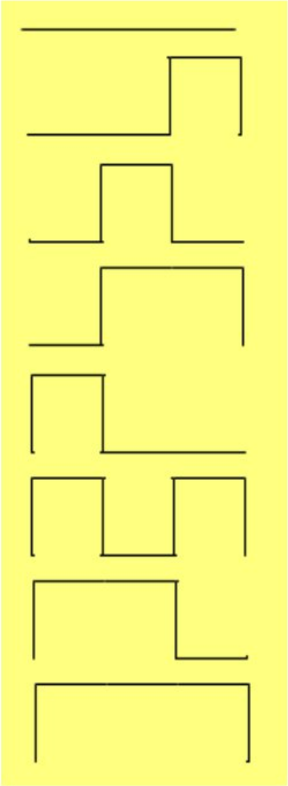
Pulse Code Modulation (PCM)

Example of binary number and 3-bit pulse code is shown below:

Quantized level	Binary number
1	000
2	001
3	010
4	011
5	100
6	101
7	110
8	111

Pulse waveform



These encoded signals now will be send through the channel.
How many bits needed to represent 16 levels?

Data Rate and Bandwidth of PCM Signals

The bit rate of PCM is given by

Bit rate = sampling rate × No. of bits / sample

$$\boxed{\boldsymbol{R = n \times f_s}}$$

where $\boldsymbol{n}$ is the number of bits in the PCM word ($L = 2^n$) and fs is the sampling rate. For no aliasing, we require that $\boldsymbol{f_s \geq 2B \text{ or } (2f_m)}$, where B is the bandwidth of the analog signal (that is to be converted to the PCM signal).

The bandwidth of (serial) binary PCM waveforms depends on the bit rate and the waveform pulse shape used to represent the data. The bandwidth of the binary encoded PCM waveform is bounded by

$$\boldsymbol{B_{PCM} \geq \frac{1}{2} R = \frac{1}{2} \times \mathbf{n} \times f_s}$$

Example :

We want to digitize the human voice with frequencies from 0 to 4000 Hz. What is the bit rate, assuming 8 bits per sample?

