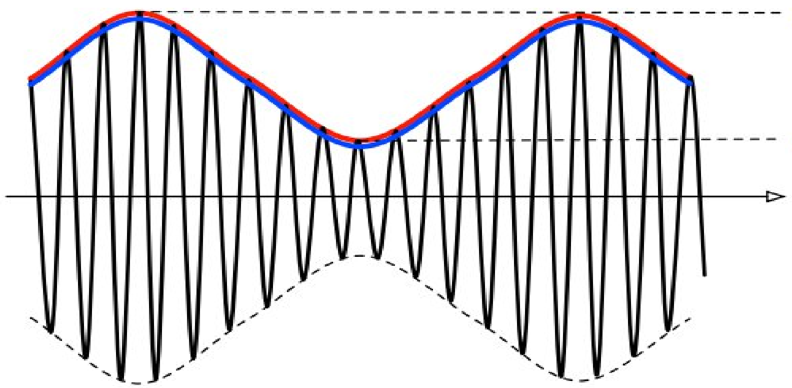
Case (i): $A_m/A_c < 1$

$A(t)$ amplitude function
$E(t)$ envelope function

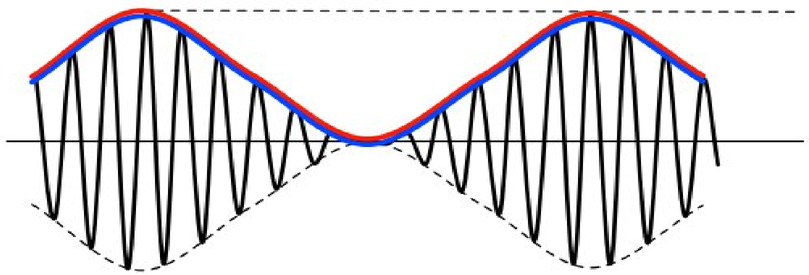


$E(t) = A(t) = [A_c + A_m \cos \omega_m t\,] \geq 0$ such that $E(t) \sim m(t)$ → $\mu < 1$

perfectly suitable to use an envelope detector

Case (ii): $A_m/A_c = 1$

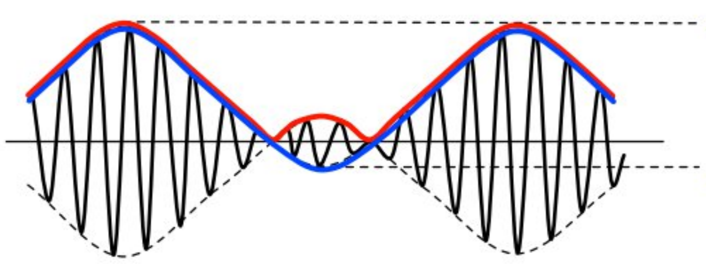
$A(t)$ amplitude function
$E(t)$ envelope function



$E(t) = A(t) = [A_c + A_m \cos \omega_m t\,] \geq 0$ such that $E(t) \sim m(t)$ → $\mu = 1$

borderline case for using an envelope detector

Case (iii): $A_m/A_c > 1$



$E(t) = | A_c + A_m \cos \omega_m t | \neq A(t)$ such that $E(t) \not\sim m(t)$ → $\mu > 1$

overmodulation: cannot use an envelope detector

When $A_c < A_m$ *hence* $\mu > 1$ (overmodulation). In this case, the option of envelope detection is no longer viable. We then need to use synchronous demodulation. Note that synchronous demodulation can be used for any value of μ . The envelope detector, which is considerably simpler and less expensive than the synchronous detector, can be used only for $\mu < 1$

We can determine the spectrum of the AM signal using the frequency shifting/ modulation property.

