Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class

Control Engineering

Chapter 3

Lecture 8

Block diagram

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3.1. DEFINITION OF BASIC ELEMENTS OF BLOCK DIAGRAM

Block diagram: The shorthand pictorial representation of the cause-and-effect relationship between the input and output of a physical system is known as block diagram. Figure 6.1 shows the representation of a block diagram.

Fig. 3.1 Block diagram

Output: The value of input multiplied by the block gain is known as output. From Fig. 3.1(b)

C(s) = G(s)R(s) (3.1)

Summing point: At summing point, two or more signals can be added or subtracted. Figure 3.2 shows a summing point.

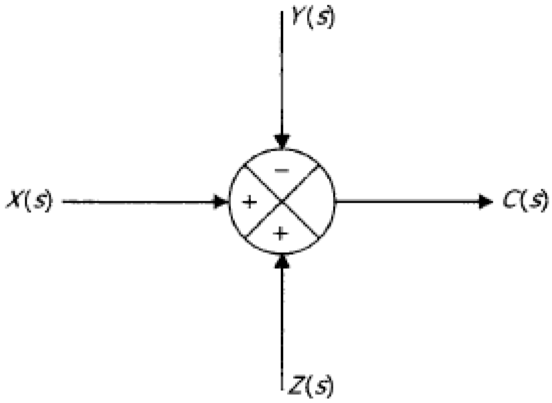


Fig 3.2 summing point

In Fig. 3.2, X(s), F(s), and ***Z(s)*** are the inputs while CXs) is the output. From Fig. 3.2, it can be written as

***C(s)* = *X(s)* - *Y(s)* + *Z(s*)** (3.2)

Take off point: The point at which the output signal of any block can be applied to two or more points is known as take-off point. Figure 3.3 shows a take-off point. The output signal is analogous to voltage not the current.

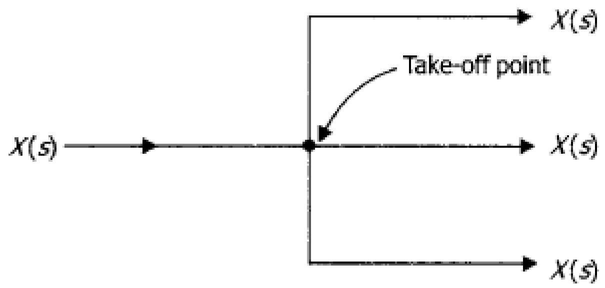


Fig 3.3 take-off point

Forward path: The direction of flow of signal from input to output is known as forward path. Figure 3.4 shows a forward path. Individual block gains are G{(s), G2(s), and G3(s) in Fig. 3.4 from input to output of the system.

Fig 3.4 forward path

Feedback path: The direction of flow of signal from output to input is known as feedback path. Figure 3.5 shows the feedback path.

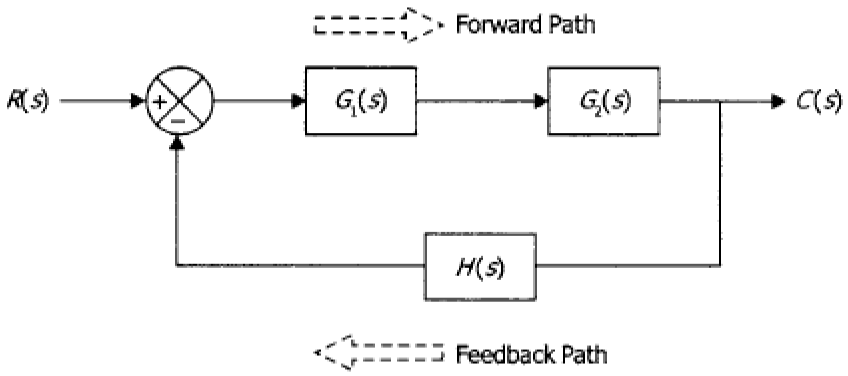


Fig 3.5 feedback path

3.2 CANONICAL FORM OF CLOSED LOOP SYSTEM

Figure 3.6 shows a block diagram which consists of a forward path having one block, a feedback path having one block, a take-off point, and a summing point. It represents a canonical form of a closed-loop system.

