Antennas and Transmission Lines

Electrical Department-3rd Stage

By

Assistant Lecturer: Adnan Ali Abdullah

2025-2024

Linear Wire Antenna

Wire Antennas (Linear or Curved) are some of the oldest , simplest , cheapest , and in many cases the most versatile for many application.

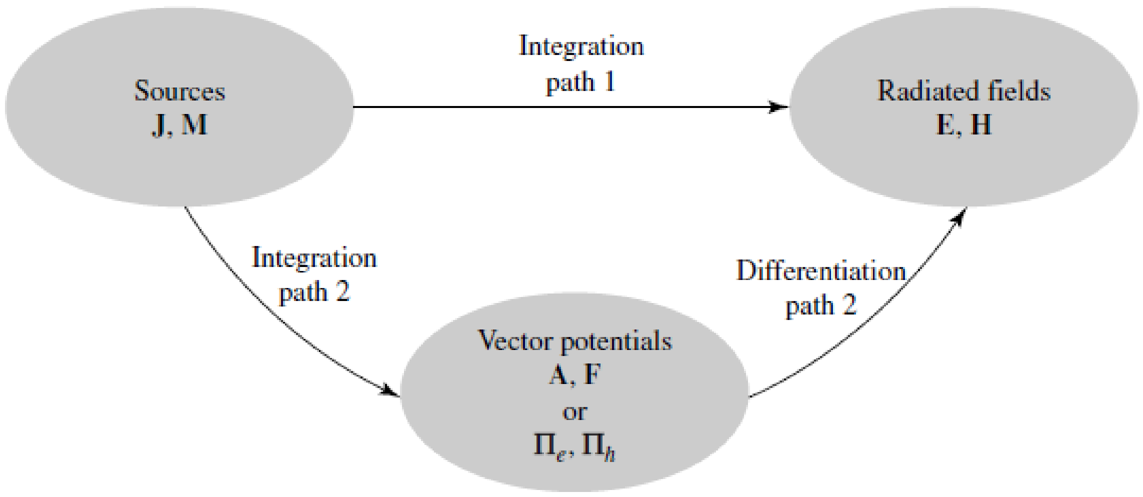
❖ Infinitesimal Dipole (Hertz Antenna) (L<<λ)

Short Dipole Current Element (L≤λ /50)

The current is assumed to be constant and given by:

$$\overrightarrow{I(z)} = I\vec{a}_z \; (I = \text{constant}) \tag{2.34}$$

Radiated fields it will be required to determine first $\vec{A}$ (magnetic vector potential) and then find $\vec{E}$ & $\vec{H}$ at P.



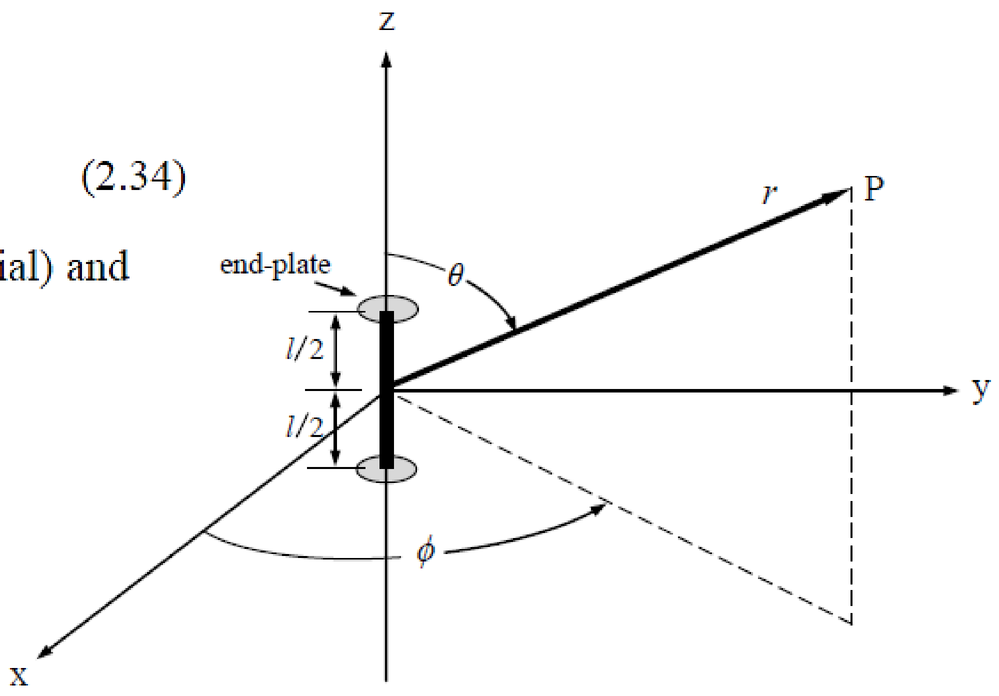


Figure 2.15 Infinitesimal dipole .

