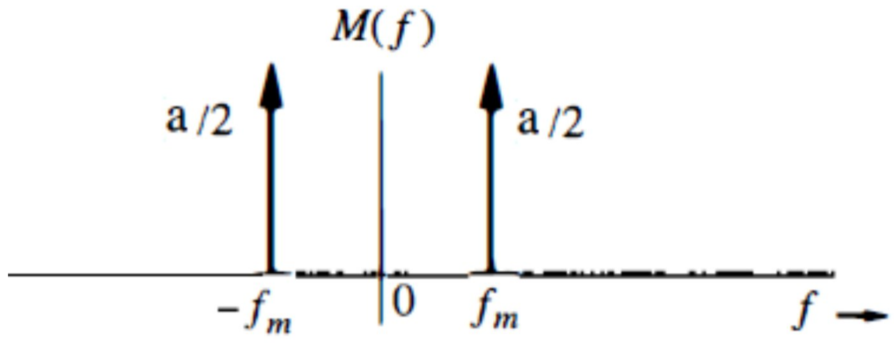
Example: For a baseband signal $m(t) = a\cos(w_m t) = a\cos(2\pi f_m t)$.

1- Find the DSB-SC signal, and sketch its spectrum. Identify the upper and lower sidebands (USB and LSB). Verify that the DSB-SC modulated signal can be demodulated by the synchronous detection or coherent detection).

Solution The spectrum of the baseband signal $m(t) = a\cos(w_m t) = a\cos(2\pi f_m t)$ is given by

$$M(\omega) = \mathscr{F}[m(t)] = a\pi[\delta(\omega-\omega_m) + \delta(\omega-\omega_m) \quad Or$$

$$M(f) = \mathscr{F}[m(t)] = \frac{a}{2}[\delta(f-f_m) + \delta(f+f_m)]$$



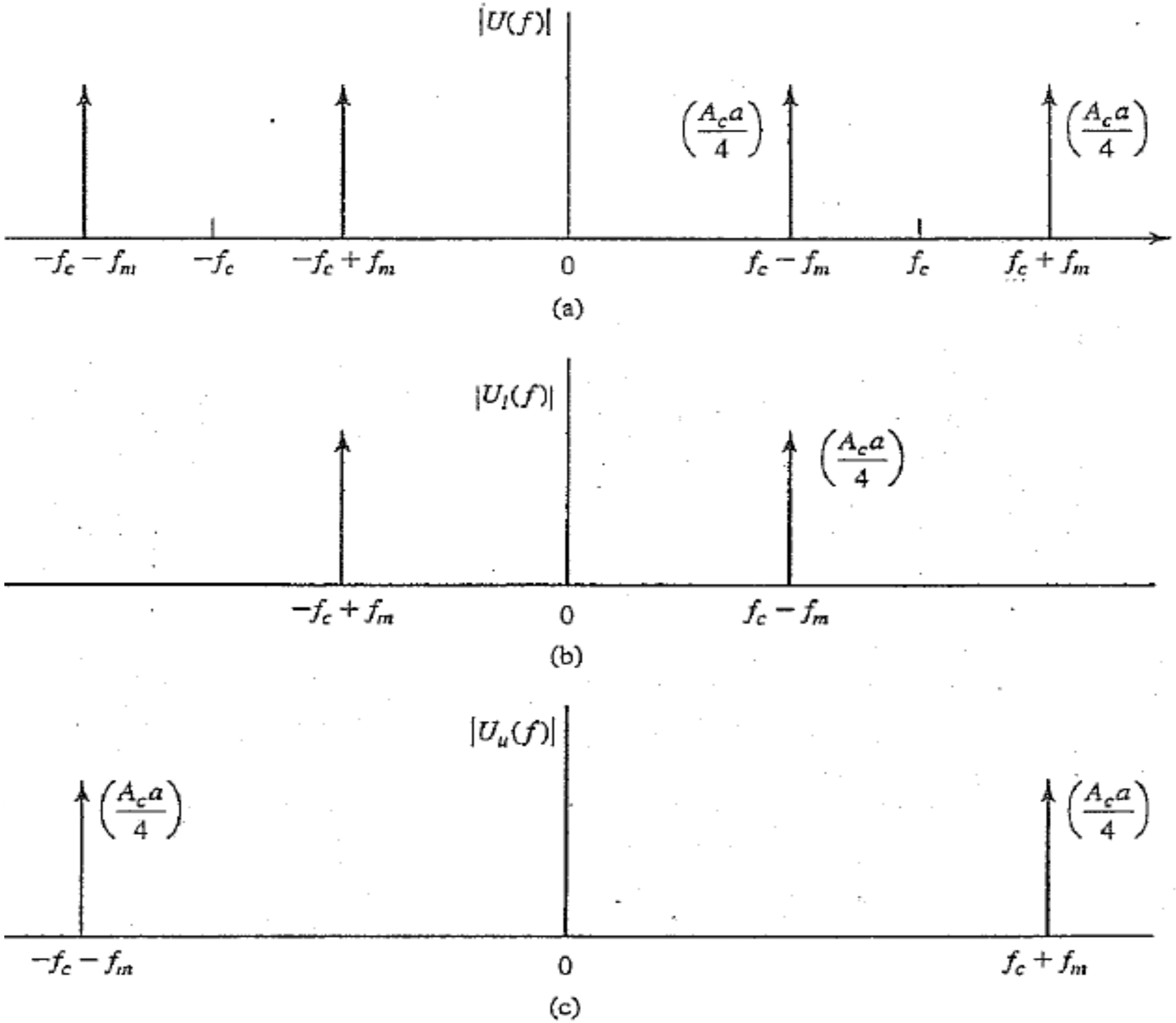
The DSB-SC AM is expressed in the time domain as

