin once in the clockwise direction, Z - P = 1. If q(s) has three poles in the RHP, from Z - P = 1 we get Z = 4 or four zeros of q(s) are in the RHP. Here we can conclude the following:

- Take G(s)H(s) as an open-loop transfer function.
- Define poles of CLTF.
- Define a function q(s) whose zeros are the poles of the CLTF.
- Map the RHP of the s-plane in q(s).
- Examine for the number of (zeros-poles) by noting encirclements about the origin.
- If there are any zeros in RHP we have concluded that the system is unstable.

Now for N = Z - P (Z must be zero for stability) there are the following values of N which are possible.

N = 0 (no encirclement)

> 0 (clockwise encirclement, i.e., Z > P)

< 0 (CCW encirclement Z < P)

Case l : If N = 0, either Z = 0 and P = 0 or Z = P.

If N = 0, P must be 0 for stability. This is the first condition.

Case 2: If N > 0, either P = 0 and N = Z o r Z > P.

In both cases the number of zeros are more than poles. It indicates that there are zeros in the RHP given

by Z = N + P. This is an unstable condition.

Case 3: If N < 0, we can conclude that either Z = 0 so N = - P or N = Z - P with P > Z. I f N < 0, for stability the number of encirclements must be equal to the number of poles. This is another condition. A given system will be stable if

- There are no clockwise encirclements about the origin and the number of poles in the RHP is zero.
- There are anti-clockwise encirclement about the origin with the number of poles equal to the number of encirclements.

10.3 Polar Plots of G(S) H(S) and Stability

From Eq. (10.2) we have

q(s) = 1 + G(s)H(s)

when $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Let us have the polar plot of the RHP of the s-plane for G(s)H(s). For any zero(s) in the RHP we get from encirclement(s) around the origin. Modifying the function to 1 + G(s)H(s) and instead of redrawing the polar plot of q(s) = 1 + G(s)H(s), we can examine the encirclement from the polar plot of 1 + G(s)H(s) = 0

i.e.,

G(s)H(s) = -1 (10.4)

i.e., encirclement about (-1, 0) instead of (0, 0). For a given G(s)H(s) the stability of an open-loop transfer function system can be carried out by

(i) choosing a Nyquist path that maps RHP and

(ii) plotting corresponding polar plot o f G(s)H(s). After getting the encirclements about (-1, 0) say N', in order to distinguish it from N which is the encirclement about (0, 0), where N ' - Z - P where Z is zeros of 1 + G(s)H(s) and P is poles o f G(s)H(s) as well as of 1 + G(s)H(s).

10.4 Nyquist Path

This path should not pass through a pole/zero, but it may contain poles and zeros. The path must map the RHP. Figure 10.8 shows a general Nyquist path based on these constraints.

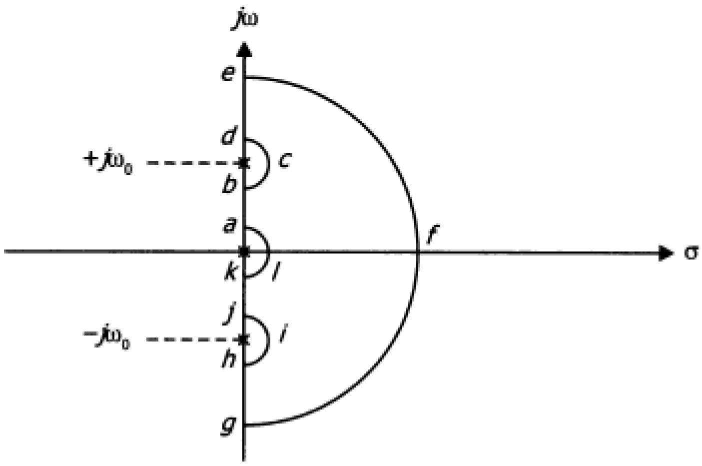

Fig. 10.8 Nyquist path

Table 10.1 gives the description of Fig. 10.8.

