Tikrit university

Collage of Engineering Shirqat

Department of Electrical Engineering

Fourth Class

Control Engineering

Chapter 5

Lecture 10

Time domain response

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Time Domain Analysis of Control Systems

The time response has utmost importance for the design and analysis of control systems because these are inherently time domain systems where time is the independent variable. During the analysis of response, the variation of output with respect to time can be studied and it is known as time response. To obtain satisfactory performance of the system, the output behaviour of the system with respect to time must be within the specified limits. From time response analysis and corresponding results, the stability of system, accuracy of system and complete evaluation can be studied very easily.

The standard test inputs have already been introduced in Chapter 1 and their Laplace transforms have been derived in Chapter 2. The aim of this chapter is to explore time response analysis of control systems, steady-state response and error, transient response of control systems.

5.1 Classification of Time Responses

Due to the application of an excitation to a system, the response of the system is known as time response and it is a function of time. There are two parts of response of any system: (i) transient response (ii) Steady-State response.

5.1.1 Transient Response

The part of the time response which goes to zero after large interval of time is known as transient response. In this case Lt ***c(t)*** = 0.

5.1.2 Steady State Response

The part of response that remains even after the transients have died out is said to be steady-state response.

The total response of a system is the sum of transient response and steady-state response:

$$C(t) = C_{tr}(t) + C_{ss}(t) \qquad (5.1)$$

Figure 5.1 shows the transient and steady-state responses along with steady-state error.

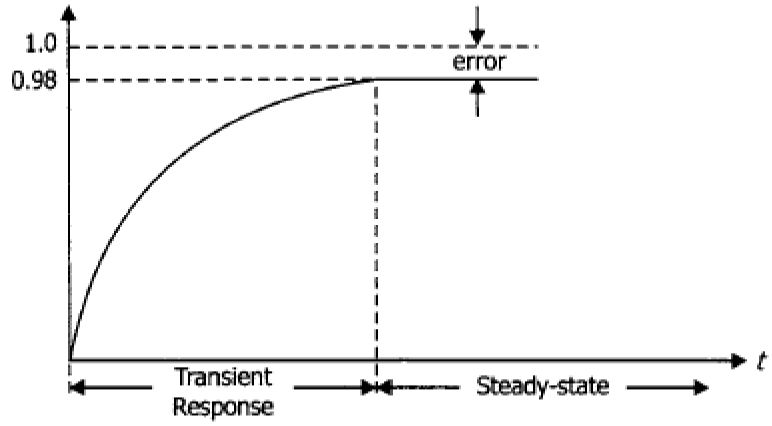

Fig. 5.1 Response of a system and steady-state error

8.2 System Time Response

After applying the excitation at the input terminals of a system, an output response c(t) is produced at the output terminals, which varies with time as shown in Fig. 5.2