Let $x(t) = \left[A_c + A_m \cos\omega_m t \right]$ such that

$$\varphi_{\mathrm{AM}}(t) = \underbrace{\left[A_c + A_m \cos\omega_m t \right]}_{x(t)} \cos\omega_c t = x(t)\cos\omega_c t$$

with

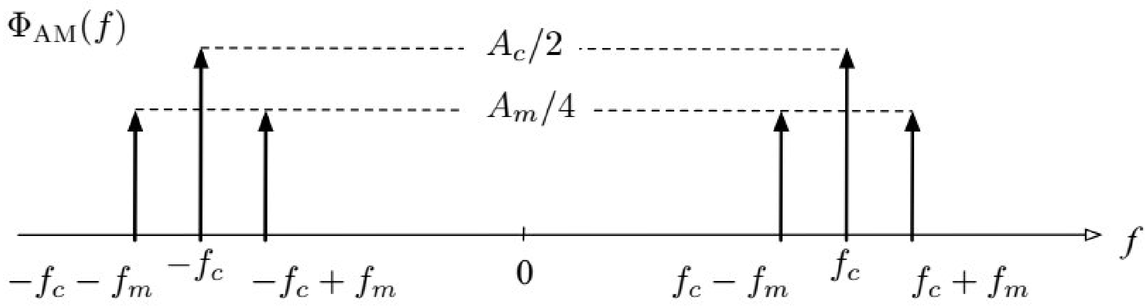
$$X(f) = A_c\,\delta(f) + \frac{A_m}{2}\left[\delta(f - f_m) + \delta(f + f_m) \right]$$

Therefore

$$\Phi_{\mathrm{AM}}(f) = \frac{1}{2}\left[X(f - f_c) + X(f + f_c) \right]$$

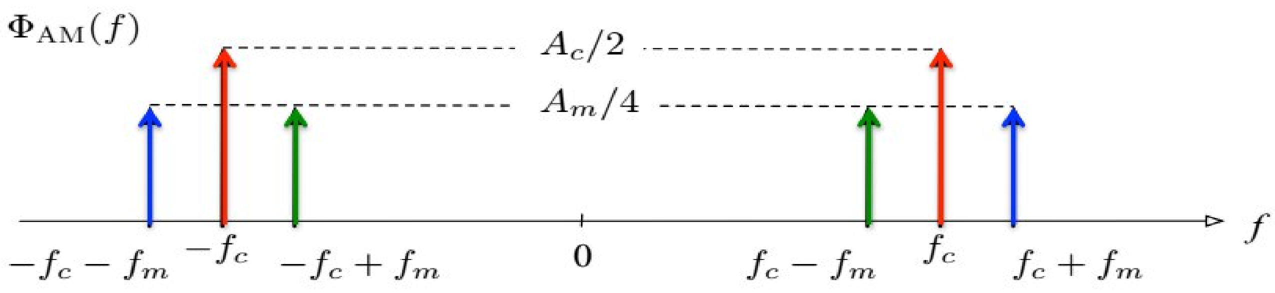
Putting everything together we have

$$\begin{aligned}\Phi_{\mathrm{AM}}(f) = \frac{A_c}{2}&\left[\delta(f - f_c) + \delta(f + f_c) \right] \\ &+ \frac{A_m}{4}\left[\delta\big(f - (f_c + f_m)\big) + \delta\big(f - (f_c - f_m)\big) \right] \\ &\quad + \frac{A_m}{4}\left[\delta\big(f + (f_c - f_m)\big) + \delta\big(f + (f_c + f_m)\big) \right]\end{aligned}$$



Alternatively, we can first expand $\varphi_{AM}(t)$ using trigonometric identities:

$$\varphi_{AM}(t) = A_c \cos\omega_c t + A_m \cos\omega_m t \cos\omega_c t,$$
$$= A_c \cos\omega_c t + \frac{A_m}{2}\cos(\omega_c + \omega_m)t + \frac{A_m}{2}\cos(\omega_c - \omega_m)t$$



Example: Sketch $\varnothing_{AM}(t)$ for modulation indices of $\mu = 0.5$ and $\mu = 1$, when $m(t) = B\cos\omega_m t$.