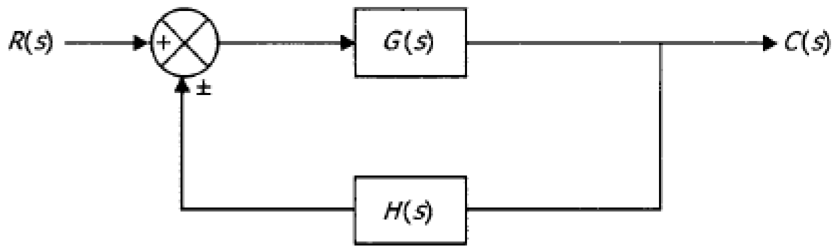


Fig 3.6 canonical form of a closed-loop system

In Fig. 3.6 R(s) is Laplace transform of reference input r(t)\ CO) is Laplace transform of controlled output c(t)\ E(s) is Laplace transform of error signal e(t)\ B(s) is Laplace transform of feedback signal b{t)\ C(s) is equivalent forward path transfer function; H(s) is equivalent feedback path transfer function.

3.3 RULES FOR BLOCK DIAGRAM REDUCTION

Any complicated system can be brought into simple form by reduction of block diagram. The following rules are used in block diagram reduction.

Rule 1: Associative law

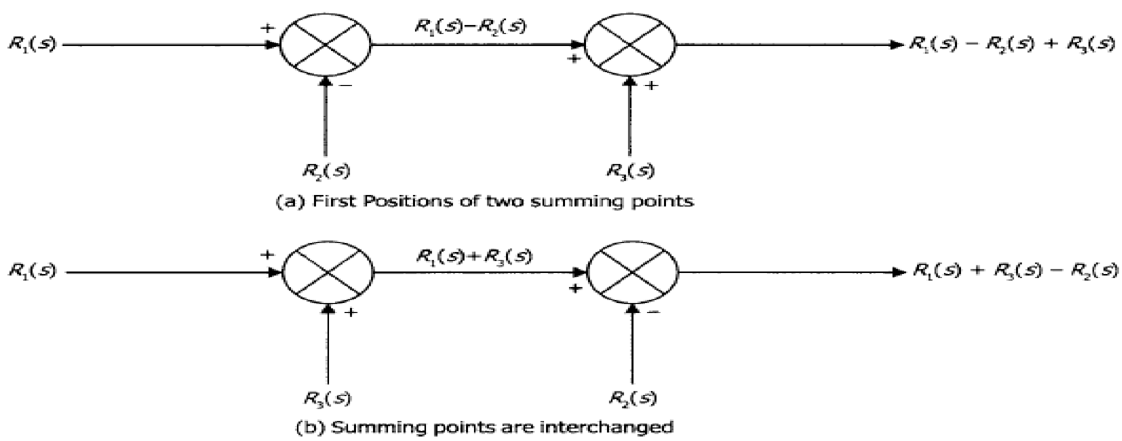


Fig. 3.7 Associative law

In Fig. 3.7(a) two summing points have been taken into account. In the first case, the output is Rx(s) - R 2{s) + R 3(s). Figure 3.7(b), the positions of summing points are interchanged. The output is R{(s) +R 3(s) - R 2(s). From Figs. 3.7(a) and 3.7 (b), we have

R { s) - R 2(s) + R 3(s) = R {(s) + R 3(s) - R 2(s) If any block is present in between the summing points, by interchanging the summing points, it can be shown that output will not be equal.

Rule 2 For blocks in cascade

Figure 3.8 shows that the three blocks are in cascade.

Fig 3.8 Blocks in cascade

From Fig. 6.8, we can write

$$R_1(s) = G_1(s)R(s)$$

$$R_2(s) = G_2(s)R_1(s) = G_1(s)\, G_2(s)R(s)$$

And $$C(s) = G_3(s)R_2(s) = G_1(s)\, G_2(s)\, G_3(s)R(s)$$

Transfer function of the system =

$$\frac{C(s}{R(s)} = G_1(s)\, G_2(s)\, G_3(s)$$

The equivalent block of Fig. 3.8 is shown in Fig. 3.9.

Fig. 3.9 Equivalent block of Fig. 3.8

The output in both cases are identical. In this case gains are multiplied.

Example 3.1 Determine the ratio C(s)/R(s) of the block diagram shown in Fig. E3.1.