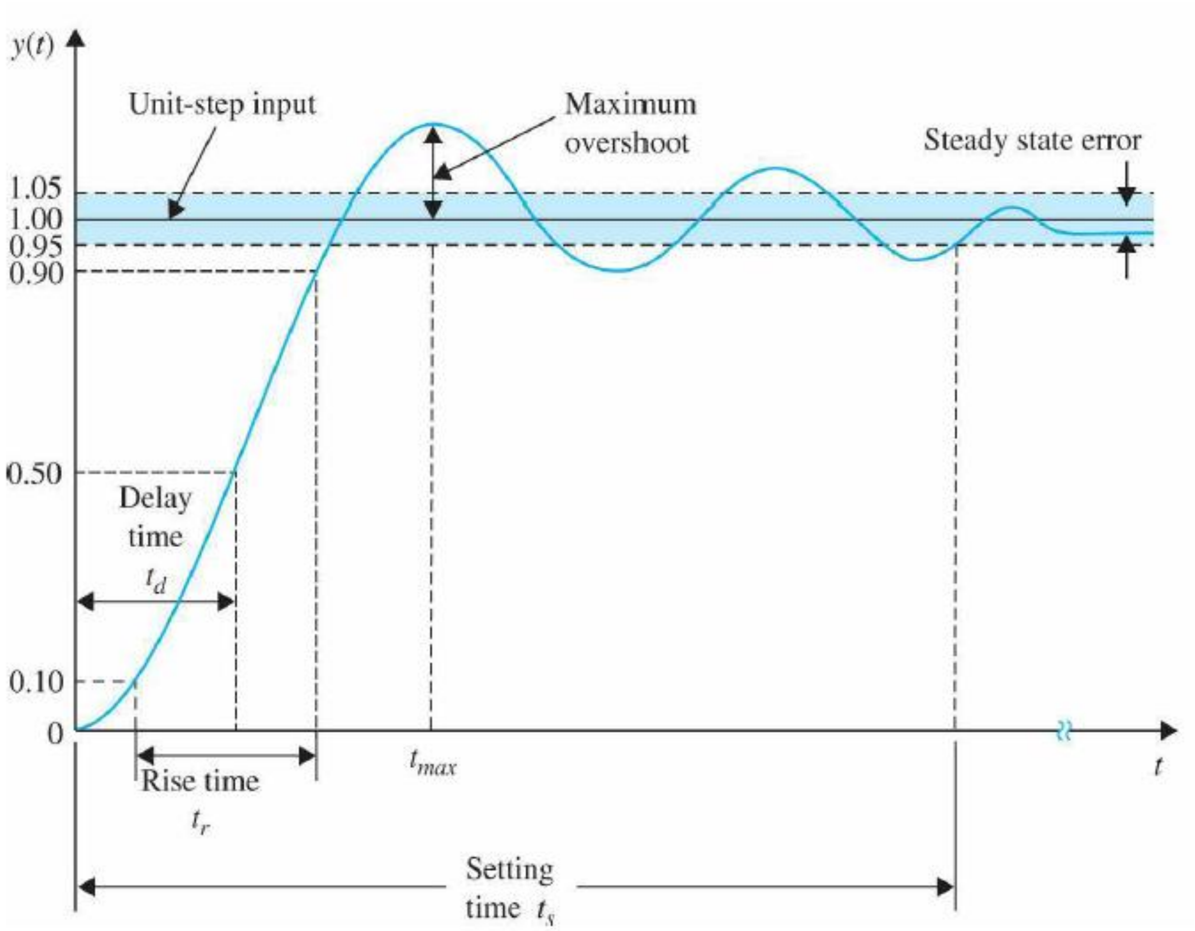

Fig. 5.2 Output vs time

Definitions of various terms related to Fig. 5.2. are given below:

Delay Time: The time that the system output takes to reach 50% of its final value is known as delay time (***td)***

Rise Time: The time that the system response takes to reach from 10% to 90% of the final value of output is called rise time (***tr***)

Settling Time: The time taken by the system response to settle down and stay within ±2% of its final value is known as settling time ***(t***s).

Peak Time: The time taken by the system response to reach the first maximum value is known as peak time ***(tp)***

Overshoot: The ratio of the maximum value of the step excited output to the final output is called overshoot

of the system. The percentage overshoot of a system is defined as follows:

$$\% \text{ overshoot} = \left.\frac{\text{Maximum output overshoot}}{\text{Final value of output}}\right|_{\text{step input}} \times 100 \qquad \textbf{(5.2)}$$

Steady-State Error: For a step excitation, the difference between the desired output and the final value of any system is termed as steady-state error of the system.

5.3 Analysis of Steady-State Error

A simple closed-loop system using negative feedback is shown in Fig. 5.3.