Table 10.1 Description of Fig. 10.8

Path	Equation	Equation No.	Limits
ab	$s = j\omega$	N_1	$0 < \omega < \omega_0$
bcd	$s = \underset{\rho \to 0}{\text{Lt}}(\rho e^{j\theta} + j\omega_0)$	N_2	$-90° < \theta < 90°$
be	$s = j\omega$	N_3	$\omega_0 < \omega < \infty$
efg	$s = \underset{R \to \infty}{\text{Lt}}(Re^{j\theta})$	N_4	$+90° < \theta < -90°$
gh	$s = j\omega$	N_5	$-\infty < \omega < \omega_0$
hij	$\underset{\rho \to 0}{\text{Lt}}(\rho e^{j\theta} - j\omega_0)$	N_6	$-90° < \theta < 90°$
jk	$s = j\omega$	N_7	$\omega_0 < \omega < 0$
kla	$\underset{\rho \to 0}{\text{Lt}}(e^{j\theta})$	N_8	$-90° < \theta < 90°$

15.4.1 Nyquist Stability Criterion

A closed-loop control system having an open-loop transfer function G(s)H(s) will be stable if and only if

N = - P

where

N = Number of counter-clockwise encirclements about (-1, 0) point in the G(s)H(s) plane

P = Number of poles G(s)H(s) in the RHP.

If N > 0, the system is unstable with the number of zeros of 1 + G(s)H(s) in RHP Z = N + P

If N< 0, i.e., (-1, 0) is not enclosed, the system is stable only for N = 0 and P = 0 or N = - P

Example 10.1 For G(s)H(s) = 1/ (S + 2) draw the Nyquist plot and decide stability.

Solution

Step 1: Pole of G(s)H(s) is at s = -2 and there are no poles either on the jw axis or on the origin

Step 2: The Nyquist path is shown in Fig. E10.1.

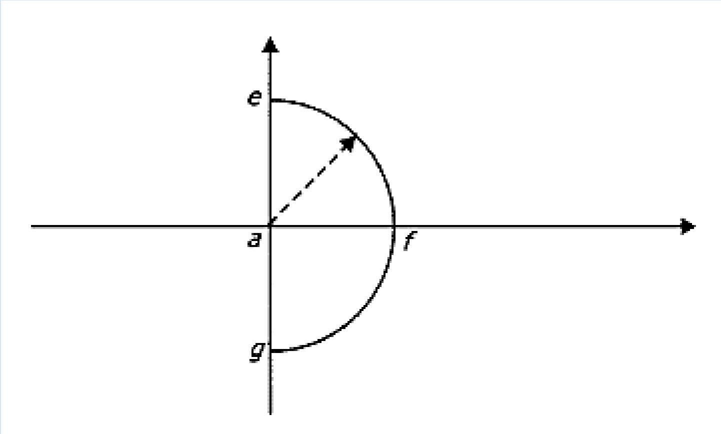

Fig. E10.1

For path ae s = jw where 0 < jw < ∞

$$G(j\omega)H(j\omega) = \frac{1}{j\omega+2} = \frac{1}{\sqrt{4+\omega^2}}\angle -\tan^{-1}\left(\frac{\omega}{2}\right)$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \frac{1}{\sqrt{4+\omega^2}} \text{ and } \angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\frac{\omega}{2}\right)$$

At $\omega = 0$, $|G(j\omega)H(j\omega)| = 0.5$ and $\angle G(j\omega)H(j\omega) = 0°$

At $\omega = \infty$, $|G(j\omega)H(j\omega)| = 0$ and $\angle G(j\omega)H(j\omega) = -90°$

The polar plot o f path *ae* is shown in Figure E10.1(a).

Step 3: The dotted mirror path is of *ga*.

Step 4: For path *efg*, $s = Re^{j\theta}$> where R—» ∞ and + **90° < θ< - 90°.**

$$G(s)H(s)\Big|_{efg} = \frac{1}{\underset{R\to\infty}{\text{Lt}}\,(Re^{j\theta}+1)} = 0$$

Therefore, the infinite semicircle efg maps onto a point.