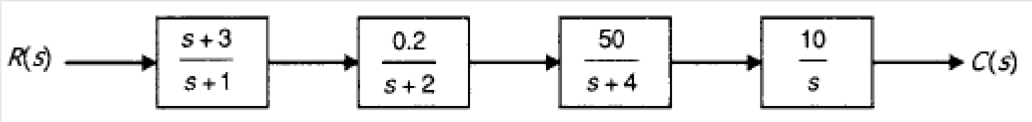


Fig. E3.1

Solution

The blocks in Fig. E3.1 are connected in cascade. The overall transfer function is given by

$$\frac{C(s)}{R(s)} = \left(\frac{s+3}{s+1}\right)\left(\frac{0.2}{s+2}\right)\left(\frac{50}{s+4}\right)\left(\frac{10}{s}\right) = \frac{100(s+3)}{s(s+1)(s+2)(s+4)}$$

Rule 3: For blocks in parallel

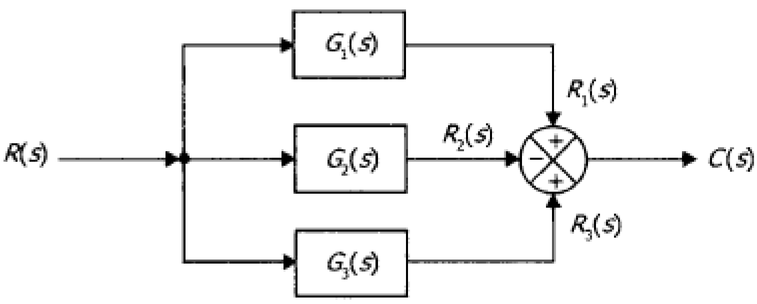


Fig. 3.10 Parallel blocks

The three blocks shown in Fig. 6.10 are in parallel.

$C(s) = R_1(s) - R_2(s) + R_3(s) = G_1(s)\ R(s) - G_2(s)\ R(s) + G_3(s)\ R(s)$

$= [G_1(s) - G_2(s) + G_3(s)]\ R(s)$

Therefore, the transfer function of the system is given by

$$\frac{C(s}{R(s)} = G_1(s) - G_2(s) + G_3(s)$$

The equivalent block of Fig. 3.10 is shown in Fig. 3.11

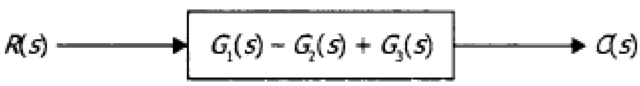


Fig. 3.11 Equivalent block of Fig. 3.10

In these cases, gains are added algebraically. This rule cannot be applied directly if a take-off occurs as shown in Fig. 3.12.

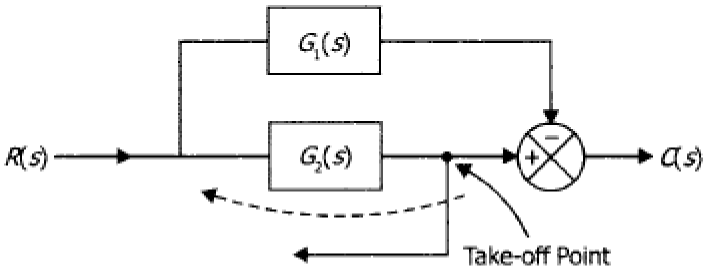


Fig. 3.12 Take-off point in between a block and a summing point

It is always advisable to shift the take-off point prior to the block G2(s) as shown by the dotted line arrow. It is always better to avoid shifting the take-off point after the summing point.

Example 3.2 Determine the ratio ***C(s)/R(s)*** of the block diagram shown in Fig. E3.2.

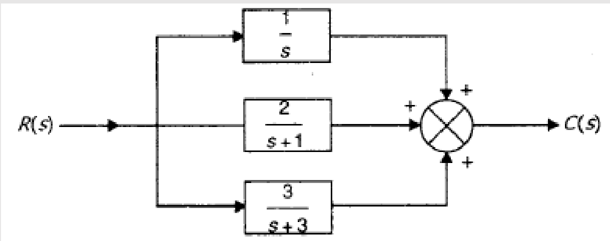


Fig. E3.2

Solution

The blocks in Fig. E3.2 are connected in parallel. The overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{1}{s} + \frac{2}{s+1} + \frac{3}{s+3} = \frac{(s+1)(s+3) + 2s(s+3) + 3s(s+1)}{s(s+1)(s+3)}$$

$$= \frac{s^2 + 4s + 3 + 2s^2 + 6s + 3s^2 + 3s}{s(s+1)(s+3)} = \frac{6s^2 + 13s + 3}{s(s+1)(s+3)}$$

Rule 4: Eliminate feedback loop

Figure 3.13 shows a feedback control.