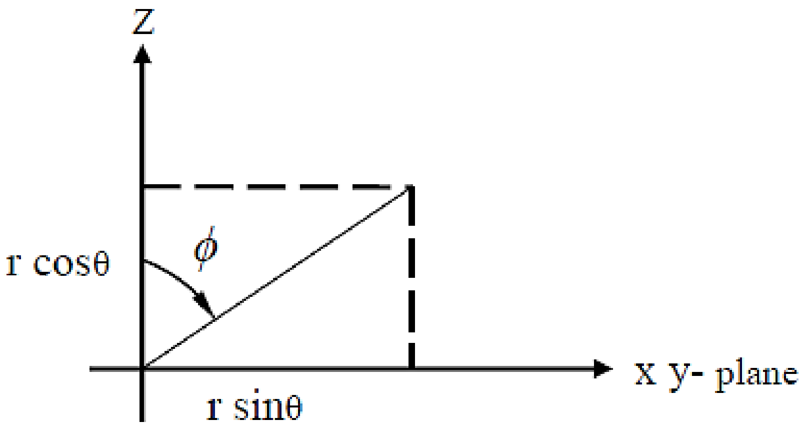
$$\vec{a}_r = cos\theta\vec{a}_z + sin\theta sin\phi\vec{a}_y + sin\theta cos\phi\vec{a}_x$$

$\vec{A}$ is given by the following relation

$$\vec{A} = \frac{\mu}{2\pi}\int_v \frac{\vec{J}}{r} e^{-j\beta r} dv \quad , \quad \vec{J} = \frac{I}{A} \tag{2.36}$$

$$dv = dx\, dydz \tag{2.37}$$

$\mu = Megnetic\ permitivity$
J=current density
$\beta = propagation\ constant = 2\pi/\lambda$



gure 2.16 Coordinate system .

The current density $\vec{J}$ Integrated over the cross sectional area of wire (which is placed on xy plane) is $\vec{J}$ use current I .

$$\int dy\, dx \frac{I}{A} = I \tag{2.38}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{\vec{I}}{r} e^{-j\beta r} dz \tag{2.39}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{I_o}{r} e^{-j\beta r} dz\, \vec{a}_z \tag{2.40}$$

$$\vec{A} = \frac{\mu I_o \ell e^{-j\beta r}}{4\pi r}\vec{a}_z \quad , \quad A_x = A_y = 0 \tag{2.41}$$

$$A_z = \frac{\mu I_o \ell e^{-j\beta r}}{4\pi r} \tag{2.42}$$

$$\nabla \times \vec{A} = \vec{B} \tag{2.43}$$

The transformation between rectangular and spherical components is given by:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} sin\theta cos\phi & sin\theta sin\phi & cos\theta \\ cos\theta cos\phi & cos\theta sin\phi & -sin\theta \\ -sin\phi & cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad (2.44)$$

$$A_r = A_z cos\theta = \frac{\mu I_o \ell}{4\pi r} e^{-j\beta r} \, cos\theta \quad (2.45)$$

$$A_\theta = -A_z sin\theta = -\frac{\mu I_o \ell}{4\pi r} e^{-j\beta r} \, sin\theta \quad (2.46)$$

$$A_\phi = 0 \quad (2.47)$$

$$\nabla \times \vec{A} = \vec{B} \quad (2.48)$$

$$\nabla \times \vec{A} = \frac{1}{r sin\theta}\left(\frac{\partial}{\partial\theta}\left(A_\phi sin\theta\right) - \frac{\partial A_\theta}{\partial\phi}\right)\vec{a}_r + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}\left(rA_\phi\right)\right)\vec{a}_\theta$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{a}_\phi \quad (2.49)$$

$$\nabla \times \vec{A} = \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{a}_\phi \quad (2.50)$$

$$\vec{H} = \frac{j\beta I_o \ell}{4\pi r} sin\theta \left(1 + \frac{1}{j\beta r}\right) e^{-j\beta r} \vec{a}_\phi \quad (2.51)$$