Solution:

In this case m_p=B

$$\mu = \frac{B}{A_C}$$

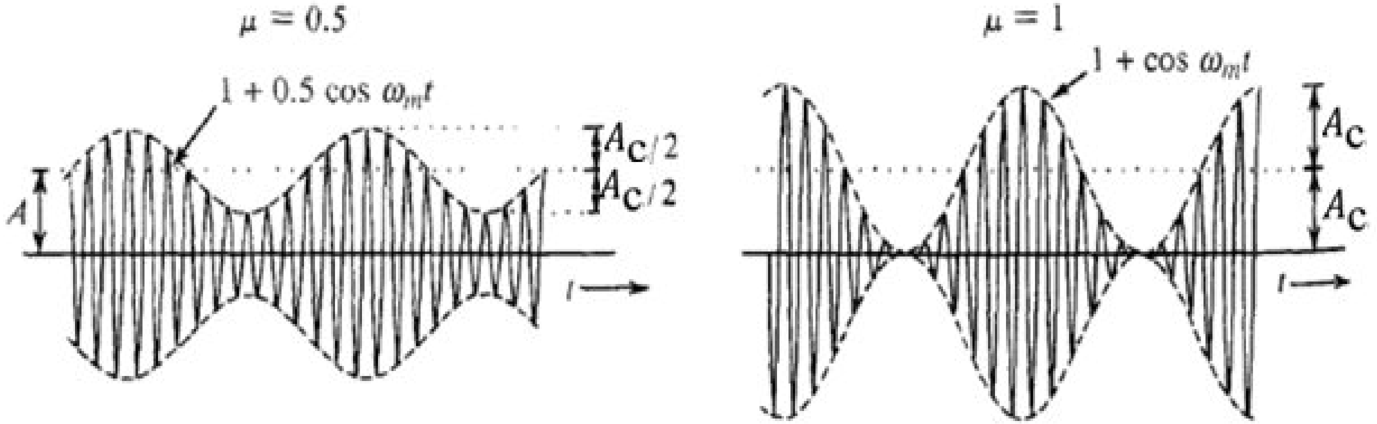
Hence B= μA_C

$$m(t) = B\cos\omega_m t = \mu A_C \cos\omega_m t$$

Therefore,

$$\varphi_{AM}(t) = [A_C + m(t)]\cos\omega_c t = A_C[1 + \mu\cos\omega_m t]\cos\omega_c t$$

Figure shows the modulated signals corresponding to $\mu = 0.5$ and $\mu = 1$, respectively.



The advantage of envelope detection in AM has its price. In AM, the carrier term does not carry any information, and hence, the carrier power is wasted

Consider the AM signal $\varphi_{AM}(t)$ modulating signal generated by the baseband message/modulating signal $m(t)$ bandlimited to B_m-Hz.

$$\varphi_{\mathrm{AM}}(t) = \underbrace{A_c \cos \omega_c t}_{\text{carrier}} + \underbrace{m(t) \cos \omega_c t}_{\text{sidebands}}$$

Objective:

- Compute the power of the AM signal P_φ, and
- Express it in terms of the carrier power P_c and the sideband power P_s

$$\varphi_{\mathrm{AM}}(t) = \underbrace{A_c \cos \omega_c t}_{\text{carrier}} + \underbrace{m(t) \cos \omega_c t}_{\text{sidebands}}$$

$$P_\varphi = \overline{A_c^2 \cos^2 \omega_c t + 2A_c m(t) \cos^2 \omega_c t + m^2(t) \cos^2 \omega_c t}$$

$$\overline{A_c^2 \cos^2 \omega_c t} = A_c^2/2$$

Remember !

The average power of $x(t) = A\cos(\omega_0 t + \theta)$

$$P = \frac{1}{T_0}\int_0^{T_0} [x(t)]^2\, dt = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} A^2 \cos^2(\omega_0 t + \theta)\, dt$$

$$= \frac{A^2\omega_0}{2\pi}\int_0^{2\pi/\omega_0} \frac{1}{2}[1 + \cos(2\omega_0 t + 2\theta)]\, dt = \frac{A^2}{2}$$

$$\varphi_{\mathrm{AM}}(t) = \underbrace{A_c \cos \omega_c t}_{\text{carrier}} + \underbrace{m(t) \cos \omega_c t}_{\text{sidebands}}$$

$$P_\varphi = \overline{A_c^2 \cos^2 \omega_c t + 2A_c m(t) \cos^2 \omega_c t + m^2(t) \cos^2 \omega_c t}$$