$$\begin{aligned} u(t) = m(t)c(t) &= A_c a\cos 2\pi f_m t\cos(2\pi f_c t + \phi_c) \\ &= \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t + \phi_c] + \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t + \phi_c] \end{aligned}$$

In the frequency domain, the modulated signal has the form

$$\begin{aligned} U(f) = &\frac{A_c a}{4}\left[e^{j\phi_c}\delta(f - f_c + f_m) + e^{-j\phi_c}\delta(f + f_c - f_m)\right] \\ &+ \frac{A_c a}{4}\left[e^{j\phi_c}\delta(f - f_c - f_m) + e^{-j\phi_c}\delta(f + f_c + f_m)\right] \end{aligned}$$

(a) The (magnitude) spectrum of a AM DSB-SC signal for a sinusoidal message signal and
(b) its lower sideband
(c) upper sidebands.



The lower and upper sideband of *u(t)* is the signal

$$u_\ell(t) = \frac{A_c a}{2} \cos[2\pi(f_c - f_m)t + \phi_c]$$

$$u_u(t) = \frac{A_c a}{2} \cos[2\pi(f_c + f_m)t + \phi_c]$$

2- Determine the power in the modulated signal, and the power in each of the sidebands.

Solution The message signal is $m(t) = a\cos 2\pi f_m t$, its power-spectral density is given by

$$S_m(f) = \frac{a^2}{4}\delta(f - f_m) + \frac{a^2}{4}\delta(f + f_m)$$

$$U(f) = \frac{A_c a}{4}\left[e^{j\phi_c}\delta(f - f_c + f_m) + e^{-j\phi_c}\delta(f + f_c - f_m)\right] + \frac{A_c a}{4}\left[e^{j\phi_c}\delta(f - f_c - f_m) + e^{-j\phi_c}\delta(f + f_c + f_m)\right]$$

Using Parseval's Relation $\int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$

$$P_u = \int_{-\infty}^{\infty} S_u(f)\,df = \frac{A_c^2 a^2}{4}$$

Because of symmetry of sidebands the powers the upper and lower sidebands, P_{us} and P_{ls}, are equal and given by

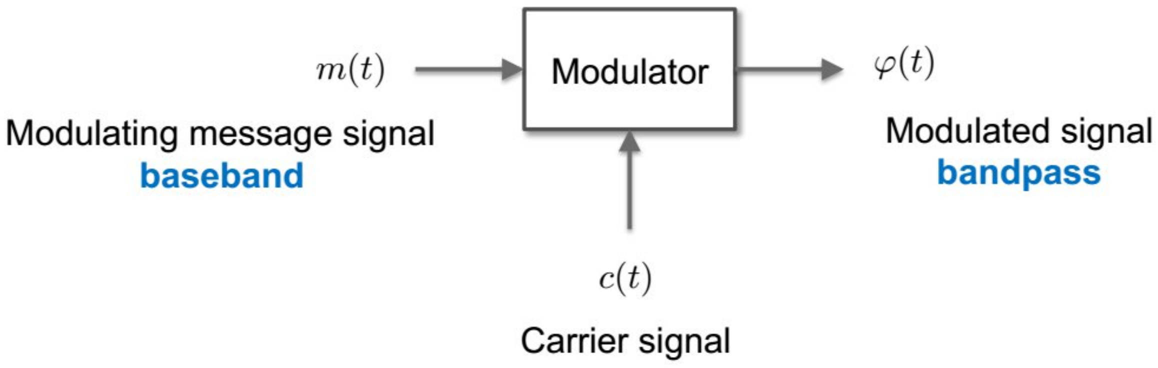
$$P_{us} = P_{ls} = \frac{A_c^2 a^2}{8}$$