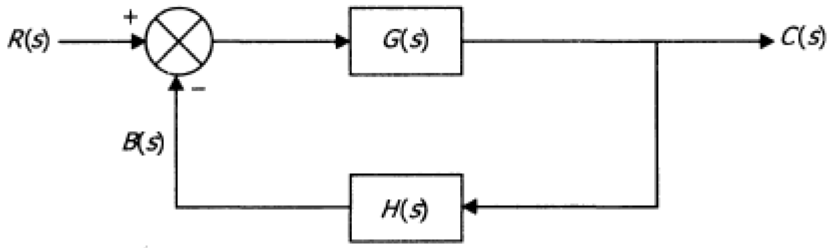

Fig. 5.3 Closed-loop system

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} \qquad (5.3)$$

For unity feedback. $E(s) = \dfrac{R(s)}{1+G(s)}$ (5.4)

For unit positive feedback, the denominator term in Eqs. (5.3) and (5.4) will be 1 - ***G(s).*** This error ***E(s)*** is in Laplace domain. The corresponding error in time domain will be ***e(t).*** During steady state, ***t*** —> ∞.

Steady-state error (e_{ss}) = $\lim_{t\to\infty} e(t)$ = $\lim_{s\to 0} sE(s)$ (5.5)

Using Eq. (5.4), we get from Eq. (5.5)

$$\lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)} \tag{5.6}$$

5.4 Type of Input and Steady-State Error

The steady-state error of a system having open loop transfer function ***G(s)H(s)*** is calculated for three types of input (i) step, (ii) ramp, and (iii) parabolic. Equation (5.6) is valid for any type of input. For the above inputs Eq. (5.6) is taken into account.

5.4.1 Step Input

Let the step input of magnitude A be applied. The Laplace transform of step input [r(t)] having magnitude A is given by

$$R(s)=\frac{A}{s} \tag{5.7}$$

Using Eq. (5.7), we have from Eq. (5.6),

$$\lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)} = \lim_{s\to 0} \frac{s\frac{A}{s}}{1+G(s)H(s)} = \frac{A}{1+\lim_{s\to 0} G(s)H(s)} \tag{5.8}$$

Fig. 5.4 shows the variation of ***C(t)*** with time.

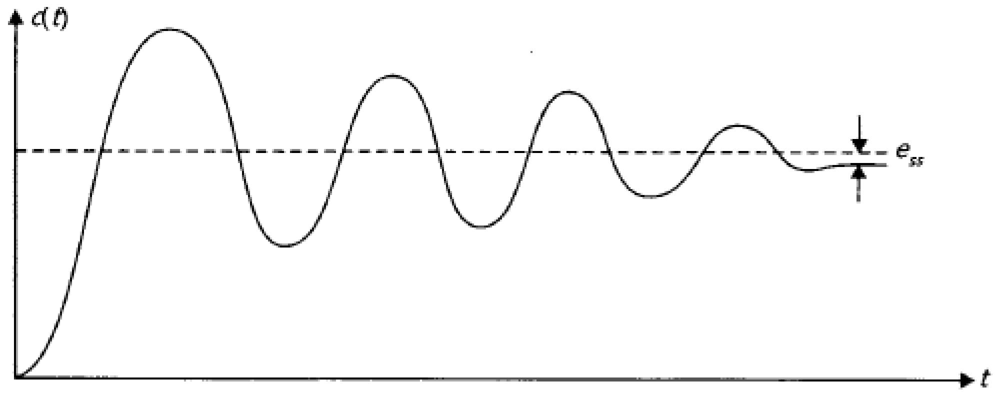

Fig. 5.4 Steady-state error for step input

For a system $\lim_{s\to 0} G(s)H(s)$ is constant and it is called positional error constant ***(Kp)***

Kp = $\lim_{s\to 0} G(s)H(s)$ (5.9)

Using Eq. (5.9), Eq. (5.8) can be written as

$$e_{ss} = \frac{A}{1+Kp} \tag{5.10}$$

For any given system, the steady-state error due to step input of value A is given by Eq. (5.10) and ***Kp*** is found by Eq. (5.9). ***Kp*** depends on the type of the system.