Step 5: Connected

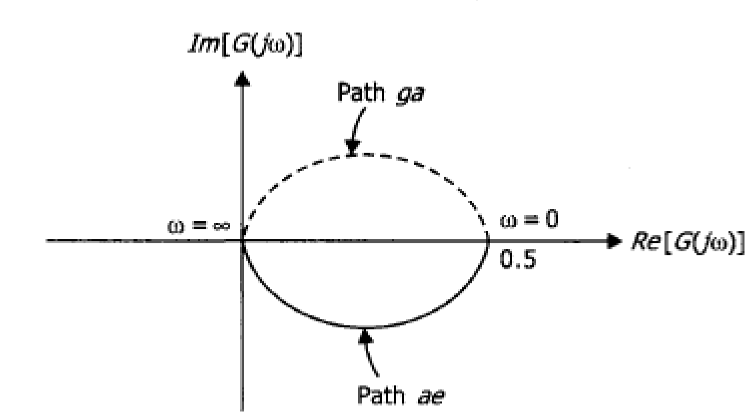

Fig E 10.1a

Step 6: There are no encirclements about (-1, 0).

Step 7: Since P = 0 (number of poles in the RHP) = Poles of G(s)H(s) here the number of zeros of 1+G(s)H(s) in the RHP is zero.

Z = 0, and hence system is stable.

10.6 Relative Stability

We have introduced the Nyquist criterion for the absolute stability analysis of the system. Using the Nyquist criterion, it is also possible to find the relative stability of the system. By relative stability we mean how close the system is to instability, and we can improve the stability of the system. The degree or extent of the system is called relative stability.

I f the Nyquist polar plot is close to -1 + j0 point, the system is on the verge of the instability. The proximity to -1 + j0 point is specified in terms of the following two quantities:

(i) Gain margin and (ii) Phase margin

10.6.1 Gain Margin

The gain margin is defined as the reciprocal of the open-loop transfer function evaluated at the frequency (w_{pc}) at which the phase angle is -180°.

$$\therefore \qquad \text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|}$$

and

$$\phi = \angle G(j\omega)H(j\omega).$$

The frequency w_{pc} is known as the phase crossover frequency at which the polar plot crosses the negative real axis. Gain margin measures the relative distance between the -1 + j0 point and the G(j(o)H(j(o) plot.

Depending upon the phase crossover point X shown in Fig. 10.8, we set the gain. If the point X is too near the -1 + j0 point, we can decide how much to reduce the gain and if the point X is too far from the -1 + j0 point, we can decide how much to increase the gain.

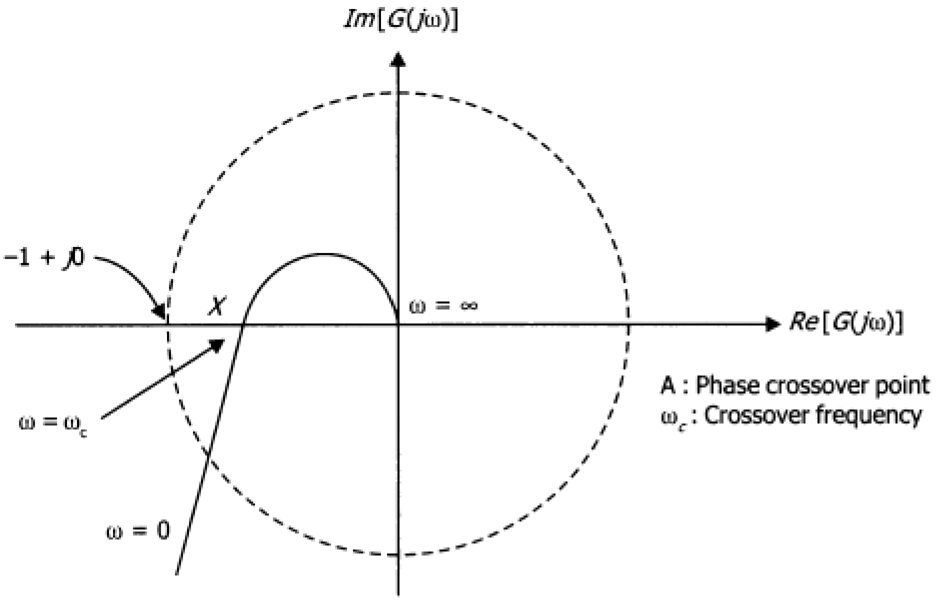

Fig 10.8 gain margin

vector is │G(jw)H (jw)│. The gain crossover frequency is the frequency at which │G(jw)H (jw)│ = 1, i.e., the point of intersection of the polar plot and the (- l, j0) circle. Phase margin = (180° + Φ). where Φ= ∠G(jw)H (jw) and │G(jw)H (jw)│ = 1.