because: $f_c \gg B_m$ and $\overline{m(t)} = 0$

$$2A_c\, \overline{m(t) \cos^2 \omega_c t} = 2A_c\, \overline{m(t)}\ \overline{\cos^2 \omega_c t} = 0$$

$$\varphi_{\mathrm{AM}}(t) = \underbrace{A_c \cos \omega_c t}_{\text{carrier}} + \underbrace{m(t) \cos \omega_c t}_{\text{sidebands}}$$

$$P_\varphi = \overline{A_c^2 \cos^2 \omega_c t + 2A_c m(t) \cos^2 \omega_c t + m^2(t) \cos^2 \omega_c t}$$

because: $f_c \gg B_m$

$$\overline{m^2(t) \cos^2 \omega_c t} = \overline{m^2(t)}\ \overline{\cos^2 \omega_c t} = \overline{m^2(t)}/2$$

Sideband and Carrier Power

$$\varphi_{\mathrm{AM}}(t) = \underbrace{A_c \cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{\text{sidebands}}$$

$$P_\varphi = \overline{A_c^2\cos^2\omega_c t + 2A_c m(t)\cos^2\omega_c t + m^2(t)\cos^2\omega_c t}$$

$$= A_c^2/2 \quad + \quad 0 \quad + \quad \overline{m^2(t)}/2$$

$$P_c = \frac{A_c^2}{2} \qquad P_s = \frac{\overline{m^2(t)}}{2}$$

$$P_\varphi = P_c + P_s$$

Define power efficiency:

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_\varphi} = \frac{P_s}{P_c + P_s}.$$