$$H_r = 0 \quad (2.52)$$

$$H_\theta = 0 \quad (2.53)$$

$$\boxed{H_\phi = \frac{j\beta I_o \ell}{4\pi r} sin\theta \left(1 + \frac{1}{j\beta r}\right) e^{-j\beta r}} \quad (2.54)$$

in the same manner

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (2.55)$$

$$\vec{J} = \sigma \vec{E} \Longrightarrow \vec{J} = 0 \quad \text{free space} \qquad (2.56)$$

$$\nabla \times \vec{H} = j\omega\varepsilon\, \vec{E} \qquad (2.57)$$

$$\boxed{\therefore E_r = \eta \frac{I_o \ell \cos\theta}{2\pi r^2}\left(1 + \frac{1}{j\beta r}\right) e^{-j\beta r}} \qquad (2.58)$$

$$\eta = 120\pi \qquad (2.59)$$

$$\boxed{E_\theta = j\eta \frac{\beta I_o \ell \sin\theta}{4\pi r}\left(1 + \frac{1}{j\beta r} - \frac{1}{(\beta r)^2}\right) e^{-j\beta r}} \qquad (2.60)$$

$$E_\phi = 0 \qquad (2.61)$$

Near Field Region ($\beta r \ll 1$ or $r \ll \lambda$)

For $\beta r \ll 1$ $(r \ll \lambda)$ the fields can be approximated by:

$$\boxed{\begin{aligned} E_r &= -j\eta \frac{I_o \ell e^{-j\beta r}}{2\pi\beta r^3} \cos\theta && (2.62) \\ E_\theta &= -j\eta \frac{I_o \ell e^{-j\beta r}}{4\pi\beta r^3} \sin\theta && (2.63) \\ H_\phi &= \frac{I_o \ell e^{-j\beta r}}{4\pi r^2} \sin\theta && (2.64) \\ E_\phi &= H_r = H_\theta = 0 && (2.65) \end{aligned}}$$

Example For infinitesimal dipole (Near field region) prove that the average power density $\vec{W}_{av} = 0$.

The solution:- $\vec{W}_{av} = \frac{1}{2} R_e\left[\vec{E} \otimes \vec{H}^*\right]$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[-j\eta \frac{I_o \ell e^{-j\beta r}}{2\pi\beta r^3} \cos\theta\, \vec{a}_r \otimes \frac{I_o \ell e^{-j\beta r}}{4\pi r^2} \sin\theta\, \vec{a}_\phi\right] = 0$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[-j\eta \frac{I_o \ell e^{-j\beta r}}{4\pi\beta r^3} \sin\theta\, \vec{a}_\theta \otimes \frac{I_o \ell e^{-j\beta r}}{4\pi r^2} \sin\theta\, \vec{a}_\phi\right] = 0$$

Far Field Region $\beta r \gg 1$

$$\boxed{\begin{aligned} E_\theta &= j\eta \frac{\beta I_o \ell e^{-j\beta r}}{4\pi r} \sin\theta && (2.66) \\ H_\phi &= j \frac{\beta I_o \ell e^{-j\beta r}}{4\pi r} \sin\theta && (2.67) \\ E_r &= E_\phi = H_r = H_\theta = 0 && (2.68) \end{aligned}}$$

Parameters of infinitesimal dipole :

1- Radiation Resistance

$$\overrightarrow{W}_{av} = \frac{1}{2} R_e[\vec{E} \otimes \vec{H}^*]$$

$$\overrightarrow{W}_{av} = \frac{1}{2} R_e\left[j\eta \frac{\beta I_o \ell e^{-j\beta r}}{4\pi r} sin\theta\, \vec{a}_\theta \otimes -j \frac{\beta I_o \ell e^{+j\beta r}}{4\pi r} sin\theta \vec{a}_\phi\right]$$

$$\overrightarrow{W}_{av} = \frac{\eta}{2} \left[\frac{\beta I_o \ell}{4\pi r} sin\theta\right]^2 \vec{a}_r$$

$$P_{rad} = \int\int \overrightarrow{W}_{av} \,.\, \overrightarrow{ds} \quad , \quad \overrightarrow{ds} = r^2 sin\theta\, d\theta d\phi\, \vec{a}_r$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} \frac{\eta}{2} \left[\frac{\beta I_o \ell}{4\pi r}\right]^2 sin^2\theta\, \vec{a}_r \,.\, r^2 sin\theta\, d\theta d\phi\, \vec{a}_r$$