5.4.2 Ramp Input

The Laplace transform of a ramp input having magnitude A is given by

$$\frac{A}{s^2} \tag{5.11}$$

Using Eq. (5.11), we get from Eq. (5.6),

$$\lim_{s\to 0} \frac{s\frac{A}{s^2}}{1+G(s)H(s)} = \lim_{s\to 0} \frac{A}{s+sG(s)H(s)} = \frac{A}{\lim_{s\to 0} sG(s)H(s)} \tag{5.12}$$

For a system $\lim_{s\to 0} s$G(s)H(s) is constant and it is termed as velocity error coefficient. It is denoted by Kv

Kv = $\lim_{s\to 0} s$G(s)H(s) (5.13)

Using Eq. (5.13), Eq. (5.12) can be written as

$$e_{ss} = \frac{A}{Kv} \tag{5.14}$$

For any system, the steady error due to the ramp input of value A is given by Eq. (5.14) and K v can be found by Eq. (5.13). K v depends on the type of the system.

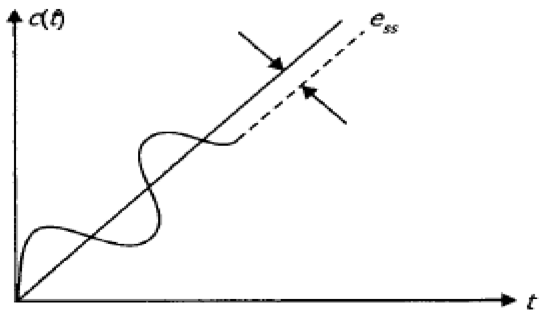

Fig 5.5 steady-state error for ramp input

5.4.3 Parabolic Input

The Laplace transform for the parabolic input of magnitude A is given by

$$R(s)=\frac{A}{s^3} \tag{5.15}$$

Using Eq. (5.15), we get from Eq. (5.6),

$$\lim_{s\to 0}\frac{s\frac{A}{s^3}}{1+G(s)H(s)}=\lim_{s\to 0}\frac{A}{s^2+s^2G(s)H(s)} =$$

$$e_{ss} = \frac{A}{\lim_{s\to 0} s^2G(s)H(s)} \tag{5.16}$$

For a system $\lim_{s\to 0}$ ***s²G(s)H(s)*** is constant and it is termed as acceleration error coefficient. It is denoted Ka

$$Ka = s^2G(s)H(s) \tag{5.17}$$

Using Eq. (5.15), Eq. (5.17) can be written as

$$e_{ss} = \frac{A}{Ka} \tag{5.18}$$

For any system, the steady-state error due to the prabolic input of value ***A*** is given by Eq. (5.18) and ***Ka*** can be determined by Eq. (5.17). ***Ka*** depends on the type of the system.

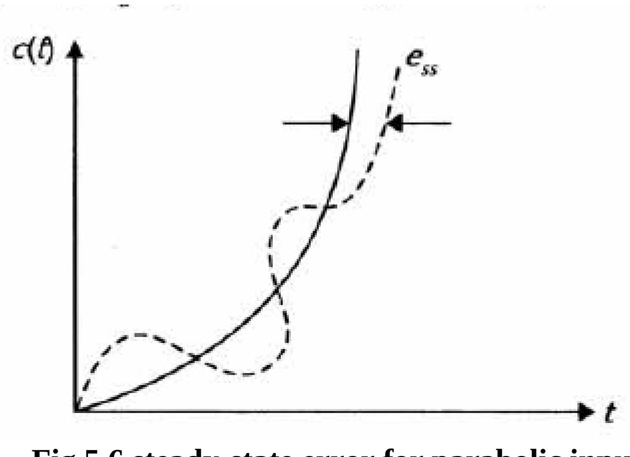

Fig 5.6 steady-state error for parabolic input

Table 5.1 summarises the above results.

