Antennas and Transmission Lines

Electrical Department-3rd Stage

By
Assistant Lecturer: Adnan Ali Abdullah

2025-2024

3- L=λ/4 dipole

$$\frac{\beta L}{2}\cos\theta = \frac{2\pi}{2\lambda}\frac{3\lambda}{4}\cos\theta = \frac{3\pi}{4}\cos\theta\ ,\ \cos\left(\frac{\beta L}{2}\right) = \cos\left(\frac{2\pi}{2\lambda}\frac{3\lambda}{4}\right) = \cos\left(\frac{3\pi}{4}\right) = -0.707$$

$$\vec{E} = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)-(-0.707)}{\sin\theta}\right]\vec{a}_\theta\ ,\ H_\phi = \frac{E_\theta}{\eta}$$

$$\vec{W}_{av} = \frac{1}{2}R_e\left[\vec{E}\otimes\vec{H}^*\right]$$

$$\vec{W}_{av} = \frac{1}{2}R_e\left[\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right]\vec{a}_\theta\otimes\frac{-j60I_m e^{j\beta r}}{\eta r}\left[\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right]\vec{a}_\phi\right]$$

$$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right]^2\vec{a}_r$$

$$P_{rad} = \int\int\vec{W}_{av}\,.\vec{ds}\ ,\ \vec{ds} = r^2\sin\theta\,d\theta d\phi\,\vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right]^2\vec{a}_r\ r^2\sin\theta\,d\theta d\phi\,\vec{a}_r$$

$$P_{rad} = \frac{1}{2\eta}(60I_m)^2(2\pi)\int_0^{\pi}\frac{\left[\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707\right]^2}{\sin\theta}\ d\theta$$

$$P_{rad} = 60I^2{}_{rms}\int_0^{\pi}\frac{\left[\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707\right]^2}{\sin\theta}d\theta\ ,\ \int_0^{\pi}\frac{\left[\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707\right]^2}{\sin\theta} = 3.0968$$

$$P_{rad} = 185.8\ I^2{}_{rms}\ \Rightarrow\ R_{rad} = 185.8\ \Omega$$

$$D = 4\pi \frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\ d\Omega} = 4\pi \frac{|E_{\theta max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E_\theta|^2 \sin\theta\ d\theta d\phi}$$

θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.318
45°	0.5599
60°	0.875
90°	1.707

$$D = 4\pi \frac{\left|\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right|^2_{max}}{2\pi\int_0^{\pi}\left|\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right|^2 \sin\theta\ d\theta}$$

$$D = 4\pi \frac{(1.707)^2}{2\pi\int_0^{\pi}\left|\frac{\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707}{\sin\theta}\right|^2 \sin\theta\ d\theta} = 4\pi \frac{(1.707)^2}{2\pi\int_0^{\pi}\frac{\left[\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707\right]^2}{\sin\theta}\ d\theta}$$

$$D = 4\pi \frac{(1.707)^2}{2\pi(3.0968)}$$

$$D = 1.88$$

HPBW

$$E(\theta,\phi)=\frac{\cos(3\frac{\pi}{4}\cos\theta)+0.707}{\sin\theta}$$

$$0.707(max)=E(\theta,\phi)$$

$$0.707(1.707)=\frac{\cos(3\frac{\pi}{4}\cos\theta)+0.707}{\sin\theta}$$

$$1.2068\sin\theta_h=\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707$$

$$\theta_h=\sin^{-1}\frac{1}{1.2068}\left[\cos\left(3\frac{\pi}{4}\cos\theta\right)+0.707\right]$$