- Modulated waveforms with suppressed carrier terms require fairly complex circuitry at the receiver to acquire **phase synchronization**, i.e., **coherent detection**, which makes the receivers expensive to manufacture.

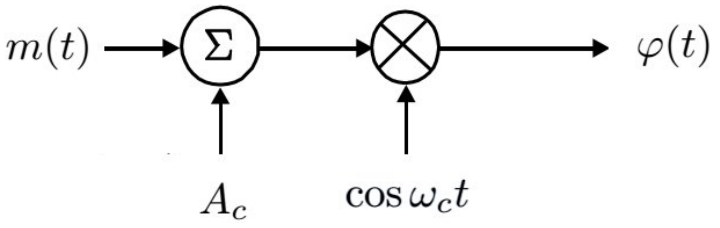
- In applications where we have one or few transmitters and a much larger number of receivers (e.g. AM/FM radio broadcasting) it makes economic sense that the **receivers** are as **simple** as possible.

- To facilitate simple demodulation we consider the idea of transmitting a **separate carrier term** in the same frequency band as a DSB-SC amplitude modulated signal.

$m(t)$ = modulating message signal
$c(t)$ = carrier signal
$\varphi(t)$ = modulated signal



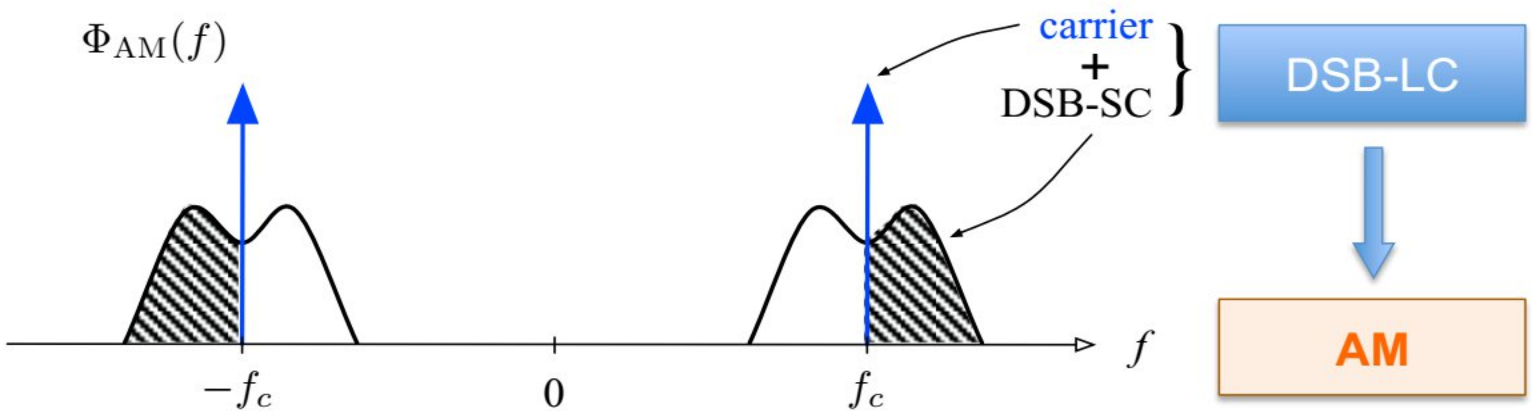
Let's consider amplitude modulation signals of the form: **φDSB-SC + carrier**



$$
\begin{aligned}
\varphi_{\text{AM}}(t) &= \varphi_{\text{DSB-SC}}(t) + \text{carrier} \\
&= m(t)\cos\omega_c t + A_c\cos\omega_c t \\
&= \left[\, A_c + m(t) \,\right]\, \cos\omega_c t
\end{aligned}
$$

with spectrum:

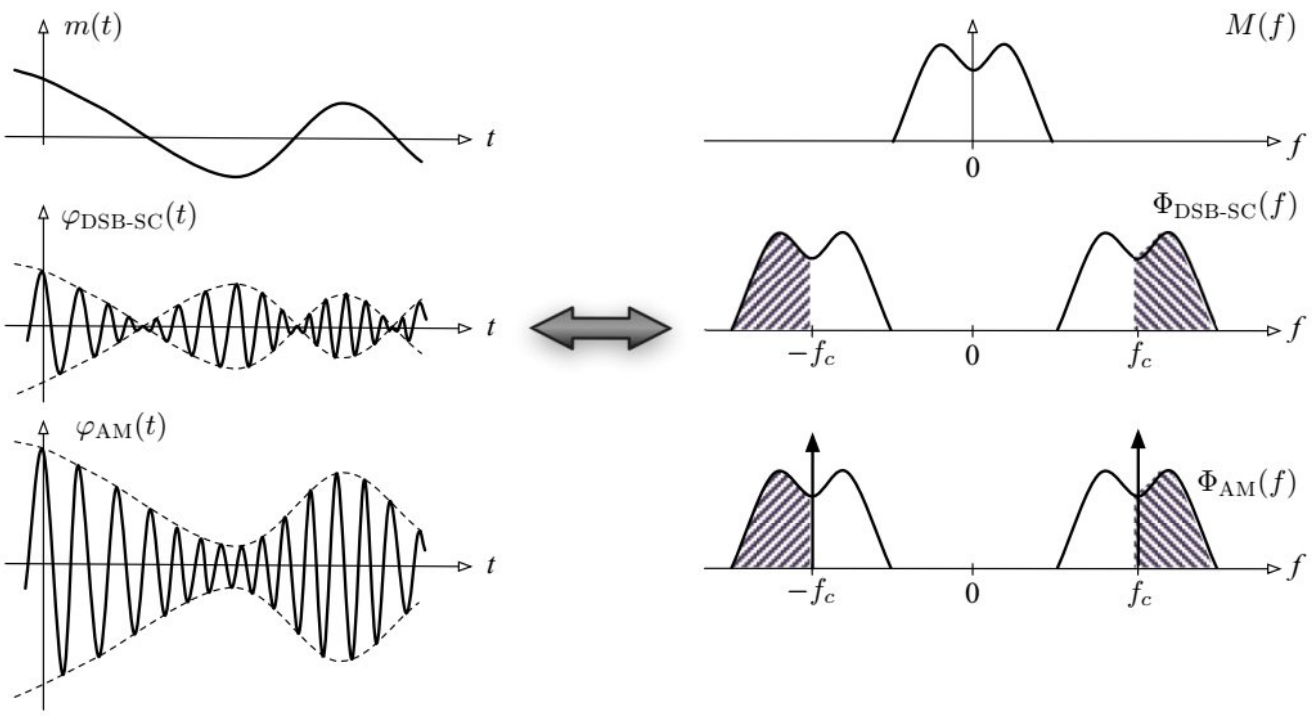
$$
\Phi_{\text{AM}} = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] + \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]
$$