Table 5.1 Type of input and steady state error

Constant	Equation	Steady-state error(e_{ss})
Position error constant (K_p)	$K_p = \underset{s\to 0}{Lt}\ G(s)\,H(s)$	$e_{ss} = \dfrac{A}{1+K_p}$
Velocity error constant (K_v)	$K_v = \underset{s\to 0}{Lt}\ sG(s)\,H(s)$	$e_{ss} = \dfrac{A}{K_v}$
Acceleration error constant (K_a)	$K_a = \underset{s\to 0}{Lt}\ s^2G(s)\,H(s)$	$e_{ss} = \dfrac{A}{K_a}$

5.5 Steady-State Error for Type 0 ,1 , and 2 Systems

The number of poles of a system having open-loop transfer function G(s) H(s) at s=0 specifies the type of the system. The general form of G(s)H(s) is

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)...(1+T_n s)}{s^j(1+T_a s)(1+T_b s)...(1+T_m s)} \tag{5.19}$$

From Eq. (8.22), there are j poles at ,s=0. Therefore, the type of the system is j.

Example 5.1 Identify the type number:

(i) $G(s) = \dfrac{K(1+3s)}{s^2}$ and $H(s) = \dfrac{1+5s}{(s^2+6s+7)}$

(ii) $G(s) = \dfrac{K}{s}$ and $H(s) = \dfrac{1}{s(s+2)}$

(iii) $G(s) = \dfrac{4}{s^2+6s+7}$ and $H(s) = s+3$

(iv) $G(s) = \dfrac{5}{s}$ and $H(s) = \dfrac{1}{s}$

Solution

(i) $G(s)H(s) = \dfrac{K(1+3s)}{s^2}\times\dfrac{(1+5s)}{(s^2+6s+7)} = \dfrac{K(1+3s)(1+5s)}{s^2(s^2+6s+7)}$

This is a type 2 system.

(ii) $G(s)H(s) = \dfrac{K}{s^2(s+2)}$. This is a type 2 system.

(iii) $G(s)H(s) = \dfrac{4(s+3)}{s^2+6s+7}$. This is a type 0 system.

(iv) $G(s)H(s) = \dfrac{5}{s^2}$. This is a type 2 system.

5.5.1 Error for Step Input

The error for a step input of value A is given by Eq. (5.10) as follows:

$$e_{ss} = \frac{A}{1+Kp} \qquad (5.20)$$

$$\boldsymbol{Kp} = \lim_{s\to 0} G(s)H(s) \qquad (5.21)$$

(i) Type 0

In this case j = 0 $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots(1+T_n s)}{(1+T_a s)(1+T_b s)\ldots(1+T_m s)}$ (5.22)

Since $\boldsymbol{Kp} = \lim_{s\to 0} G(s)H(s)$

$$K_p = \frac{K.1\ldots1}{1.1\ldots1} \qquad Kp=K \qquad (5.23)$$

Using Eq. (5.23), Eq. 5.20 can be written as

$$e_{ss} = \frac{A}{1+K}$$

Figure (5.7) shows the plot of c(t) versus t for type 0 system and also shows the error ess.