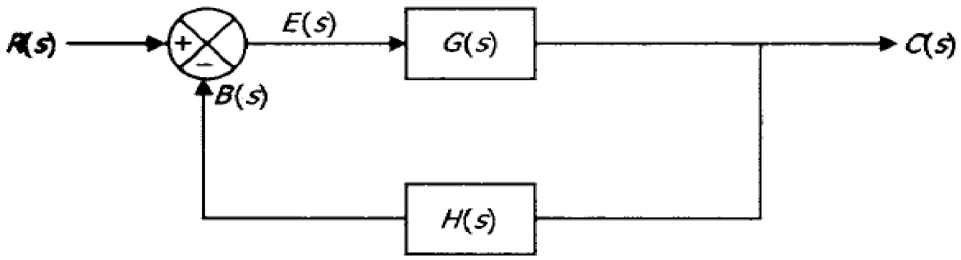


Fig. 3.13 Feedback control system

In Fig. 3.13, E(s) = R(s) - B(s)

C(s) = G(s)E(s) = G(s)[R(s) - B(s)] = G(s) R(s) - G(s) B(s)

= G(s)R(s) - G(s)H(s)C(s) [.-. B(s) = H(s)C(s)]

= C(s) [l + G(s) H(s)] = G(s)R(s)

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad (3.3)$$

Equation (3.3) is valid for negative feedback.

Similarly, for positive feedback we will get

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)} \qquad (3.4)$$

In general, we have

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)} \qquad \begin{bmatrix} + \text{ for negative feed back and} \\ - \text{ for positive feedback} \end{bmatrix} \qquad (3.5)$$

Example 3.3 Determine the ratio ***C(s)/R(s)*** of the block diagram shown in of Fig. E3.3. Also find the characteristic equation.

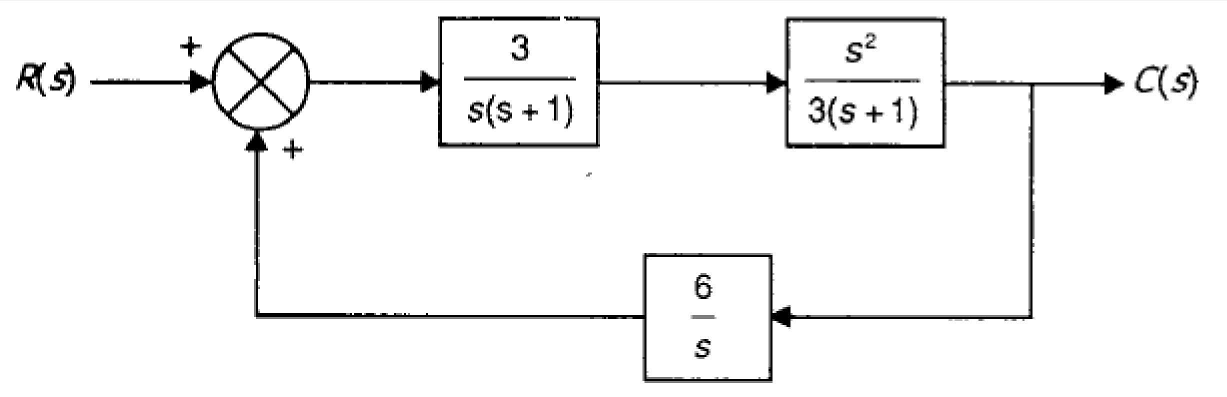


Fig. E3.3

Solution

Here $$G(s) = \left[\frac{3}{s(s+1)}\right]\left[\frac{s^2}{3(s+2)}\right] = \frac{s}{(s+1)(s+2)}$$

and $$H(s) = \frac{6}{s}$$

Now $$G(s)H(s) = \frac{6}{(s+1)(s+2)}$$

Since the feedback is positive, the overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)} = \frac{\frac{s}{(s+1)(s+2)}}{1-\frac{6}{(s+1)(s+2)}} = \frac{s}{(s+1)(s+2)-6} = \frac{s}{s^2+3s-4}$$

Rule 5: Shifting a summing point before a block

Figure 3.14(a) shows a system in which summing point needs to be shifted before ***G*** as shown by the **dotted** line in Fig. 3.14.