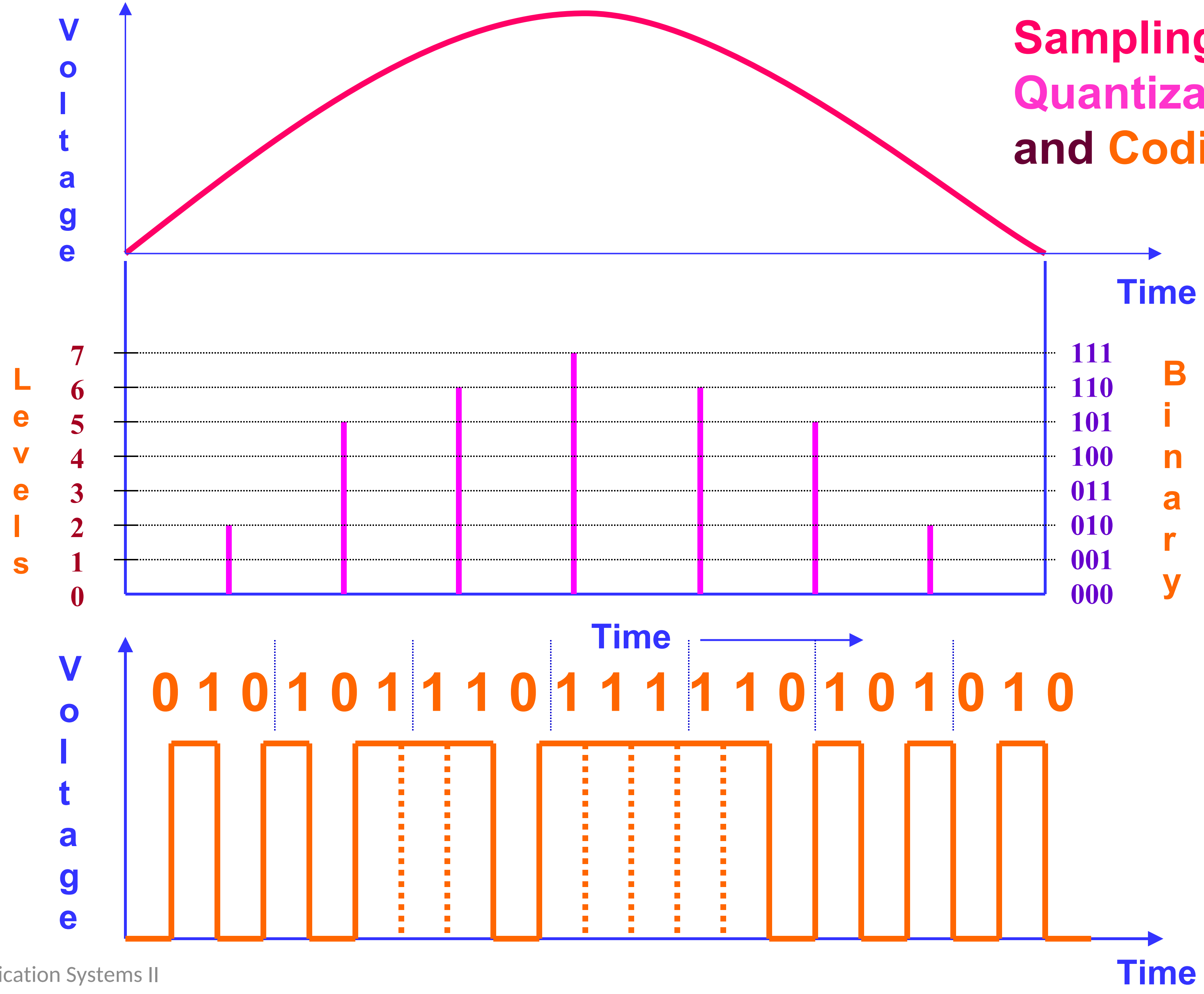
Solution:

Sampling rate $\boldsymbol{f_s}$ = 4000 x 2 = 8000 samples/s

Bit rate = $\boldsymbol{n\ f_s}$ = sampling rate x number of bits per sample

= 8000 x 8 = 64,000 bps = 64 Kbps

Sampling, Quantization and Coding



Example:

For voice message, $B = 0.3\sim3.4\ kHz\ and\ n = 8\ bits$, calculate: quantization levels, sampling frequency, bit rate, and the bandwidth of PCM.

Solution:

Quantization levels = $\text{L} = 2^n = 2^8 = 256\ levels$

The message bandwidth $B = 3.4 - 0.3 = 3.1\ kHz$

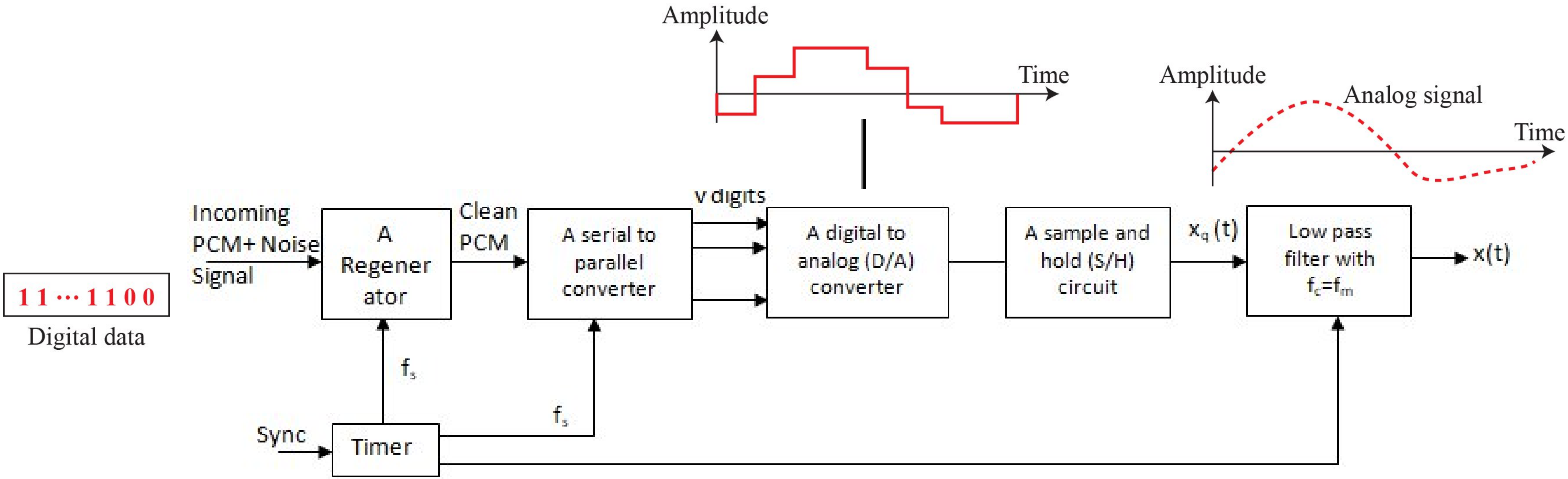
Sampling frequency = $f_S \geq 2B \rightarrow f_S \geq 2(3.1) \rightarrow f_S \geq 6.2 \rightarrow f_S \approx 8\ ksample/sec$

Bit rate of PCM = $R_{PCM} = nf_s = 8(8) = 64\ kbits/sec$

Bandwidth of PCM =$B_{PCM} = \frac{R_{PCM}}{2} = \frac{64}{2} = 32\ kHz$

Decoder of PCM

- Decoding the PCM signal includes :-
- Restore a pulse from a code word and It works similar to the demodulation process and converts the binary pulses to the original form or the analog signal. Low-pass filter : Cutoff frequency must be same as the sender's frequency. *Components of a PCM decoder are shown in figure below*



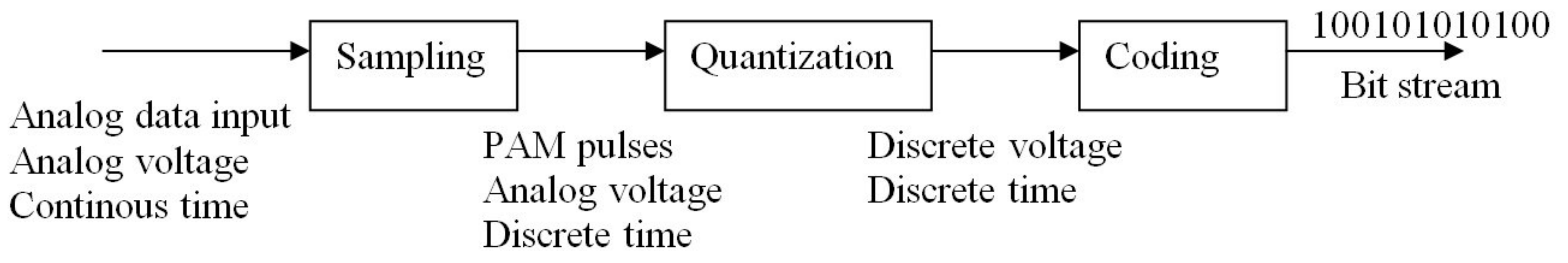
To recover an analog signal from a digitized signal, following steps are needed:

- A hold circuit that holds the amplitude value of a pulse till the next pulse arrives.
- Pass this signal through LPF with a cutoff frequency equal to the highest frequency in the pre-sampled signal.
- The higher the value of L, the less distorted a signal is recovered

PCM

Example: A system is designed to sample analog voltage signals using a 3-bit PCM codec. Assume that the voltage values are in the range of 0 to 5 Volts.

- Sketch and briefly describe the various stages of a PCM codec.