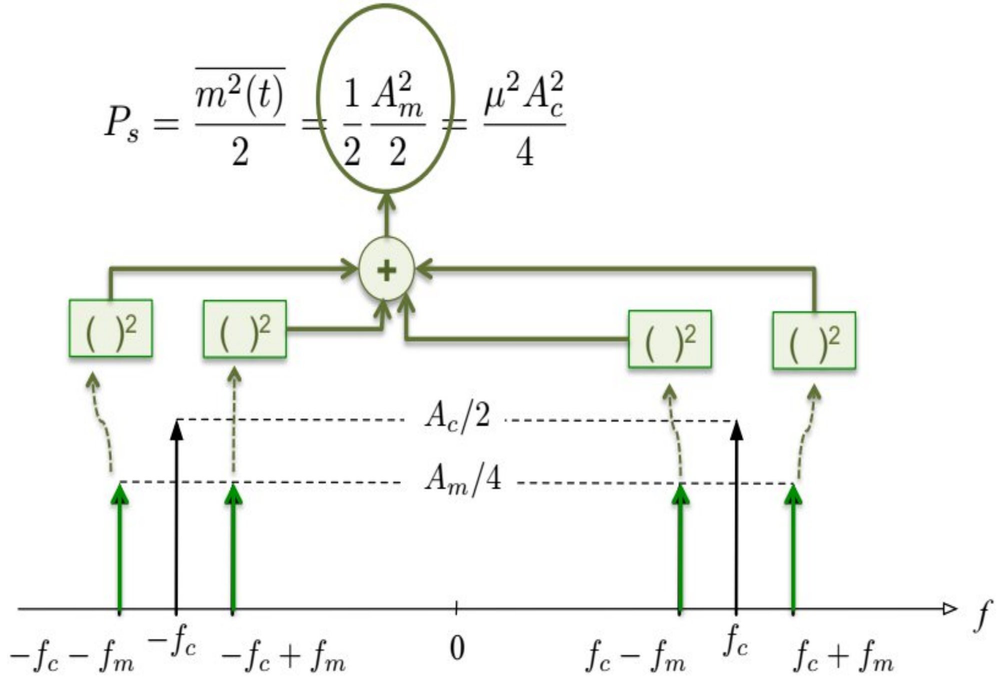
Example (single-tone modulation)**:**

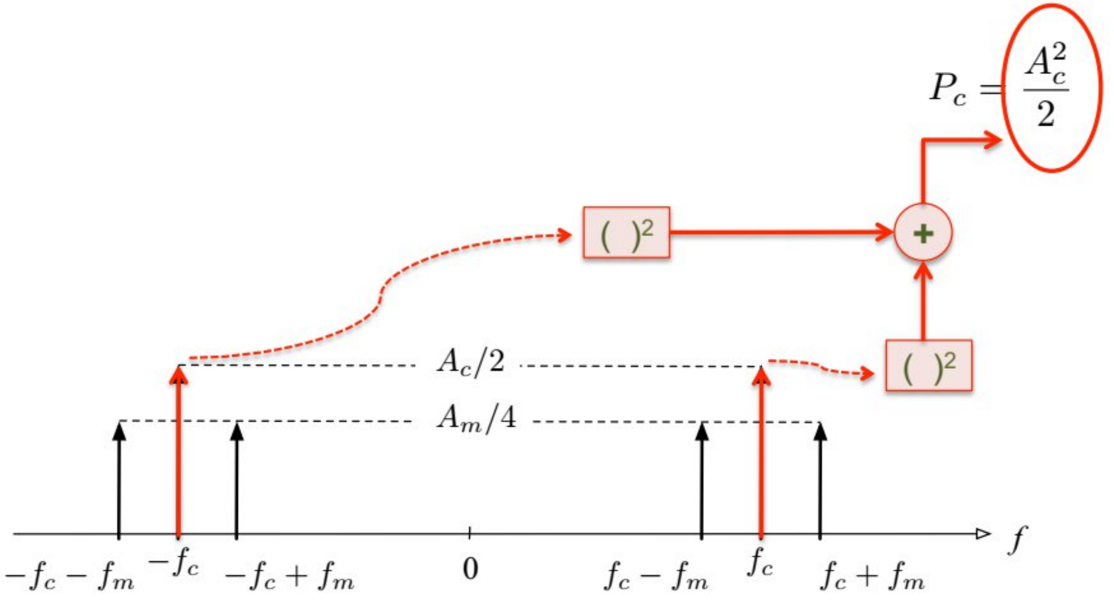
- Modulating signal: $m(t) = A_m \cos \omega_m t$,
- Carrier: $c(t) = A_c \cos \omega_c t$ with $fc \gg f_m$
- Modulation index: $\mu = A_m / A_c$
- Sideband Power:

$$P_s = \frac{\overline{m^2(t)}}{2} = \frac{1}{2}\frac{A_m^2}{2} = \frac{\mu^2 A_c^2}{4}$$

Carrier Power:

$$P_c = \frac{A_c^2}{2}$$





Example: One input of a conventional AM modulator is a 500 KHz carrier with amplitude of 20 Vp. The second input is a 10 KHz modulating signal that is of sufficient amplitude to cause a change in the output wave of ± 7.5 Vp. Express the modulated wave and draw its spectrum. Determine (i) The USB and LSB frequencies, (ii) Peak amplitude of the modulated carrier and the upper and lower side frequency voltages, (iii) Modulation coefficient and percent modulation.

Solution: Given that $A_c = 20\ \text{V}_\text{p}$, $f_c = 500\text{X}10^3$ Hz, $A_m = 7.5\ \text{V}_\text{p}$ and $f_m = 10\text{X}10^3$ Hz

Then $\mu = \dfrac{A_m}{A_c} = \dfrac{7.5}{20} = 0.375$

From the standard AM equation,
$$\varphi_{\text{AM}}(t) = A_c \cos\omega_c t + A_m \cos\omega_m t \cos\omega_c t,$$
$$= \boxed{A_c \cos\omega_c t} + \boxed{\frac{A_m}{2}\cos(\omega_c + \omega_m)t} + \boxed{\frac{A_m}{2}\cos(\omega_c - \omega_m)t}$$

$$\varphi_{\text{AM}}(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\cos 2\pi(f_c - f_m)t + \frac{\mu A_c}{2}\cos 2\pi(f_c + f_m)t$$
, we obtain

$$\varphi_{\text{AM}}(t) = 20\cos 2\pi 500\text{X}10^3 t + 3.75\cos(2\pi 510\text{X}10^3)t + 3.75\cos(2\pi 490\text{X}10^3)t$$

(i) The LSB frequency = 490 KHz, and the USB frequency = 510 KHz

(ii) **Time domain:**

The modulated carrier magnitude = 20 Vp.