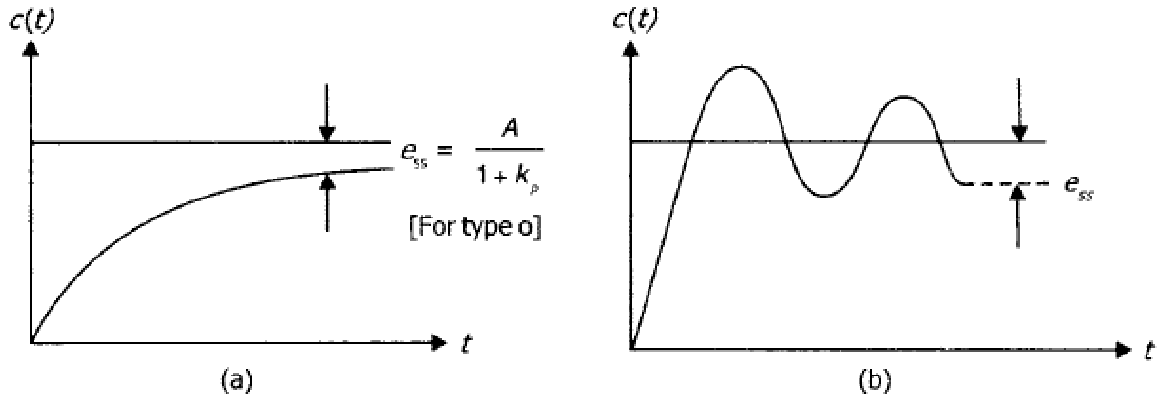

Fig. 5 .7 Error for step input and type 0 system

(ii) Type 1

In this case j = 1

$$G(s)\,H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots(1+T_n s)}{s(1+T_a s)(1+T_b s)\ldots(1+T_m s)}$$

$$K_p = \operatorname*{Lt}_{s\to 0} G(s)H(s) = \infty$$

$$e_{ss} = \frac{A}{1+K_p} = 0$$

ess is type 1 system for step input as shown in Fig. 5.8

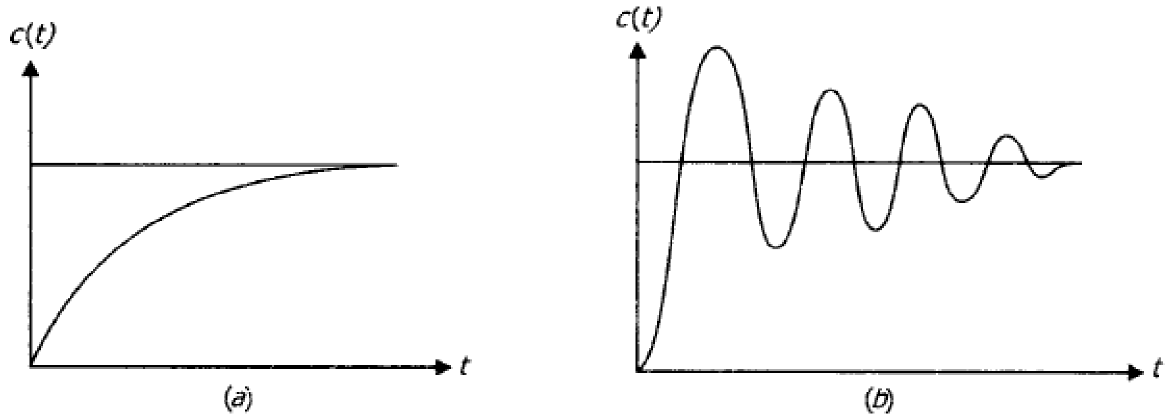

Fig. 8 .8 Error for step input and type 1 system

Similarly, Kp = ∞ and ess = 0 for step input and for higher-order systems.

5.5.2 Error for Ramp Input

The steady-state error for ramp input of value A is given by equation

$e_{ss} = \frac{A}{Kv}$ $\qquad$ $Kv = \lim_{s\to 0} sG(s)H(s)$

(i) For type 0

$$G(s)\,H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots(1+T_n s)}{(1+T_a s)(1+T_b s)\ldots(1+T_m s)}$$

$$K_v = \underset{s\to 0}{\text{Lt}}\; sG(s)H(s) = 0 \text{ and } e_{ss} = \frac{A}{K_v} = \frac{A}{0} = \infty$$

Type 0 system fails to keep track of a ramp input due to increment of errors continuously

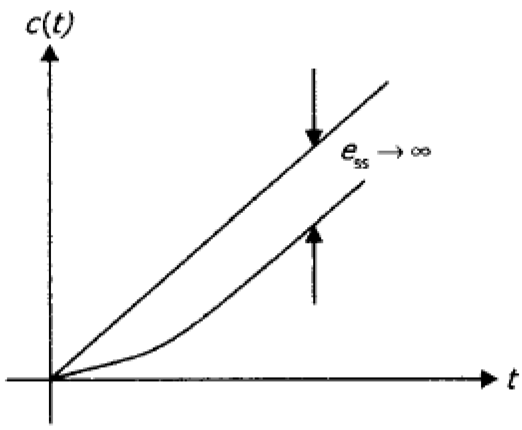

Fig. 8 .9 Error for ramp input and type 0 system

(ii) For Type 1

$$\therefore \quad G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)...(1+T_n s)}{s(1+T_a s)(1+T_b s)...(1+T_m s)}$$