Sampling: The input analog data signal is sampled at twice the highest significant frequency of the data signal. Each sample represents a narrow pulse whose amplitude is proportional to the value of the original data signal. This process is known as Pulse Amplitude Modulation (PAM).

Quantization: The PAM pulses are quantized, which consists of the assignment of discrete values to the PAM pulses. The discrete values are approximations of the amplitudes of the PAM pulses.

Coding: Each discrete value is encoded in an ***n***-bit integer. Therefore, 2^n signal levels are available for approximating the PAM pulses.

Example: A system is designed to sample analog voltage signals using a 3-bit PCM codec. Assume that the voltage values are in the range of 0 to 5 Volts.

- Name and briefly describe the type of errors/noise that may be introduced by a PCM codec.

1. **Aliasing Error:** When sampling is performed at a rate less than the Nyquist rate.
2. **Quantization Error:** Caused by the fact that a countable number of discrete signal levels are used to approximate an uncountable number of signal levels. This error is reduced by increasing the number of levels, i.e. the number of quantization steps. This requires the use of more bits

- Into how many groups (levels) does this 3-bit codec divide the input voltage?

$$L = \boldsymbol{2^{\wedge}n} = 2^3 = 8 \text{ groups.}$$

- What is the resolution of this 3-bit codec?

$$\text{Resolution} = \Delta v = \frac{\boldsymbol{Vmax-Vmin}}{\boldsymbol{L}} = 5/2^3 = 0.625 \text{ Volt.}$$

- What is the quantization error of this PCM signal?

$$\text{Quantization Error} = \text{Resolution}/2 = 0.3125 \text{ Volt.}$$

PCM

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- Give the analog voltages and the corresponding digital values that will be used by this 3-bit codec.

Answer:

0		0.625		1.25		1.875		2.5		3.125		3.75		4.375		5
|	000	|	001	|	010	|	011	|	100	|	101	|	110	|	111	|

- What bit stream output will occur from the following voltage inputs?

1.25 3.21 5.5 3.75 0.6

Answer:

1.25	3.21	5.5	3.75	0.6
001	101	111	101	000

Example: A system is designed to sample analog voltage signals using a 3-bit PCM codec. Assume that the voltage values are in the range of 0 to 5 Volts.

- If the frequency of the input signal is 4 KHz, what sampling rate should be used to avoid aliasing?

Answer:
Sampling Rate >= 8 KHz

- Suppose that the input voltage is sampled at 50% more than the Nyquist rate, what bit rate would be generated out of the codec.

Answer:
Bit Rate = 8000 x 1.5 × 3 = 36 Kbps $\boldsymbol{R = n \times fs}$

- Suppose that the codec resolution has been improved so that the bit rate output from the codec becomes 56 Kbps. What will be in that case the bandwidth required to transmit the above digitized analog voltage if the channel has a signal-to-noise ratio of 30 decibels.

Answer: $C = BW \log_2(1+SNR)$
$BW = 56000/(\log_2(1+SNR)) = 56000/(\log_2(1+1000)) = 5.6$ KHz

Differential Pulse Code Modulation (DPCM)

In PCM each sample of the waveform is encoded independently of all the other samples. PCM is not a very efficient system **because it generates so many bits and requires so much bandwidth to transmit.** However, most source signals sampled at the Nyquist rate or faster exhibit significant correlation between successive samples. In other words, the average change in amplitude between the successive samples is relatively small. Consequently, an encoding scheme that exploits the redundancy in the samples will result in a lower bit rate for the source output.

Samples of a bandlimited signal are correlated -> previous sample gives information about the next one.

Example: if previous samples are small, the next one will be small with high probability.

• This can be used to improve PCM performance: to decrease the number of bits used (and, hence, the bandwidth) or to increase SQNR for a given bandwidth.

• Main idea: **quantize and transmit the difference between two adjacent samples rather than sample values**.

• Since two adjacent samples are correlated (bandlimited signal!), their difference is small and requires less bits to transmit.