The modulated LSB magnitude = The modulated USB magnitude = 3.75 Vp.

Frequency domain:

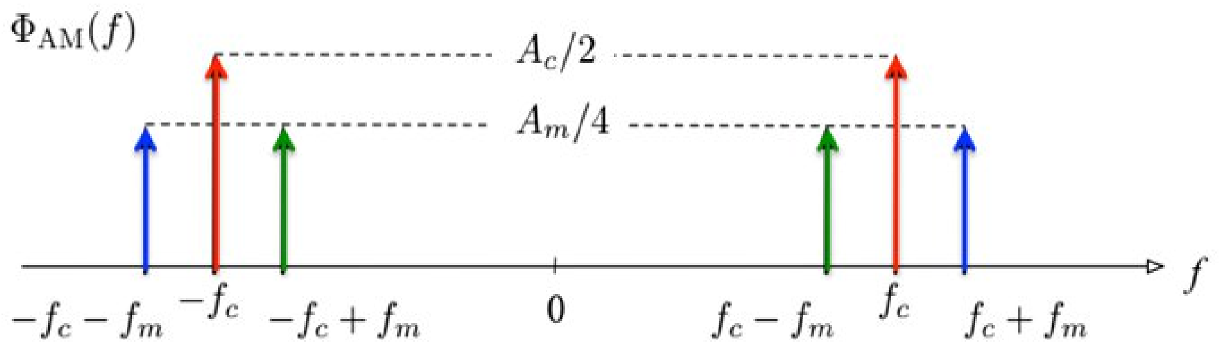
The frequency domain representation of the modulated signal is given by

$$\Phi_{AM}(f) = 10\left[\delta(f - 500X10^3) + \delta(f + 500X10^3)\right] + 1.875\left[\delta(f - 490X10^3) + \delta(f + 490X10^3)\right] + 1.875\left[\delta(f - 510X10^3) + \delta(f + 510X10^3)\right]$$

The spectral magnitude of the modulated carrier = 10 Vp.

The spectral magnitude of the modulated LSB = 1.875 Vp

The spectral magnitude of the modulated USB = 1.875 Vp.



$$\Phi_{AM}(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_m}{4}\left[\delta\big(f - (f_c + f_m)\big) + \delta\big(f - (f_c - f_m)\big)\right] + \frac{A_m}{4}\left[\delta\big(f + (f_c - f_m)\big) + \delta\big(f + (f_c + f_m)\big)\right]$$

(iii) The Modulation index = $\mu = \frac{A_m}{A_c} = \frac{7.5}{20} = 0.375$,

The percent modulation = 37.5 %

Power efficiency:

$$\eta = \frac{P_s}{P_c + P_s} = \frac{\mu^2 A_c^2/4}{A_c^2/2 + \mu^2 A_c^2/4} = \frac{\mu^2}{2 + \mu^2}$$

Given the power efficiency:

$$\eta = \frac{P_s}{P_c + P_s} = \frac{\mu^2 A_c^2/4}{A_c^2/2 + \mu^2 A_c^2/4} = \frac{\mu^2}{2 + \mu^2}$$

- To use an envelope detector we must have $\mu \leq 1$
- Maximum power efficiency at $\mu = 1$

$$\eta_{\max} = \left.\frac{\mu^2}{2 + \mu^2}\right|_{\mu=1} = \frac{1}{3}.$$

$$\eta_{max} = 33\%$$

Example: Determine η and the percentage of the total power carried by the sidebands of the AM wave for tone modulation when (a) μ =0.5 and (b) μ =0.3.

Solution: For $\mu = 0.5$,

$$\eta = \frac{\mu^2}{2+\mu^2}100\% = \frac{(0.5)^2}{2+(0.5)^2}100\% = 11.11\%$$

Hence, only about 11% of the total power is in the sidebands

For $\mu = 0.3$,

$$\eta = \frac{(0.3)^2}{2+(0.3)^2}100\% = 4.3\%$$

Hence, only 4.3% of the total power is the useful power (power in sidebands).