$$\theta_h=57.99^o$$

$$\text{HPBW}=2(90-\theta_h)$$

$$\text{HPBW}=2(90^o-57.99^o)$$

$$\text{HPBW}=64^o$$

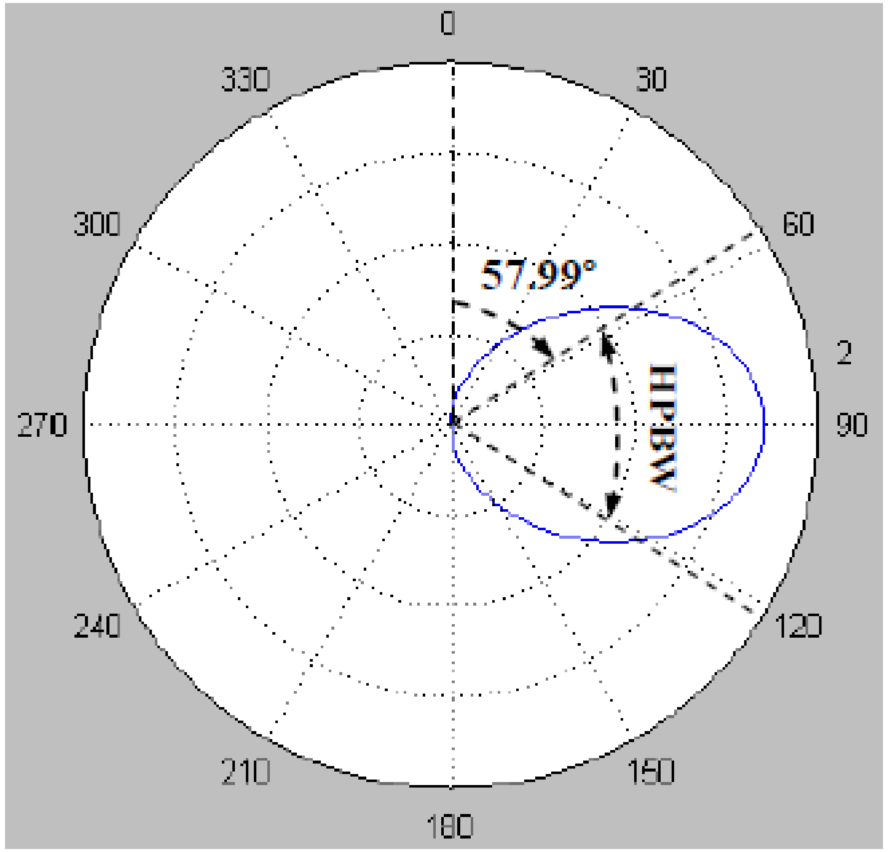


Figure 2.21 Radiation pattern of the antenna .

4- L=λ dipole

$$\frac{\beta L}{2}\cos\theta = \frac{2\pi}{2\lambda}\lambda\cos\theta = \pi\cos\theta \quad , \quad \cos\left(\frac{\beta L}{2}\right) = \cos\left(\frac{2\pi}{2\lambda}\lambda\right) = \cos(\pi) = -1$$

$$\vec{E} = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos(\pi\cos\theta)-(-1)}{\sin\theta}\right]\vec{a}_\theta \quad , \quad H_\phi = \frac{E_\theta}{\eta}$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\vec{E} \otimes \vec{H}^*\right]$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60I_m e^{j\beta r}}{\eta r}\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]\vec{a}_\phi\right]$$

$$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]^2\vec{a}_r$$

$$P_{rad} = \int\int \vec{W}_{av} \,.\vec{ds} \quad , \quad \vec{ds} = r^2\sin\theta \, d\theta d\phi \, \vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]^2\vec{a}_r \; r^2\sin\theta \, d\theta d\phi \, \vec{a}_r$$

$$P_{rad} = \frac{1}{2\eta}(60I_m)^2(2\pi)\int_0^{\pi}\frac{[\cos(\pi\cos\theta)+1]^2}{\sin\theta} \, d\theta$$

$$P_{rad} = 60{I^2}_{rms}\int_0^{\pi}\frac{[\cos(\pi\cos\theta)+1]^2}{\sin\theta} \, d\theta \quad , \quad \int_0^{\pi}\frac{[\cos(\pi\cos\theta)+1]^2}{\sin\theta} = 3.31$$

$$P_{rad} = 198.6 \, {I^2}_{rms} \quad \Rightarrow \quad R_{rad} = 198.6 \, \Omega$$

$$D = 4\pi \frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\ d\Omega} = 4\pi \frac{|E_{\theta max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E_\theta|^2 \sin\theta\ d\theta d\phi}$$

θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.174
45°	0.5572
60°	1.1547
90°	2

$$D = 4\pi \frac{\left|\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right|^2_{max}}{2\pi\int_0^{\pi}\left|\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right|^2 \sin\theta\ d\theta}$$

$$D = 4\pi \frac{(2)^2}{2\pi\int_0^{\pi}\left|\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right|^2 \sin\theta\ d\theta} = 4\pi \frac{(2)^2}{2\pi\int_0^{\pi}\frac{[\cos(\pi\cos\theta)+1]^2}{\sin\theta}\ d\theta}$$

$$D = 4\pi \frac{4}{2\pi(3.31)}$$

$$D = 2.41$$

Table 2.1 Length , HPBW, Rrad and directivity (D) for a dipole antenna ($L > \frac{\lambda}{10}$).