$$P_{rad} = \frac{\eta}{2} \left[\frac{\beta I_o \ell}{4\pi}\right]^2 \int_0^{2\pi} \int_0^{\pi} sin^3\theta\, d\theta d\phi$$

$$P_{rad} = \frac{\eta}{2} \beta^2 I_o{}^2 \left[\frac{I_o \ell}{4\pi}\right]^2 \left(\frac{4}{3}\right) (2\pi)$$

where $I_o = \sqrt{2} I_{rms}$, $\beta = \frac{2\pi}{\lambda}$, $\eta = 120\pi$

$$P_{rad} = \frac{120\pi}{2} \left(\frac{2\pi}{\lambda}\right)^2 \left(\sqrt{2} I_{rms}\right)^2 \left[\frac{I_o \ell}{4\pi}\right]^2 \left(\frac{4}{3}\right) (2\pi)$$

$$P_{rad} = I^2{}_{rms} \left(80\pi^2 \left(\frac{\ell}{\lambda}\right)^2\right) \longleftrightarrow P_{rad} = I^2{}_{rms} R_{rad}$$

$$R_{rad} = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2 \quad \text{only for} \quad L \leq \frac{\lambda}{50}$$

2- Directivity

$$D_g(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{\iint U(\theta,\phi)\ d\Omega} = 4\pi \frac{sin\theta^2}{\int_0^{2\pi}\int_0^{\pi} sin\theta^2\ \ sin\theta\ d\theta d\phi}$$

$$D_g(\theta,\phi) = 4\pi \frac{sin\theta^2}{\int_0^{2\pi}\int_0^{\pi} sin\theta^3 d\theta d\phi} = 4\pi \frac{sin\theta^2}{2\pi\left(\frac{4}{3}\right)}$$

$$D_0 = 4\pi \frac{(1)^2}{2\pi\left(\frac{4}{3}\right)}$$

$$D_0 = 1.5$$

3- HPBW

$$E(\theta,\phi) = j\eta \frac{\beta I_o \ell e^{-j\beta r}}{4\pi r} sin\theta$$

$$E(\theta,\phi)_n = sin\theta$$

$$0.707(max) = E(\theta,\phi)$$

$$0.707(1) = sin\theta_h$$

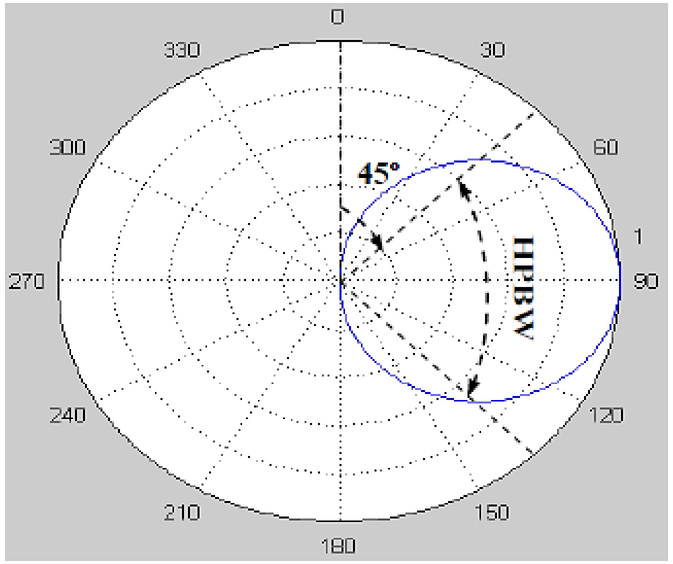
$$sin\theta_h = 0.707$$

$$\theta_h = sin^{-1} 0.707 = 45^o$$

$$\text{HPBW} = 2(90 - \theta_h)$$

$$\text{HPBW} = 2(90^o - 45^o)$$

$$\text{HPBW} = 90^o$$



For Infinitesimal dipole $\left(L \leq \frac{\lambda}{50}\right)$

$$R_{rad} = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2$$

$$D_0 = 1.5$$

$$\text{HPBW} = 90^o$$

Example Derive an expression for the radiation resistance of a short dipole antenna.