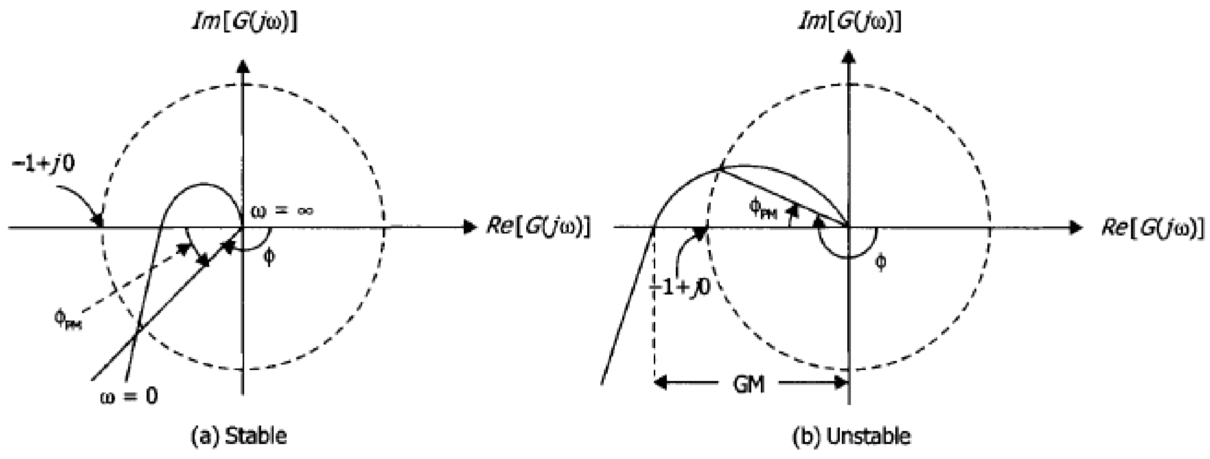

Fig. 10.9 Phase margin

Here we are measuring ϕ in the clockwise direction, hence it is negative. For stability, ϕ_{PM} must be positive as shown in Fig. 10.9(a). Therefore, ϕ is less than 180°. If ϕ_{PM} is negative, ϕ is greater than 180° and the system is unstable shown Fig. 10.9(b).

Example 10.2 For G(s)H(s) = 30/[(s + 3) (s^2 + 2s + 2)], draw the Nyquist plot and decide the stability.

Solution

Step 1: Poles of G(s)H(s) are at s = -3 and s = - 1 ± j. There is no pole at the origin and on the jw axis

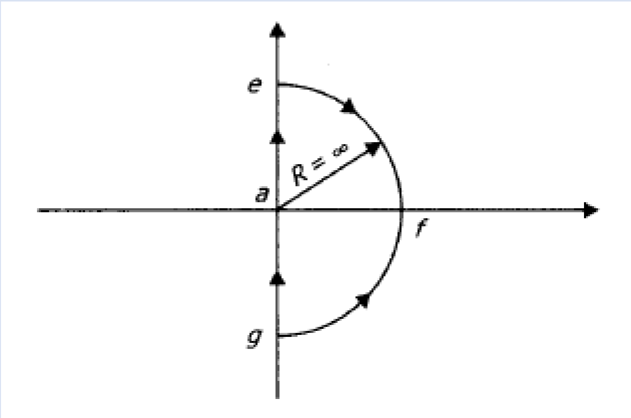

Fig. E10.2

Step 2: The Nyquist path is shown in Fig. E10.8.