Length	HPBW	R_{rad}	D
L = λ/50	90°	very small	1.5
L = λ/4	87°	6.84	1.52
L = λ/2	78°	73	1.64
L =3λ/4	64°	185.8	1.88
L = λ	48°	198	2.41

HPBW

$$E(\theta,\phi) = \frac{\cos(\pi\cos\theta)+1}{\sin\theta}$$

$$0.707(max) = E(\theta,\phi)$$

$$0.707(2) = \frac{\cos(\pi\cos\theta)+1}{\sin\theta}$$

$$1.414\sin\theta_h = \cos(\pi\cos\theta) + 1$$

$$\theta_h = \sin^{-1}\frac{1}{1.414}[\cos(\pi\cos\theta) + 1]$$

$$\theta_h = 66.08^o$$

$$\text{HPBW} = 2(90 - \theta_h)$$

$$\text{HPBW} = 2(90^o - 66^o)$$

$$\text{HPBW} = 48^o$$

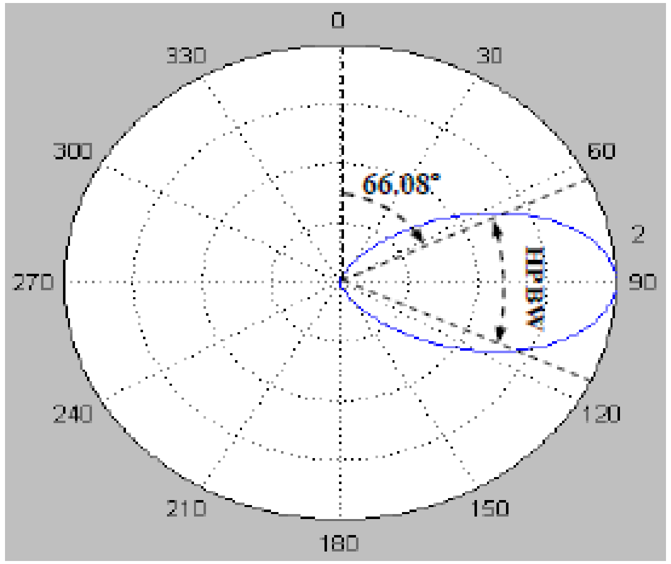


Figure 2.22 Radiation pattern of the antenna .

Problems and Solutions

.1 A λ/4 transmitting dipole antenna of 4mm radius situated with its at origin radiates a power of 200W at a frequency of 100 MHz . determine:

1-The current fed to the dipole.

2-Directivity

3-HPBW

4-The electric field at (2 Km , 30° , 60°). Assume $\sigma_{copper} = 58$ MS/m

If a second λ/4 dipole is placed at 1 Km away from the transmitting antenna in the direction of maximum radiation, find the received power at the terminal of the second (receiving) dipole.

The solution:- $E_\theta = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$

1- $\frac{\beta L}{2} = \frac{2\pi}{2\lambda}\frac{\lambda}{4} = \frac{\pi}{4}$, $\cos(\frac{\beta L}{2}) = \cos(\frac{2\pi}{2\lambda}\frac{\lambda}{4}) = \cos(\frac{\pi}{4}) = 0.707$

$\vec{E} = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta$, $H_\phi = \frac{E_\theta}{\eta}$

$\overrightarrow{W}_{av} = \frac{1}{2} R_e\left[\vec{E} \otimes \vec{H}^*\right]$

$\overrightarrow{W}_{av} = \frac{1}{2} R_e\left[\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60I_m e^{j\beta r}}{\eta r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\phi\right]$

$\overrightarrow{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\vec{a}_r$

$P_{rad} = \int\int \overrightarrow{W}_{av} \cdot \overrightarrow{ds}$, $\overrightarrow{ds} = r^2\sin\theta\, d\theta d\phi\, \vec{a}_r$

$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\vec{a}_r\, r^2\sin\theta\, d\theta d\phi\, \vec{a}_r$

$P_{rad} = \frac{1}{2\eta}(60I_m)^2(2\pi)\int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}\, d\theta$

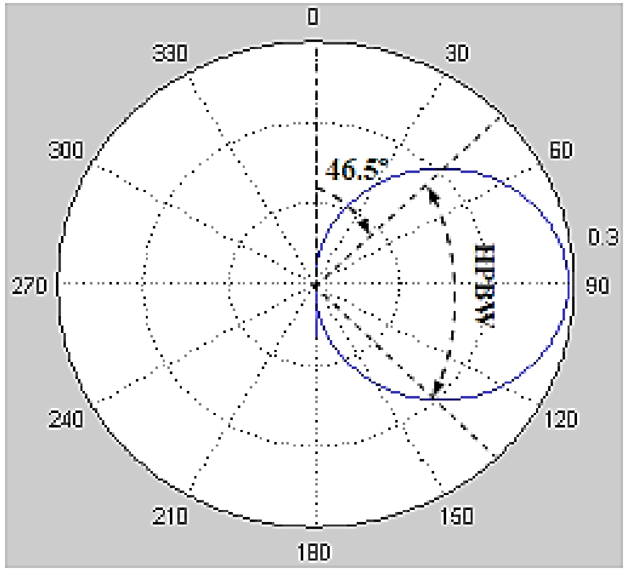
$P_{rad} = 60I^2{}_{rms}\int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}\, d\theta$, $\int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta} = 0.114$