The solution:- $\overrightarrow{W}_{av} = \frac{1}{2} R_e\left[\vec{E} \otimes \vec{H}^*\right]$

$$\overrightarrow{W}_{av} = \frac{1}{2} R_e\left[j\eta \frac{\beta I_o \ell e^{-j\beta r}}{4\pi r} sin\theta\, \vec{a}_\theta \otimes -j\frac{\beta I_o \ell e^{+j\beta r}}{4\pi r} sin\theta \vec{a}_\phi\right]$$

$$\overrightarrow{W}_{av} = \frac{\eta}{2}\left[\frac{\beta I_o \ell}{4\pi r} sin\theta\right]^2 \vec{a}_r$$

$$P_{rad} = \int\int \overrightarrow{W}_{av} \,.\, \overrightarrow{ds} \;,\; \overrightarrow{ds} = r^2 sin\theta\, d\theta d\phi\, \vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2}\left[\frac{\beta I_o \ell}{4\pi r}\right]^2 sin^2\theta\, \vec{a}_r \,.\, r^2 sin\theta\, d\theta d\phi\, \vec{a}_r$$

$$P_{rad} = \frac{\eta}{2}\left[\frac{\beta I_o \ell}{4\pi}\right]^2 \int_0^{2\pi}\int_0^{\pi} sin^3\theta\; d\theta d\phi$$

$$P_{rad} = \frac{\eta}{2}\, \beta^2 {I_o}^2 \left[\frac{I_o \ell}{4\pi}\right]^2 \left(\frac{4}{3}\right)(2\pi) \quad \text{where} \quad I_o = \sqrt{2} I_{rms} \,,\, \beta = \frac{2\pi}{\lambda} \,,\, \eta = 120\pi$$

$$P_{rad} = \frac{120\pi}{2}\left(\frac{2\pi}{\lambda}\right)^2 \left(\sqrt{2} I_{rms}\right)^2 \left[\frac{I_o \ell}{4\pi}\right]^2 \left(\frac{4}{3}\right)(2\pi)$$

$$P_{rad} = I^2{}_{rms}\left(80\pi^2\left(\frac{\ell}{\lambda}\right)^2\right) \longleftrightarrow P_{rad} = I^2{}_{rms}\, R_{rad}$$

$$R_{rad} = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

❖ Finite Length Dipole (L> $\lambda/10$)

Assume sinusoidal current distribution

$$I = I_m \sin\beta\left(\frac{L}{2} - z\right) \quad z > 0 \qquad (2.7($$

$$I = I_m \sin\beta\left(\frac{L}{2} + z\right) \quad z < 0 \qquad (2.71$$

The current is zero at the ends of the dipole since the current in the $\vec{a}_z$-direction The magnetic potential vector $\vec{A}$ is also in the z-direction .