Example : Find the power content of the DSB-SC wave with single tone message and find the power efficiency

Solution

$$\varphi_{DSB-SC}(t) = m(t)\ c(t) = A_m \cos(2\pi f_m t) A_c \cos(2\pi f_c t) = [\frac{A_m A_c}{2}\cos(2\pi(f_c + f_m)t + \frac{A_m A_c}{2}\cos(2\pi(f_c - f_m)t]$$

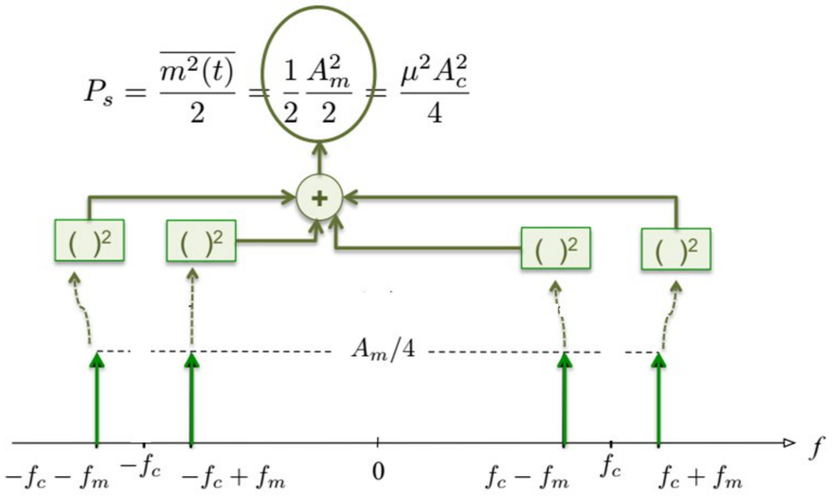
$$P_{USB} = \frac{\left(\frac{A_m A_c}{2}\right)^2}{2} = \frac{{A_c}^2{A_m}^2}{8}$$

and same for $P_{LSB} = \frac{{A_c}^2{A_m}^2}{8}$

$$P_{Total} = P_{USB} + P_{LSB} = \frac{{A_c}^2{A_m}^2}{4}$$

Power Efficiency $\eta = \frac{Useful\ Power}{Total\ Power} = \frac{P_{USB}}{P_{Total}} = \frac{P_{LSB}}{P_{Total}}$

$$= \frac{{A_c}^2{A_m}^2}{8} / \frac{{A_c}^2{A_m}^2}{4} = 50\%$$



- A scheme in which only one sideband is transmitted is known as **single-sideband . (SSB) transmission,** which requires only one-half the bandwidth of the DSB signal. i.e. we send either the upper sideband or the lower sideband of a DSB-SC signal.

Why SSB Modulation?

- The basic AM has a carrier which does not carry information—Inefficient power usage
- The basic AM has two sidebands contain the same information—Additional loss of power
- DSBSC has two sidebands, containing the same information—Loss of power
- Standard AM and DSB-SC amplitude modulation techniques are wasteful of bandwidth because they both require transmission bandwidths of $2B_m$Hz where B_m is the bandwidth of the baseband modulating signal ***m(t)***.
- Therefore, the basic AM and DSBSC are bandwidth and power inefficient while SSB is bandwidth and power efficient.

- **Modulation of SSB is** similar to DSBSC. Only change the settings of the BPF (center frequency, bandwidth).
- **Demodulation:** similar to DSBSC (coherent or synchronous detection)

Two methods are commonly used to generate SSB signals. The **first** method uses sharp cut-off filters to eliminate the undesired sideband, and the **second** method uses phase-shifting modulator to achieve the same goal.