$200 = 6.84\, I^2{}_{rms} \Rightarrow I_{rms} = \sqrt{\frac{200}{6.84}} = 5.4$ A

2- $D = 4\pi \frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\ d\Omega} = 4\pi \frac{|E_{\theta max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E_\theta|^2 \sin\theta\ d\theta d\phi}$

$$D = 4\pi \frac{\left|\frac{\cos(\frac{\pi}{4}\cos\theta)-0.707}{\sin\theta}\right|^2_{max}}{2\pi \int_0^{\pi} \frac{\cos(\frac{\pi}{4}\cos\theta)-0.707^2}{\sin\theta}\ \sin\theta\ d\theta} = 4\pi \frac{(0.293)^2}{2\pi \int_0^{\pi} \left|\frac{\cos(\frac{\pi}{4}\cos\theta)-0.707}{\sin\theta}\right|^2 \sin\theta\ d\theta}$$

$$D = 4\pi \frac{(0.293)^2}{2\pi \int_0^{\pi} \frac{[\cos(\frac{\pi}{4}\cos\theta)-0.707]^2}{\sin\theta}\ d\theta} = 4\pi \frac{(0.293)^2}{2\pi(0.114)} = 1.506$$



3- HPBW

$$E(\theta,\phi) = \frac{\cos(\frac{\pi}{4}\cos\theta)-0.707}{\sin\theta}$$

$$0.707(max) = E(\theta,\phi)$$

$$0.707(0.293) = \frac{\cos(\frac{\pi}{4}\cos\theta)-0.707}{\sin\theta}$$

$$0.207\sin\theta_h = \cos\left(\frac{\pi}{4}\cos\theta\right) - 0.707$$

$$\theta_h = \sin^{-1}\frac{1}{0.207}\left[\cos\left(\frac{\pi}{4}\cos\theta\right) - 0.707\right]$$

$$\theta_h = 46.5^o$$

$$\text{HPBW} = 2(90 - \theta_h)$$

$$\text{HPBW} = 2(90^o - 46.5^o)$$

$$\text{HPBW} = 87^o$$

θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.14
45°	0.2018
60°	0.25
90°	0.293

.2 A half wave dipole is radiating into free space. The coordinate system is defined so that the origin is at the center of the dipole and the z-axis is aligned with the dipole. input power to the dipole is 100W, assuming an overall efficiency of 50%, find the power density at (500 m , 60° , 0°).

The solution:- $E_\theta = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$

half wave dipole(H) $= \frac{\lambda}{2} \Longrightarrow \frac{\beta L}{2} = \frac{2\pi}{2\lambda}\frac{\lambda}{2} = \frac{\pi}{2}$, $\cos(\frac{\beta L}{2}) = \cos(\frac{2\pi}{2\lambda}\frac{\lambda}{2}) = 0$

$$\vec{E} = \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\vec{a}_\theta \ , \ H_\phi = \frac{E_\theta}{\eta}$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\vec{E}\otimes\vec{H}*\right]$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60I_m e^{j\beta r}}{\eta r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\vec{a}_\phi\right]$$

$$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2\vec{a}_r$$

$$P_{rad} = \int\int \vec{W}_{av} \cdot \vec{ds} \quad , \quad \vec{ds} = r^2\sin\theta \, d\theta d\phi \, \vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2\vec{a}_r \ r^2\sin\theta \, d\theta d\phi \, \vec{a}_r$$

$$P_{rad} = \frac{1}{2\eta}(60I_m)^2(2\pi)\int_0^{\pi}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} \, d\theta$$

$$P_{rad} = 60I^2{}_{rms}\int_0^{\pi}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} \, d\theta \ , \ \int_0^{\pi}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} = 1.218$$

$$P_{rad} = 73\, I^2{}_{rms} \Longrightarrow R_{rad} = 73$$

$$\eta = \frac{R_{rad}}{R_{rad}+R_{Loss}} \Longrightarrow \eta = \frac{R_{rad}}{R_t} \Longrightarrow R_t = \frac{R_{rad}}{\eta} = \frac{73}{0.5} = 146\,\Omega$$

$$P_{in} = R_t\, I^2{}_{rms} \Longrightarrow I_{rms} = \sqrt{\frac{100}{146}} = 0.83 \text{ A}$$

$$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60I_m}{r}\right)^2\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2\vec{a}_r$$

$$\vec{W}_{av} = \frac{1}{2(120\pi)}\left(\frac{60\times0.83\sqrt{2}}{500}\right)^2\left[\frac{\cos(\frac{\pi}{2}\cos60)}{\sin60}\right]^2\vec{a}_r$$

$$\vec{W}_{av} = 17.5\mu \ \vec{a}_r \frac{\text{w}}{\text{m}^2}$$

.3 A 0.75m transmitting dipole antenna of 4mm radius situated with its at origin radiates a power of 200W at a frequency of 100 MHz , if the directivity of the antenna is 1.52 , determine , assume $\sigma_{copper} = 58$ MS/m .