$$\therefore \quad K_v = \underset{s\to 0}{\text{Lt}}\ sG(s)H(s) = \underset{s\to 0}{\text{Lt}} \frac{K(1+T_1 s)(1+T_2 s)...(1+T_n s)}{(1+T_a s)(1+T_b s)...(1+T_m s)} = K$$

and

$$e_{ss} = \frac{A}{K_v} = \frac{A}{K} = \text{constant}$$

The steady-state error for a type 1 system for ramp input is constant as shown in Fig. 5.10.

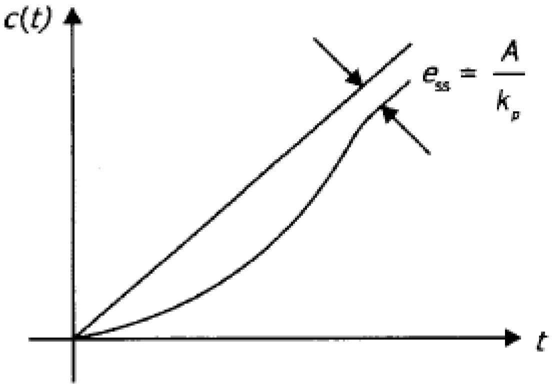

Fig. 8 .10 Error for ramp input and type 1 system

(iii) For Type 2

$$\therefore \quad G(s)\,H(s) = \frac{K(1+T_1 s)(1+T_2 s)...(1+T_n s)}{s^2(1+T_a s)(1+T_b s)...(1+T_m s)}$$

$$\therefore \quad K_v = \underset{s\to 0}{\text{Lt}}\ sG(s)H(s) = \underset{s\to 0}{\text{Lt}}\ s\left[\frac{K(1+T_1 s)(1+T_2 s)...(1+T_n s)}{s^2(1+T_a s)(1+T_b s)...(1+T_m s)}\right] = \infty$$

The steady-state error for type 2 for ramp input is zero as shown in Fig. 5.11.

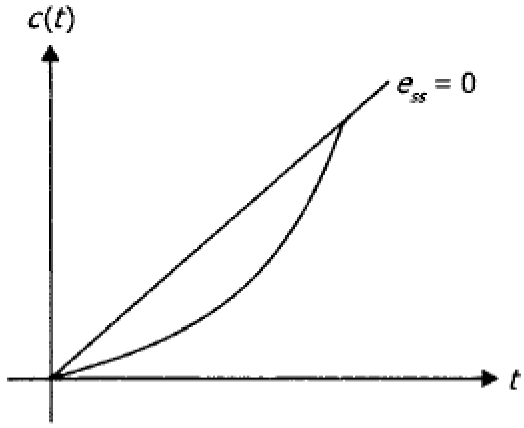

Fig. 5 .11 Error for ramp input and type 2 system

Similarly, Kv=∞ and ess = 0 for ramp input and higher-order systems.

8.5.3. Error for Parabolic Input

The steady-state error for a parabolic input of value A is given by

$$e_{ss} = \frac{A}{Ka} \qquad Ka = = \lim_{s \to 0} s^2 G(s)H(s)$$

(ii) For Type 0

$$K_a = \underset{s\to 0}{\text{Lt}}\ s^2 G(s)H(s)$$

$$\underset{s\to 0}{\text{Lt}}\ s^2 G(s)H(s) = \underset{s\to 0}{\text{Lt}}\ s^2 \left[\frac{K(1+T_1 s)(1+T_2 s)\dots(1+T_n s)}{(1+T_a s)(1+T_n s)\dots(1+T_n s)}\right] = 0$$

$$\therefore \qquad e_{ss} = \frac{a}{K_a} = \frac{A}{0} = \infty$$

The type 0 system fails to keep track of a parabolic input due to increment of errors continuously,