For path ae

s = jw and 0 < jw < ∞

$$G(j\omega)H(j\omega) = \frac{20}{(j\omega+3)(j^2\omega^2+2j\omega+2)} = \frac{20}{(j\omega+3)(-\omega^2+2j\omega+2)}$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \frac{20}{\sqrt{\omega^2+9}\sqrt{(2-\omega^2)+4\omega^2}}$$

$$\text{and} \quad \angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{3} - \tan^{-1}\left(\frac{2\omega}{2-\omega^2}\right) = -\tan^{-1}\frac{\omega}{3} - 2\tan^{-1}\omega$$

At $\omega = 0,\ |G(j\omega)H(j\omega)| = \frac{10}{3}$ and $\angle G(j\omega)H(j\omega) = 0°$

At $\omega = \infty,\ |G(j\omega)H(j\omega)| = 0$ and $\angle G(j\omega)H(j\omega) = -270°$

The polar plot of path ae is shown in Fig. El0.8(a).

Step 3: The dotted mirror path is of ga.

Step 4: For path efg S= $Re^{j\theta}$

where R —> ∞and + 90° < θ < - 90°.

$$G(s)H(s)\Big|_{efg} = \frac{20}{\underset{R\to\infty}{\text{Lt}}\ (Re^{j\theta}+3)\ [(Re^{j\theta})^2 + Re^{j\theta}+2]} = 0$$

Therefore, the infinite semicircle *efg* maps onto a point.

Step 5: Connected

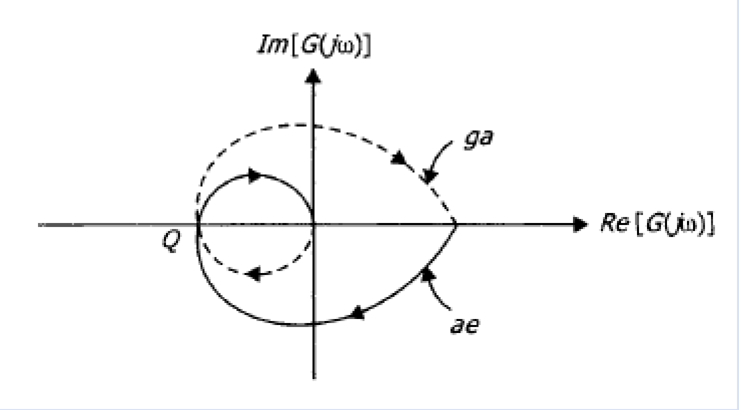

Fig. E10.8(a)

Step 6: To get Q o f Fig. El0.8(a),

$$G(j\omega)H(j\omega) = \frac{20}{(j\omega+3)(2-\omega^2+2j\omega)} = \frac{20(3-j\omega)(2-\omega^2-2j\omega)}{(\omega^2+9)[(2-\omega^2)^2+4\omega^2]}$$

$$= \frac{20(6-7\omega^2)}{(\omega^2+9)[(2-\omega^2)^2+4\omega^2]} + j\frac{20(\omega^2-8)}{(\omega^2+9)[(2-\omega^2)^2+4\omega^2]}$$

Equating the imaginary part to zero, we get $\omega = 2\sqrt{2}$.

∴ Point $$Q = \frac{20(6-7\times 8)}{(8+9)[(2-8)^2+4\times 8]} = -0.865$$

The point (–1, 0) is outside the plot. Hence there is no encirclement about the origin and $N = 0$.

Step 7: Since $P = 0$ (number of poles in the RHP) = Poles of $G(s)H(s)$, here the number of zeros of $1 + G(s)H(s)$ in the RHP is 0,

∴ $Z = 0$ and hence the system is stable.

(ii) Gain margin

$$\text{GM} = \frac{1}{|OQ|} = \frac{1}{0.865} = 1.156$$

$$= 20\log\frac{1}{|OQ|} = 20\log 1.156 = +1.259\ \text{dB}$$

Example 10.3 For G(s) = 1 /[s(s - 2)], sketch the Nyquist plot and determine the stability of the system.

Solution

Step 1: Poles of G(s)H(s) are at s = 0, s = 2. There is a pole at the origin and no pole on the jw axis.