1-The current fed to the dipole.

2-The electric field at (2 Km , 30° , 60°).

3-HPBW of the antenna.

4-The power gain.

The solution:- $E_\theta = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$

1- $\lambda = \frac{c}{F} = \frac{3\times10^8}{100\times10^6} = 3\text{m}$, $\frac{L}{\lambda} = \frac{0.75}{3} \Rightarrow L = 0.25\lambda \Rightarrow L = \frac{\lambda}{4}$

$\cos\left(\frac{\beta L}{2}\right) = \cos\left(\frac{2\pi}{2\lambda}\frac{\lambda}{4}\right) = \cos\left(\frac{\pi}{4}\right) = 0.707$

$\vec{E} = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta$, $H_\phi = \frac{E_\theta}{\eta}$

$\vec{W}_{av} = \frac{1}{2} R_e\left[\vec{E}\otimes\vec{H}^*\right]$

$\vec{W}_{av} = \frac{1}{2} R_e\left[\frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60 I_m e^{j\beta r}}{\eta r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\phi\right]$

$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60 I_m}{r}\right)^2\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\vec{a}_r$

$$D = 4\pi\frac{\left|\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right|^2_{max}}{2\pi\int_0^\pi\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\sin\theta\, d\theta}$$

$\int_0^\pi\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}d\theta = 4\pi\frac{(0.293)^2}{2\pi(1.52)}$, $\int_0^\pi\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}d\theta = 0.1129$

$P_{rad} = \int\int\vec{W}_{av}\cdot\vec{ds}$, $\vec{ds} = r^2\sin\theta\, d\theta d\phi\,\vec{a}_r$

$$P_{rad} = \int_0^{2\pi}\int_0^\pi\frac{1}{2\eta}\left(\frac{60 I_m}{r}\right)^2\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]^2\vec{a}_r\; r^2\sin\theta\, d\theta d\phi\,\vec{a}_r$$

$$P_{rad} = 60 I^2_{rms}\int_0^\pi\frac{\left[\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707\right]^2}{\sin\theta}\,d\theta$$

$200 = 6.774\, I^2_{rms} \Rightarrow I_{rms} = \sqrt{\frac{200}{6.774}} = 5.4$ A

$$2\text{- } \vec{E} = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}\right]\vec{a}_\theta$$

$$|E_\theta| = \frac{60(\sqrt{2}\times 5.4)}{2000}\left[\frac{\cos\left(\frac{\pi}{4}\cos 30\right)-0.707}{\sin 30}\right] = 32.3\text{m V/m}$$

3- HPBW

$$E(\theta,\phi) = \frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}$$

$$0.707(max) = E(\theta,\phi)$$

$$0.707(0.293) = \frac{\cos\left(\frac{\pi}{4}\cos\theta\right)-0.707}{\sin\theta}$$

$$0.207\sin\theta_h = \cos\left(\frac{\pi}{4}\cos\theta\right) - 0.707$$

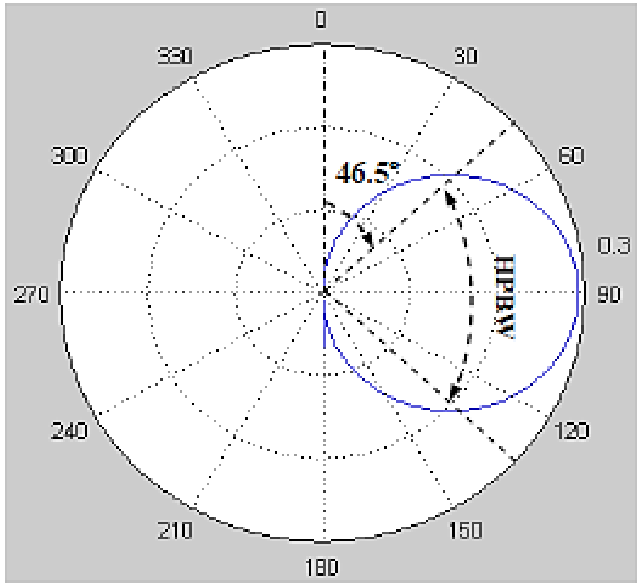
$$\theta_h = \sin^{-1}\frac{1}{0.207}\left[\cos\left(\frac{\pi}{4}\cos\theta\right) - 0.707\right]$$

$$\theta_h = 46.5^o$$

$$\text{HPBW} = 2(90 - \theta_h)$$

$$\text{HPBW} = 2(90^o - 46.5^o)$$

$$\text{HPBW} = 87^o$$



θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.14
45°	0.2018
60°	0.25
90°	0.293