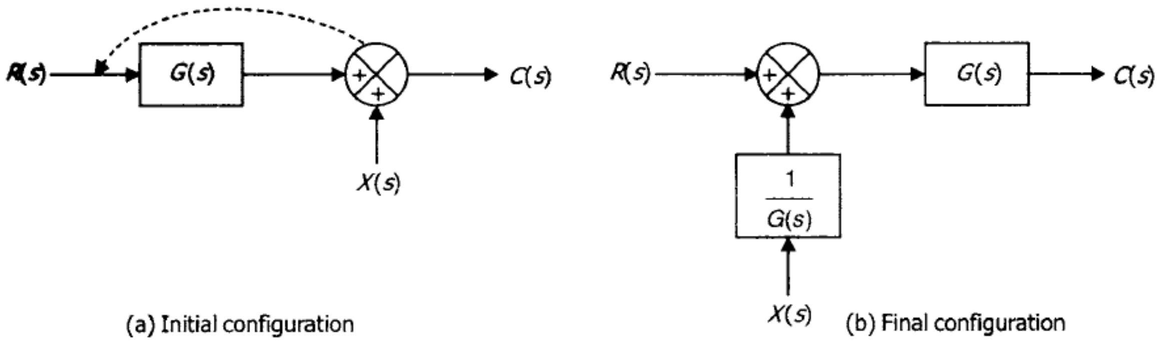


Fig. 3.14 Shifting of summing point before a block

From Fig. 3.14(a), C(s) = G(s)R(s)+X(s)

From Fig. 3.14(b), $G(s) = G(s)\left[R(s) + \frac{X(s)}{G(s)}\right] = G(s)R(s) + X(s)$

Therefore, the output in both cases are identical.

Rule 6: Shifting of summing point after a block

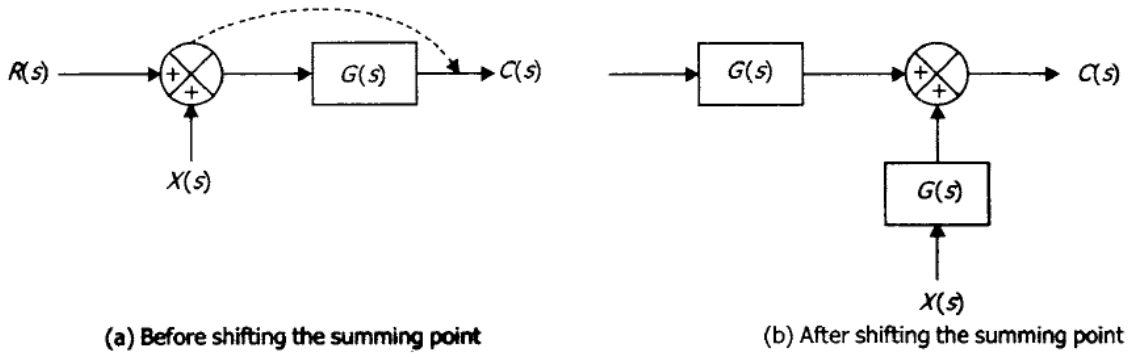


Fig. 3.15 Shifting of summing point after block

From Fig. 3.15(a), ***C(s)= G(s)[R(s) + X(s)]***

From Fig. 3.15(b), ***C(s) = G(s) R(s)+ G(s)X (s)***

Therefore, the output in both cases are same.