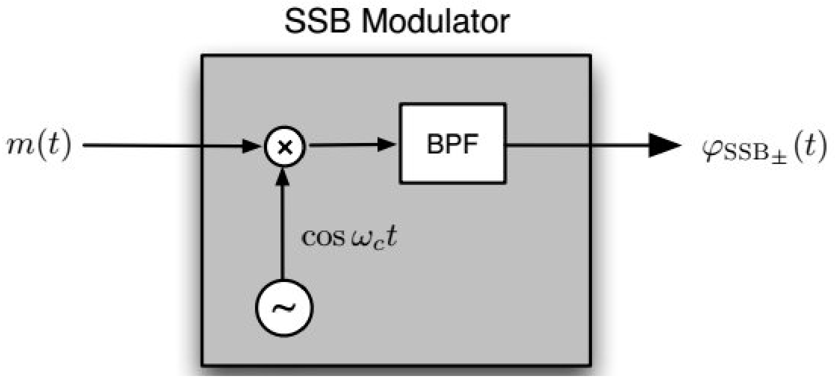
Selective Filtering Method

This is the most commonly used method of generating SSB signals. In this method, a DSBSC signal is passed through a **sharp cutoff filter** to eliminate the undesired sideband. Such an operation requires an ideal filter, which is unrealizable. It can, however, be realized closely if there is some separation between the passband and the stopband.

Two step process:

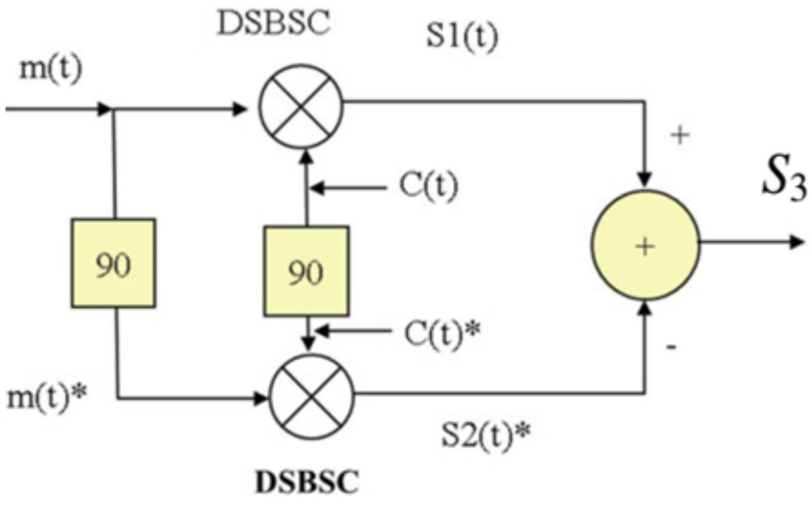
i. Generate a DSB-SC signal;

ii. Filter out one of the sidebands using a BPF



Fortunately, the voice signal provides this condition, because its spectrum shows little power content at the origin . In addition, tests have shown that for speech signals, frequency components below 300 Hz are not important. In other words, we may suppress all speech components below 300 Hz without affecting the intelligibility appreciably.

Phase-Shifting Modulator : SSB modulation uses two product modulators as shown below



$$S_3 = A_c A_m \cos(\omega_c + \omega_m)t$$

- $m(t) = A_m \cos(\omega_m t)$

$m(t)^* = A_m \sin(\omega_m t)$ – (Hilbert Transform)

- $C(t) = A_m \cos(\omega_c t)$

$C(t)^* = A_c \sin(\omega_c t)$ – (Hilbert Transform)

Solving for S_1, S_2, and S_3, we obtain:

- $S_1(t) = A_c A_m \cos(\omega_m t) \cos(\omega_c t)$
- $S_2(t) = A_c A_m \sin(\omega_m t) \sin(\omega_c t)$

$$S_3(t) = S_1(t) - S_2(t)$$
$$= A_c A_m \cos(\omega_m t) \cos(\omega_c t) - A_c A_m \sin(\omega_m t) \sin(\omega_c t)$$

Using the following formula:

cos$A$ cos$B$ = 1/2cos(A + B) + 1/2cos(A − B)
sin$A$ sin$B$ =1/2cos(A − B) − 1/2cos(A + B)

Solving for S_3, we get:

$$S_3 = A_c A_m \cos(\omega_c + \omega_m)t$$

Time domain signal for SSB

$$\varphi_{SSB_+}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

$$\varphi_{SSB_-}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

Frequency domain

$$\Phi_{SSB_+}(f) = M_+(f - f_c) + M_-(f + f_c)$$

$$\Phi_{SSB_-}(f) = M_-(f - f_c) + M_+(f + f_c)$$