$$4\text{- } R_{Loss} = \frac{L}{2\pi a}\sqrt{\frac{\omega\mu}{2\sigma}} = \frac{0.75}{2\pi(0.004)}\sqrt{\frac{2\pi\times 100\times 10^6\times 4\pi\times 10^{-7}}{2\times 58\times 10^6}} = 0.0778$$

$$\eta = \frac{R_{rad}}{R_{rad}+R_{Loss}} = \frac{6.774}{6.774+0.0778} = 98.86\%$$

$$G = \eta D = 0.9886\times 1.52 = 1.502$$

.4 The radiation intensity of an antenna is given by:

$$U(\theta,\phi) = \begin{Bmatrix} B_o \, sin\theta \, sin^2\phi & 0 \le \theta \le \pi \quad 0 \le \phi \le \pi \\ 0 & \text{elsewhere} \end{Bmatrix}$$

Determine the directivity of the antenna.

The solution:- $D_g(\theta,\phi) = 4\pi \dfrac{U(\theta,\phi)}{\iint U(\theta,\phi)\ d\Omega}$

$$D_g(\theta,\phi) = 4\pi \frac{B_o \, sin\theta \, sin^2\phi}{\int_0^{\pi}\int_0^{\pi} B_o \, sin\theta \, sin^2\phi \ \ sin\theta d\theta d\phi} = 4\pi \frac{sin\theta \, sin^2\phi}{\int_0^{\pi}\int_0^{\pi} sin^2\theta \, sin^2\phi \ d\theta d\phi}$$

$$D_g(\theta,\phi) = 4\pi \frac{sin\theta \, sin^2\phi}{\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)} = \frac{16}{\pi} sin\theta \, sin^2\phi$$

5 For a dipole of length 2.5m operating at 120 MHz .Determine

1- Radiation resistance.

2- Directivity.

3- HPBW.

4- Radiation efficiency (assume the antenna to be made of a copper wire of 4mm radius).

5- Gain.

6-The electric field intensity at a distance of 13 Km in the direction of maximum radiation (assume $I_m = 1A$). Assume $\sigma_{copper} = 58$ MS/m

The solution:- $E_\theta = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$

1- $\lambda = \frac{c}{F} = \frac{3\times10^8}{120\times10^6} = 2.5\text{m}$, $\frac{L}{\lambda} = \frac{2.5}{2.5} \Rightarrow L = \lambda$

$$\cos\left(\frac{\beta L}{2}\right) = \cos\left(\frac{2\pi}{2\lambda}\lambda\right) = \cos(\pi) = -1$$

$$\vec{E} = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]\vec{a}_\theta \quad , \quad H_\phi = \frac{E_\theta}{\eta}$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\vec{E}\otimes\vec{H}^*\right]$$

$$\vec{W}_{av} = \frac{1}{2} R_e\left[\frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]\vec{a}_\theta \otimes \frac{-j60 I_m e^{j\beta r}}{\eta r}\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]\right]$$

$$\vec{W}_{av} = \frac{1}{2\eta}\left(\frac{60 I_m}{r}\right)^2\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]^2\vec{a}_r$$

$$P_{rad} = \int\int \vec{W}_{av}\cdot\vec{ds} \quad , \quad \vec{ds} = r^2\sin\theta\, d\theta d\phi\, \vec{a}_r$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}\left(\frac{60 I_m}{r}\right)^2\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]^2\vec{a}_r\; r^2\sin\theta\, d\theta d\phi\,\vec{a}_r$$

$$P_{rad} = 60 I^2_{rms}\int_0^{\pi}\frac{[\cos(\pi\cos\theta)+1]^2}{\sin\theta}d\theta, \quad \int_0^{\pi}\frac{[\cos(\pi\cos\theta)+1]^2}{\sin\theta}d\theta = 3.316$$

$$P_{rad} = 198.96\, I^2_{rms} \Rightarrow R_{rad} = 198.96\,\Omega$$

2- $D = 4\pi\frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\, d\Omega} = 4\pi\frac{|E_{\theta max}|^2}{\int_0^{2\pi}\int_0^{\pi}|E_\theta|^2\sin\theta\, d\theta}$

θ	$\lvert E(\theta,\phi)\rvert$
0	0
30°	0.174
45°	0.5572
60°	1.1547
90°	2

$$D = 4\pi\frac{\left|\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right|^2_{max}}{2\pi\int_0^{\pi}\left|\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right|^2\sin\theta\, d\theta} = 4\pi\frac{(2)^2}{2\pi(3.316)} = 2.41$$