(ii) For Type 1

$$K_a = \underset{s\to 0}{\text{Lt}}\ sG(s)H(s) = \underset{s\to 0}{\text{Lt}}\ s^2 \left[\frac{K(1+T_1 s)(1+T_2 s)\dots(1+T_n s)}{s(1+T_a s)(1+T_b s)\dots(1+T_m s)}\right] = 0$$

$$e_{ss} = \frac{a}{K_a} = \frac{A}{0} = \infty$$

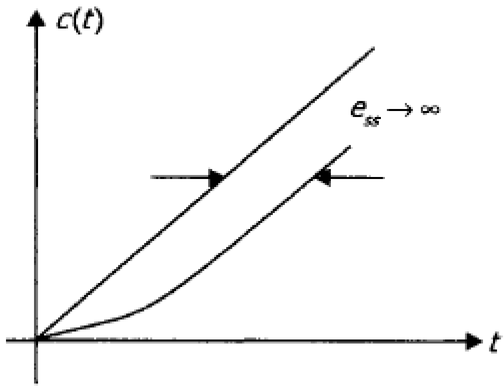

Fig. 5 .12 Error for parabolic input and type 1 system

The same conclusion as for type 0 can be drawn

(iii) For Type 2

$$\underset{s\to 0}{\text{Lt}}\ s^2 G(s)H(s) = \underset{s\to 0}{\text{Lt}}\ s^2 \left[\frac{K(1+T_1 s)(1+T_2 s)\dots(1+T_n s)}{s^2(1+T_a s)(1+T_b s)\dots(1+T_m s)}\right] = K$$

$$e_{ss} = \frac{A}{K_a} = \frac{A}{K} \text{ constant}$$

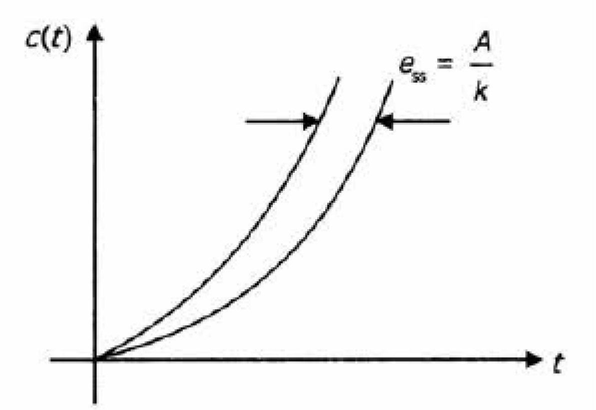

Fig. 5 .13 Error for parabolic input and type 2 system

Table 8.2 summarises the above conclusions.

Table 8.2 Error for different types of input

Type	Step input		Ramp input		Parabolic input	
	K_p	e_{ss}	K_v	e_{ss}	K_a	e_{ss}
Type 0	K	$\frac{A}{1+K}$	0	∞	0	∞
Type 1	∞	0	K	$\frac{A}{K}$	0	∞
Type 2	∞	0	∞	0	K	$\frac{A}{K}$

Example 5.2: Find the error coefficients for a system having

$$G(s)H(s) = \frac{(s+3)}{s(1+0.60s)(1+0.35s)}$$

Solution

$$K_p = \underset{s\to 0}{\text{Lt}}\ G(s)H(s) = \underset{s\to 0}{\text{Lt}}\ \frac{(s+3)}{s(1+0.60s)(1+0.35s)} = \infty$$

$\therefore$

$$K_v = \underset{s\to 0}{\text{Lt}}\ sG(s)H(s) = \underset{s\to 0}{\text{Lt}}\ \frac{(s+3)}{(1+0.60s)(1+0.35s)} = 3$$

$$K_a = \underset{s\to 0}{\text{Lt}}\ s^2 G(s)H(s) = \underset{s\to 0}{\text{Lt}}\ \frac{s(s+3)}{(1+0.60s)(1+0.35s)} = 0$$