Rule 7: Shifting of take-off point before a block

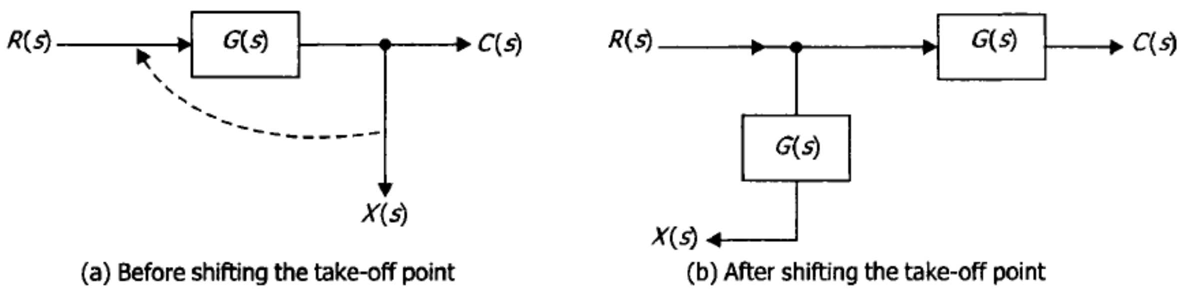


Fig. 3.16 Shifting of take-off point before a block

From Fig. 3.15(a), X(s) = C(s) = G(s)R(s)

From Fig. 3.16(b), X(s) = G(s)R(s)

Rule 8: Shifting of take-off point after a block

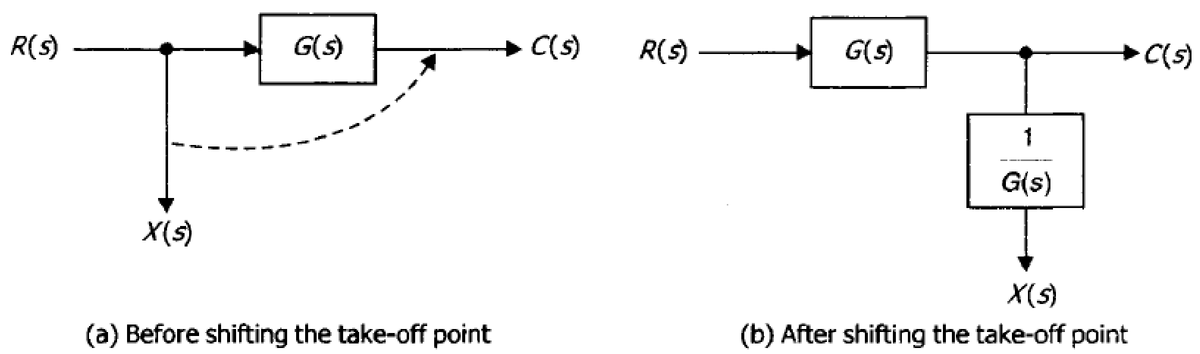


Fig. 3.17 Shifting of take-off point before a block

From Fig. 3.17(a), X(s) = R(s)

From Fig. 3.17(b), $X(s) = \frac{1}{G(s)} C(s) = \frac{1}{G(s)} G(s) R(s) = R(s)$

Therefore, X(s) in both cases are same.

Rule 9: Shifting of take-off point before summing block

Figure 3.18 (a) shows the initial situation of the take-off point. In this case, X(s) = ***R(s)*** ± y(s)

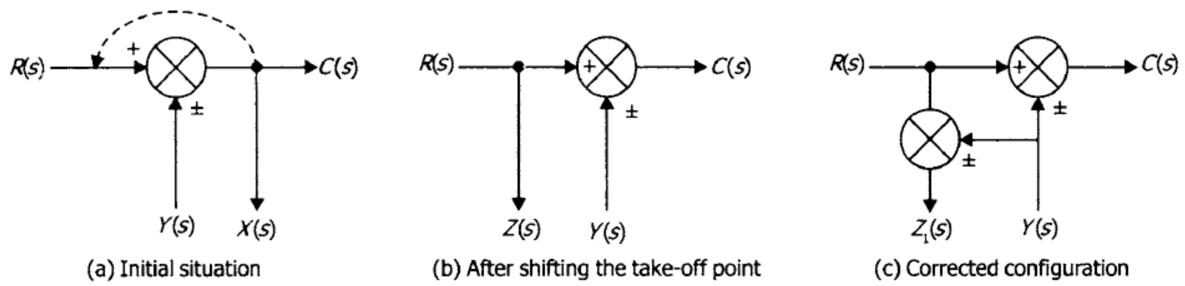


Fig. 3.18 Shifting of take-off point before a summing block

From Fig. 3.18(b) it can be written *Z(s) =R(s).* Therefore, *Z(s)≠X(s).* In Fig. 3.18(c), the signal *Y(s)* is added to *Z(s)* by summing point with the same sign. In this case