$$A_z = \frac{\mu}{4\pi}\int_{-H}^{H} \frac{Ie^{-j\beta R}}{R}\, dz$$

$\longrightarrow$ R = (r − z cosθ) for phase

$\longrightarrow$ R = r for amplitude

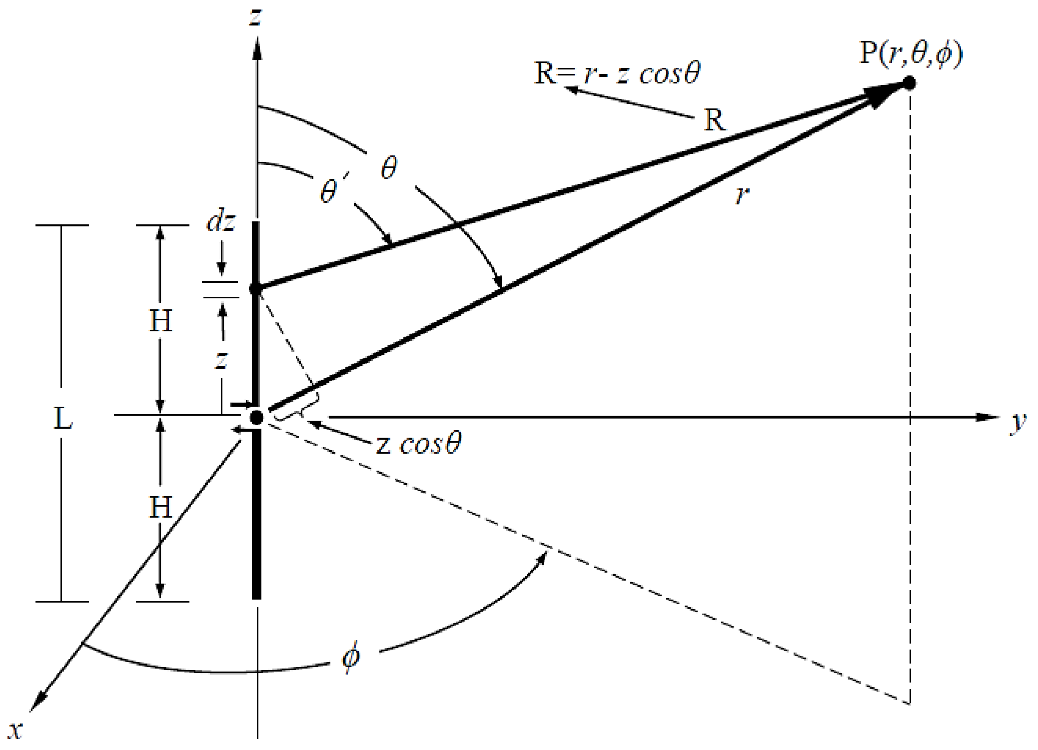


Figure 2.18 Finite Length dipole .

$$A_z = \frac{\mu}{4\pi r}\int_{-H}^{0} I_m \sin\beta(H+z)e^{-j\beta(r - z\cos\theta)}\, dz + \frac{\mu}{4\pi r}\int_{0}^{H} I_m \sin\beta(H-z)e^{-j\beta(r - z\cos\theta)}\, dz \qquad (2.73)$$

where L : dipole length

H : Half dipole length

$$\frac{L}{2} = H \qquad (2.74)$$

The general expression for the distance electric field of a dipole antenna ($L > \frac{\lambda}{10}$) is given by:

$$E_\theta = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$$

1- ($\lambda/2$) Dipole

One of the most commonly used antennas is the half-wavelength (l = λ/2) dipole. Because its radiationresistan ce is 73 ohms, which is very near the 50-ohm or 75-ohm characteristic impedances of some transmission lines, its matching to the line is simplified especially at resonance. Because of its wide acceptance in practice, we will examine in a little more detail its radiation

$$\frac{\beta L}{2}cos\theta = \frac{2\pi}{2\lambda}\frac{\lambda}{2}cos\theta = \frac{\pi}{2}cos\theta \quad , \quad cos(\frac{\beta L}{2}) = cos(\frac{2\pi}{2\lambda}\frac{\lambda}{2}) = cos(\frac{\pi}{2}) = 0$$

$$\vec{E} = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}\right]\vec{a}_\theta \quad , \quad H_\phi = \frac{E_\theta}{\eta}$$

$$\overrightarrow{W}_{av} = \frac{1}{2}\,R_e\left[\vec{E}\otimes\vec{H}^*\right]$$

$$\overrightarrow{W}_{av} = \frac{1}{2}\,R_e\left[\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60I_m e^{j\beta r}}{\eta r}\left[\frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}\right]\vec{a}_\phi\right]$$

$$\overrightarrow{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}\right]^2\vec{a}_r$$

$$P_{rad} = \int\int \overrightarrow{W}_{av}\,.\overrightarrow{ds} \quad , \quad \overrightarrow{ds} = r^2 sin\theta\, d\theta d\phi\,\vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}\right]^2\vec{a}_r\; r^2 sin\theta\, d\theta d\phi\,\vec{a}_r$$

$$P_{rad} = \frac{1}{2\eta}(60I_m)^2(2\pi)\int_0^{\pi}\frac{cos^2(\frac{\pi}{2}cos\theta)}{sin\theta}\; d\theta$$

$$P_{rad} = 60I^2{}_{rms}\int_0^{\pi}\frac{cos^2(\frac{\pi}{2}cos\theta)}{sin\theta}\; d\theta \quad , \quad \int_0^{\pi}\frac{cos^2(\frac{\pi}{2}cos\theta)}{sin\theta} = 1.218$$

$$P_{rad} = 73\;I^2{}_{rms} \;\Rightarrow\; R_{rad} = 73\;\Omega$$

These equations are used to determine Prad, Wav, Rr for ($\lambda/2$) Dipole

Directivity

$$D = 4\pi \frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\ d\Omega} = 4\pi \frac{|E_{\theta max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E_\theta|^2\ sin\theta\ d\theta\ d\phi}$$

$$D = 4\pi \frac{\left|\frac{cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right|^2}{2\pi \int_0^{\pi} \left|\frac{cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right|^2\ sin\theta\ d\theta} = 4\pi \frac{(1)^2}{2\pi \int_0^{\pi} \left|\frac{cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right|^2\ sin\theta\ d\theta}$$