3- HPBW

$$E(\theta,\phi)=\frac{cos(\pi cos\theta)+1}{sin\theta}$$

$$0.707(max)=E(\theta,\phi)$$

$$0.707(2)=\frac{cos(\pi cos\theta)+1}{sin\theta}$$

$$1.414 sin\theta_h=cos(\pi cos\theta)+1$$

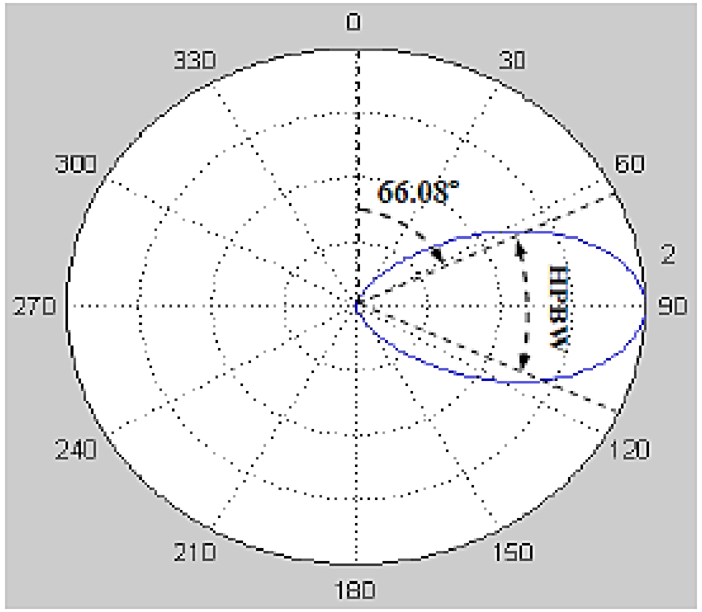
$$\theta_h=sin^{-1}\frac{1}{1.414}[cos(\pi cos\theta)+1]$$

$$\theta_h=66.08^o$$

$$\text{HPBW}=2(90-\theta_h)$$

$$\text{HPBW}=2(90^o-66^o)$$

$$\text{HPBW}=48^o$$



4- $$R_{Loss}=\frac{L}{2\pi a}\sqrt{\frac{\omega\mu}{2\sigma}}=\frac{2.5}{2\pi(0.004)}\sqrt{\frac{2\pi\times120\times10^6\times4\pi\times10^{-7}}{2\times58\times10^6}}=0.285$$

$$\eta=\frac{R_{rad}}{R_{rad}+R_{Loss}}=\frac{198.96}{198.96+0.285}=99.8\%$$

5- $G=\eta D=0.998\times2.41=2.405$

6- $$E_\theta=\frac{j60I_m e^{-j\beta r}}{r}\left[\frac{cos(\pi cos\theta)+1}{sin\theta}\right]\Rightarrow|E_\theta|=\frac{60\times1}{13000}(2)^2=9.23\text{m V/m}$$

14 Transmitting and receiving antenna operating at 1GHz with gains 20 and 15 dB respectively are separated by a distance of 1Km . find the maximum Power delivered to the load when the total power is 150W.

The solution:- $G_t = 20dB = 10^{\left(\frac{20}{20}\right)} = 10$, $G_r = 15dB = 10^{\left(\frac{15}{20}\right)} = 5.6$

$$\lambda = \frac{C}{F} = \frac{3\times10^8}{10^9} = 0.3\text{m}$$

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

$$P_r = 150 \times 10 \times 5.6 \times \left(\frac{0.3}{4\pi\times1000}\right)^2$$

$$P_r = 4.8\mu \text{ W}$$

.16 A- The maximum radiation intensity of a 90% efficiency antenna is 200mW/unit solid angle, find the directivity and gain when the point power is 40π mW.

The solution:- $\eta = \dfrac{P_{rad}}{P_{in}}$

$$P_{rad} = \eta \times P_{in}$$

$$P_{rad} = 0.90 \times 40\pi \times 10^{-3} = 113\,\text{m}$$

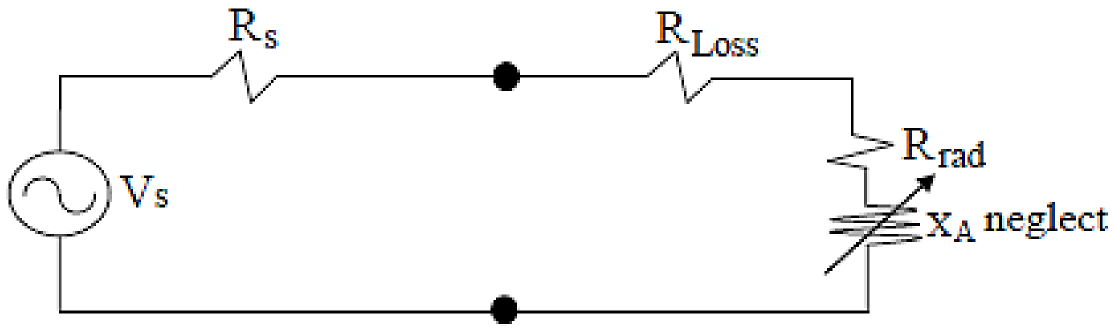
$$D = 4\pi \frac{U(\theta,\phi)|_{max}}{\int\int U(\theta,\phi)\ d\Omega} = 4\pi \frac{U(\theta,\phi)|_{max}}{P_{rad}}$$