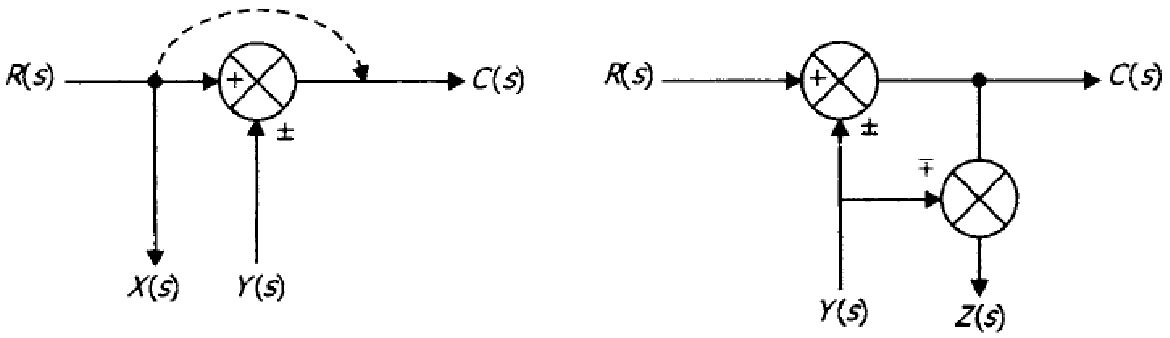
Z(s) = R(s)±Y(s)

X(s) = Z(s)

Therefore, Fig. 3.18(c) is equivalent to Fig. 3.18(a).

Rule 10: Shifting of take-off point after summing block

Figure 3.19(a) shows the initial situation of the take-off point. In this case, X(s) = R(s).



(a) Initial Situation

(b) Final configuration after shifting the take-off point

Fig. 3.19 Shifting of take-off point after a summing block

From Fig. 3.18(a), *X(s) = R(s)*

From Fig. 3.18(b), Z(s) = R(s) + – Y(s) – +Y(s) = R(s)

Therefore, Fig. 3.18(b) is equivalent to Fig. 3.18(a).

3.4 PROCEDURE FOR REDUCTION OF BLOCK DIAGRAM

Step 1: Reduce the cascade blocks.

Step 2: Reduce the parallel blocks.

Step 3: Reduce the internal feedback loops.

Step 4: It is advisable to shift take-off points towards right and summing points towards left. It is always better to avoid rule 9 and rule 10.

Step 5: Repeat steps **1** to step **4** until the simple form is obtained.

Step 6: Find transfer function of the overall system using the formula ***C(s)/R(s).***

Example 3.4 Determine the ratio ***C(s)/R(s)*** of the block diagram shown in Fig. E3.4.

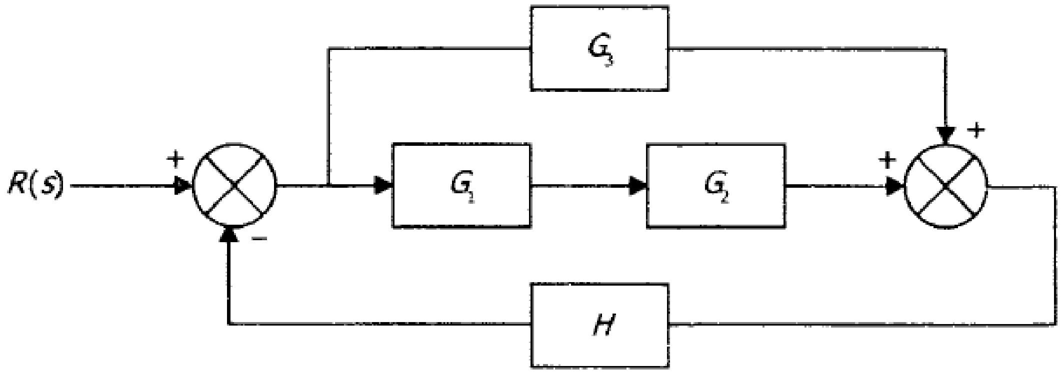


Fig. E3.4

Solution

G_1 and G_2 are connected in cascade and their equivalent is connected in parallel with G_3

$$G = G_1G_2 + G_3$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} = \frac{G_1G_2 + G_3}{1 - (G_1G_2 + G_3)H}$$