$$D = 4\pi \frac{1}{2\pi(1.218)} = 1.64$$

HPBW

$$E(\theta,\phi) = \frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}$$

$$0.707(max) = E(\theta,\phi)$$

$$0.707(1) = \frac{cos(\frac{\pi}{2}cos\theta)}{sin\theta}$$

$$0.707sin\theta_h = cos\left(\frac{\pi}{2}cos\theta\right)$$

$$\theta_h = sin^{-1}\left[\sqrt{2}\,cos\left(\frac{\pi}{2}cos\theta\right)\right]$$

$$\theta_h = 51^o$$

$$\text{HPBW} = 2(90 - \theta_h)$$

$$\text{HPBW} = 2(90^o - 51^o)$$

$$\text{HPBW} = 78^o$$

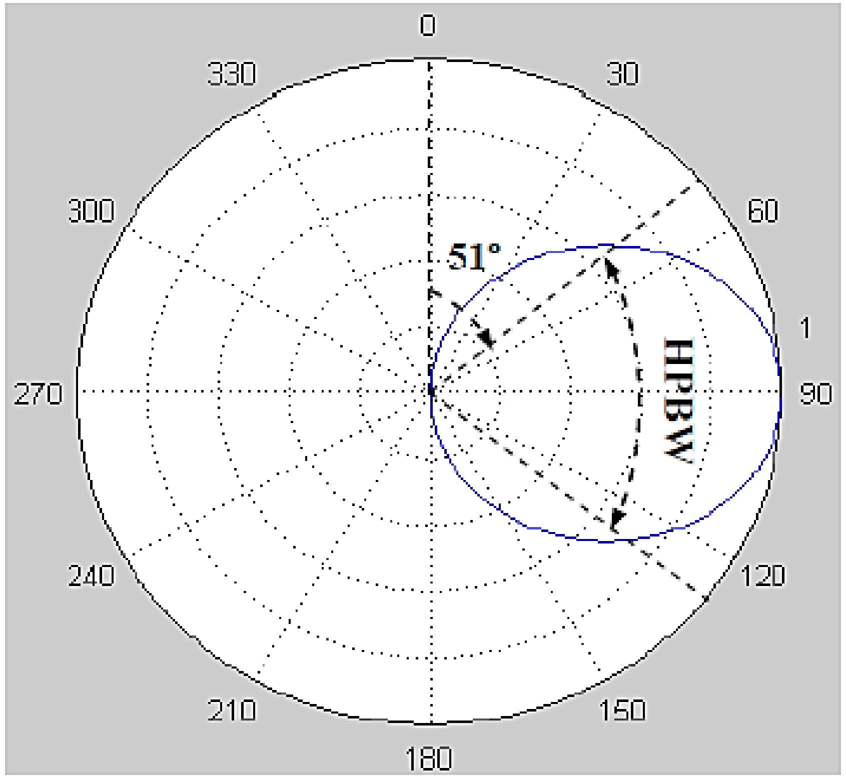


Figure 2.19 Radiation pattern of the antenna .

θ	$E(\theta,\phi)$
0	0
30°	0.417
45°	0.627
60°	0.816
90°	1

2- $\lambda/4$ dipole

These equations are used to determine Prad, Wav, Rr for ($\lambda/4$) Dipole

$$\frac{\beta L}{2}\cos\theta = \frac{2\pi}{2\lambda}\frac{\lambda}{4}\cos\theta = \frac{\pi}{4}\cos\theta \quad , \cos\left(\frac{\beta L}{2}\right) = \cos\left(\frac{2\pi}{2\lambda}\frac{\lambda}{4}\right) = \cos\left(\frac{\pi}{4}\right) = 0.707$$

$$\vec{E} = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta \quad , \quad H_\phi = \frac{E_\theta}{\eta}$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\vec{E} \otimes \vec{H}^*\right]$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60I_m e^{j\beta r}}{\eta r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\phi\right]$$

$$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\vec{a}_r$$

$$P_{rad} = \int\int \vec{W}_{av} \cdot \vec{ds} \quad , \quad \vec{ds} = r^2\sin\theta \, d\theta d\phi \, \vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\vec{a}_r \; r^2\sin\theta \, d\theta d\phi \, \vec{a}_r$$

$$P_{rad} = \frac{1}{2\eta}(60I_m)^2(2\pi)\int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}\, d\theta$$

$$P_{rad} = 60I^2{}_{rms}\int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}d\theta \; , \; \int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta} = 0.114$$

$$P_{rad} = 6.84\, I^2{}_{rms} \Rightarrow R_{rad} = 6.84\,\Omega$$

Directivity

$$D = 4\pi \frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\ d\Omega} = 4\pi \frac{|E_{\theta max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E_\theta|^2 \sin\theta\, d\theta\, d\phi}$$

θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.14
45°	0.2018
60°	0.25
90°	0.293

$$D = 4\pi \frac{\left|\frac{\cos(\frac{\pi}{4}\cos\theta) - 0.707}{\sin\theta}\right|^2}{2\pi \int_0^{\pi} \frac{\cos(\frac{\pi}{4}\cos\theta) - 0.707}{\sin\theta}^2 \sin\theta\, d\theta}$$

$$D = 4\pi \frac{(0.293)^2}{2\pi \int_0^{\pi} \left|\frac{\cos(\frac{\pi}{4}\cos\theta) - 0.707}{\sin\theta}\right|^2 \sin\theta\, d\theta} = 4\pi \frac{(0.293)^2}{2\pi \int_0^{\pi} \frac{\left[\cos(\frac{\pi}{4}\cos\theta) - 0.707\right]^2}{\sin^2\theta} \sin\theta\, d\theta}$$

$$D = 4\pi \frac{(0.293)^2}{2\pi \int_0^{\pi} \frac{\left[\cos(\frac{\pi}{4}\cos\theta) - 0.707\right]^2}{\sin\theta}\, d\theta}$$

$$D = 4\pi \frac{(0.293)^2}{2\pi(0.114)}$$

$$D = 1.506$$

HPBW

$$E(\theta,\phi)=\frac{cos(\frac{\pi}{4}cos\theta)-0.707}{sin\theta}$$

$$0.707(max)=E(\theta,\phi)$$

$$0.707(0.293)=\frac{cos(\frac{\pi}{4}cos\theta)-0.707}{sin\theta}$$

$$0.207sin\theta_h=cos\left(\frac{\pi}{4}cos\theta\right)-0.707$$

$$\theta_h=sin^{-1}\frac{1}{0.207}\left[cos\left(\frac{\pi}{4}cos\theta\right)-0.707\right]$$

$$\theta_h=46.5^o$$

$$\text{HPBW}=2(90-\theta_h)$$

$$\text{HPBW}=2(90^o-46.5^o)$$

$$\text{HPBW}=87^o$$

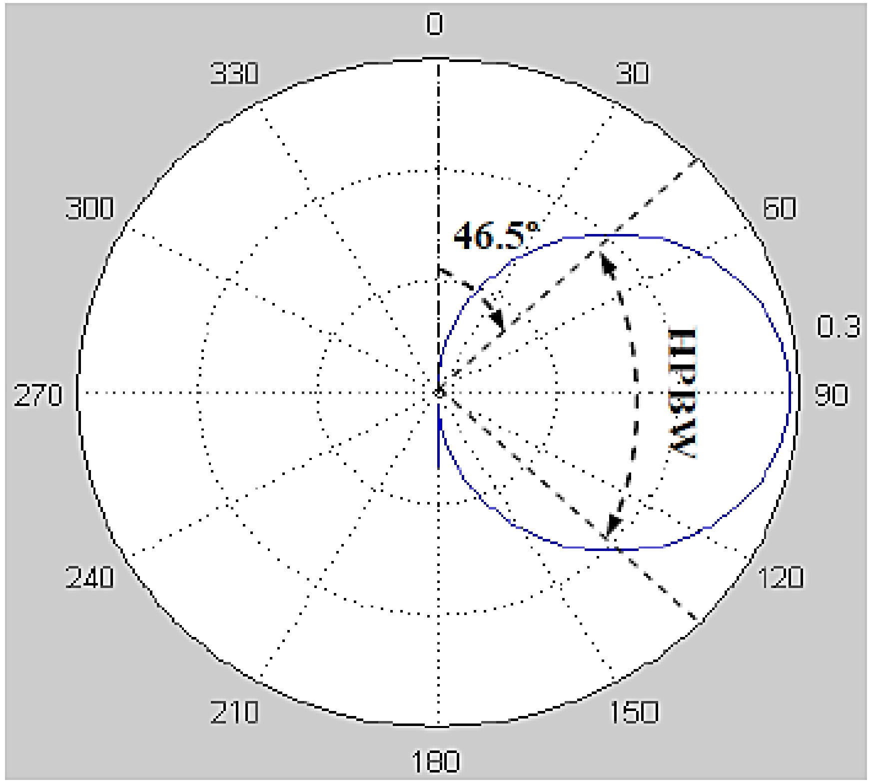


Figure 2.20 Radiation pattern of the antenna .

θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.14
45°	0.2018
60°	0.25
90°	0.293

Thank You