$$D = 4\pi \frac{200\times10^{-3}}{113\times10^{-3}} = 22.22$$

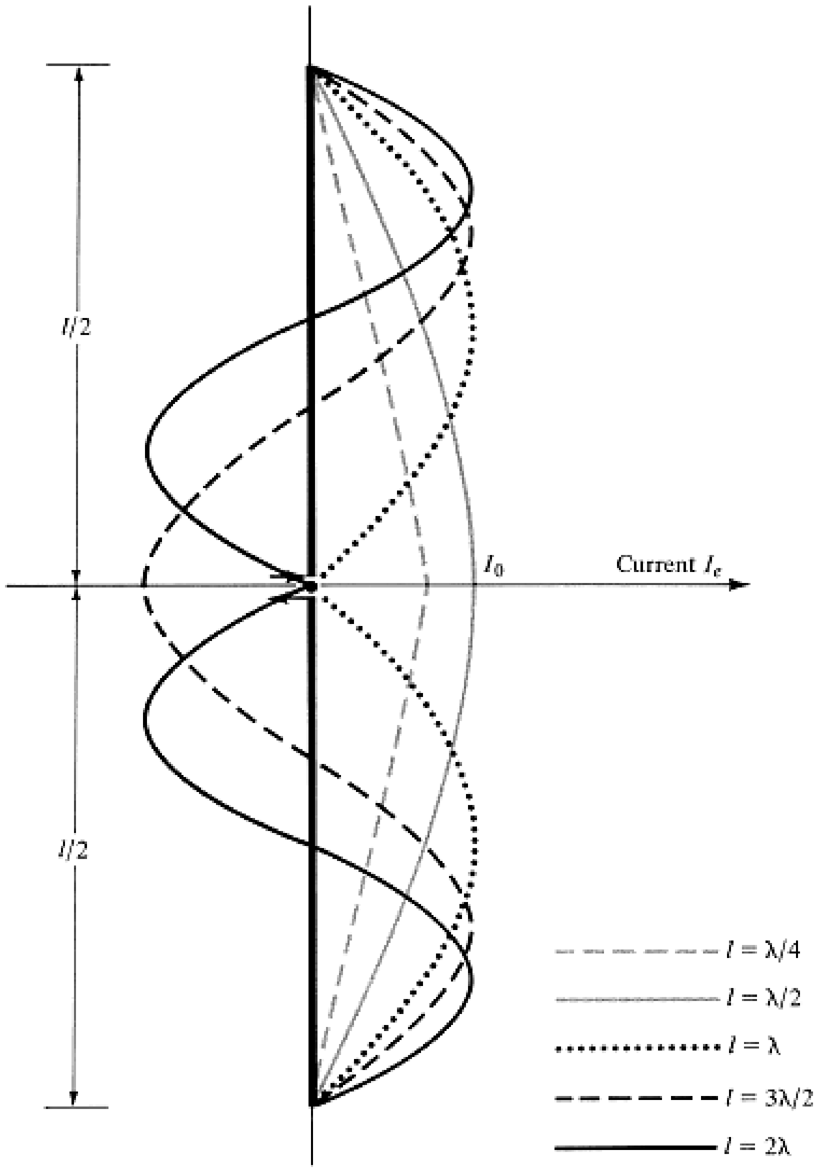
$$G = \eta D = 0.90 \times 22.22 = 20$$

B- A λ/2 dipole with loss resistance of 1Ω is connected to a generator whose internal resistance is 50 Ω. assuming that the peak voltage of the generator is 2V. find the power radiated by the antenna.

The solution:-



$$R_{rad} = 73\Omega \text{ for } \lambda/2$$
$$R_{loss} = 1\Omega$$
$$R_s = R_{in} = 50\Omega$$
$$V_s = 2\text{V}$$
$$I_p = \frac{V_s}{R_{in}+R_{rad}+R_{loss}}$$
$$I_p = \frac{2}{50+73+1} = 16.13 \text{ mA}$$
$$I_{rms} = \frac{16.13\times10^{-3}}{\sqrt{2}} = 11.4 \text{ mA}$$
$$P_{rad} = {I_{rms}}^2 \, R_{rad}$$
$$P_{rad} = (11.4 \times 10^{-3})^2 \times 73$$
$$P_{rad} = 9.48 \text{ mW}$$



Current distributions along the length of a linear wire antenna.