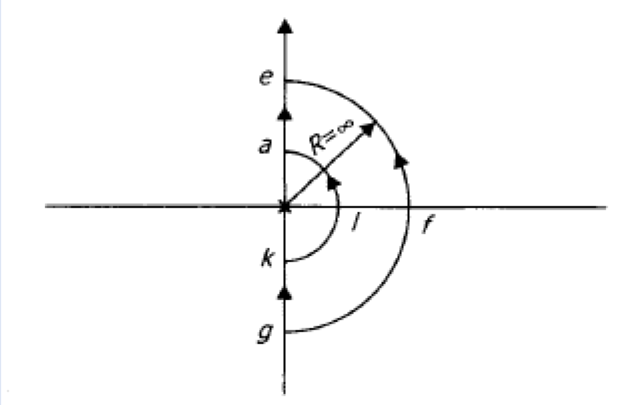

Fig. E10.3

Step 2: The Nyquist path is shown in Fig. E10.3.

For path *ae*

s = jw and 0 < jw < ∞

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(j\omega-2)} = \frac{1}{\omega\sqrt{\omega^2+4}}\angle\left[-90^\circ + \tan^{-1}\left(-\frac{\omega}{2}\right)\right]$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \frac{1}{\omega\sqrt{\omega^2+4}} \text{ and } \angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\left(-\frac{\omega}{2}\right)$$

At $\omega = 0,\ |G(j\omega)H(j\omega)| = \infty$ and $\angle G(j\omega)H(j\omega) = 90^\circ$

At $\omega = \infty,\ |G(j\omega)H(j\omega)| = 0$ and $\angle G(j\omega)H(j\omega) = 180^\circ$

The polar plot of path ae is shown in Fig. E10.3(a).

Step 3: The dotted mirror path is of gk.

Step 4: For path efg

$S = Re^{j\theta}$

where R —> ∞ and + 90° < θ < - 90°.

$$G(s)H(s)\big|_{efg} = \frac{1}{\underset{R\to\infty}{\text{Lt}}\ Re^{j\theta}(Re^{j\theta}-1)} = 0$$

Therefore, the infinite semicircle efg maps onto a point.

For path kla

$$s = \rho e^{j\theta}$$

where $\rho \to \infty$ and $-90° < \theta < +90°$.

$$G(s)H(s)\Big|_{kla} = \frac{1}{\underset{\rho \to 0}{\text{Lt}}\ \rho e^{j\theta}(\rho e^{j\theta} - 1)} = \infty\, \rho e^{-j\theta}$$

Therefore, the phase angle of G(s)H(s)\kla varies from + 90° to - 90°, since 0 varies from - 90° to +90

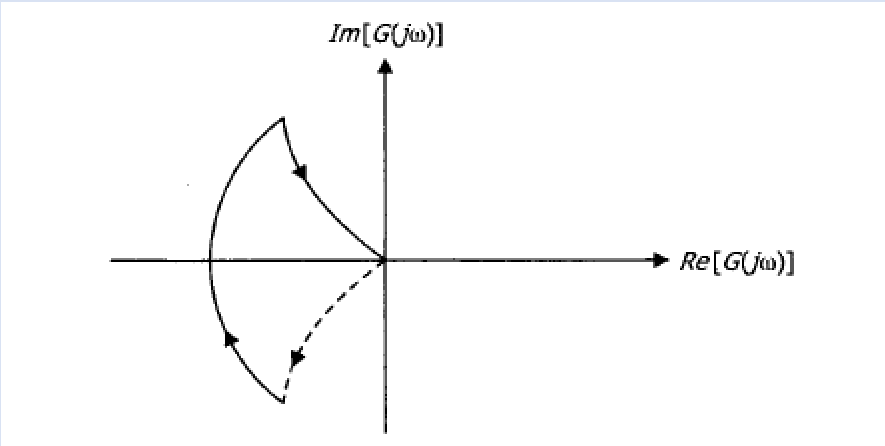

Fig. E10.3(a)

Step 5: Connected

Step 6: There are encirclements about (-1 ,0) and N =1.

Step 7: Since P =1 (number o f poles in the RHP) = Poles of G(s)H(s).

Here the number of zeros of (1+ G(s)H(s)) in the RHP = 2, and hence the system is unstable.