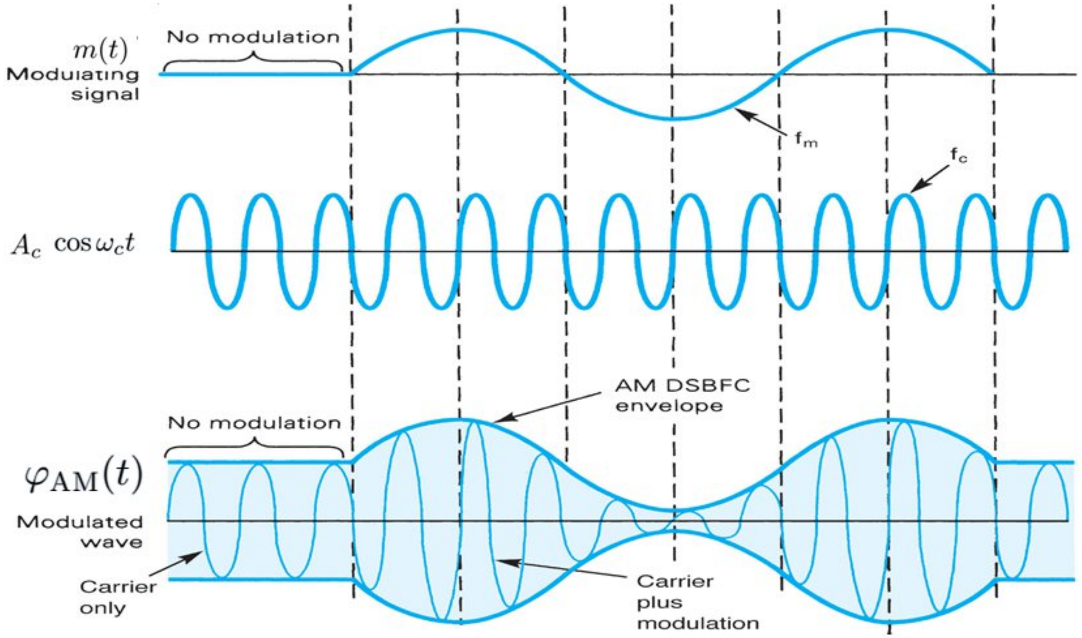


AM Frequency Spectrum and Bandwidth

AM: Time-domain analysis

$$\varphi_{\mathrm{AM}}(t) = \left[\, A_c + m(t) \,\right] \, \cos \omega_c t$$





AM modulator is nonlinear device and the output envelope is a complex wave made up of:

(a) DC voltage

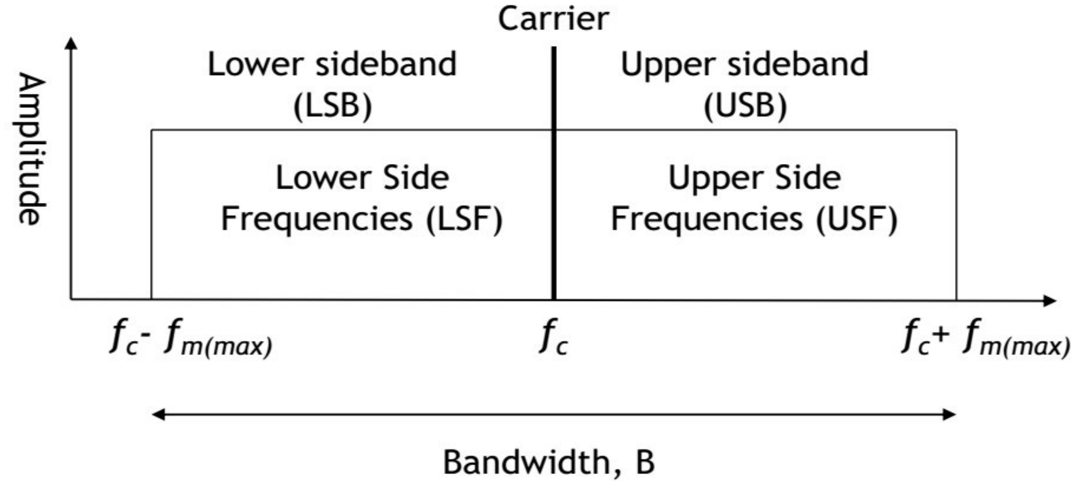
(b) carrier frequency

(c) the sum ($f_c + f_m$) and difference ($f_c - f_m$) frequencies.

Bandwidth, B = difference between highest USB and lowest LSB

i.e $B = 2f_{m(max)}$.

below shows the frequency spectrum for an AM waveform



Example:

Describe the meaning of each term in the following expression:

$$v_{am}(t) = 10\sin(2\pi\,500kt) - 5\cos(2\pi 515kt) + 5\cos(2\pi 485kt)$$

Ans: $v_{am}(t)$ is an AM signal generated by an AM modulator in which the modulating signal is a sinusoid with a frequency of 15 kHz. The expression $10\sin(2\pi\,500kt)$ is the unmodulated carrier signal having an amplitude of 10 volts and a frequency of 500 kHz. The expression $-5\cos(2\pi 515kt)$ is the upper side band signal having an amplitude of 5 volts and a frequency of 515 kHz. The expression $5\cos(2\pi 485kt)$ is the lower side band signal having an amplitude of 5 volts and a frequency of 485 kHz.

Example: For an AM DSBFC modulator with a carrier frequency,100 kHz and a maximum modulating signal frequency 5 kHz, find

a. Frequency limit for upper and lower sideband.

b. Bandwidth.

c). Upper and lower side frequencies produced when the modulating signal is a single-frequency 3 KHz tone.

d. Draw the output frequency spectrum.

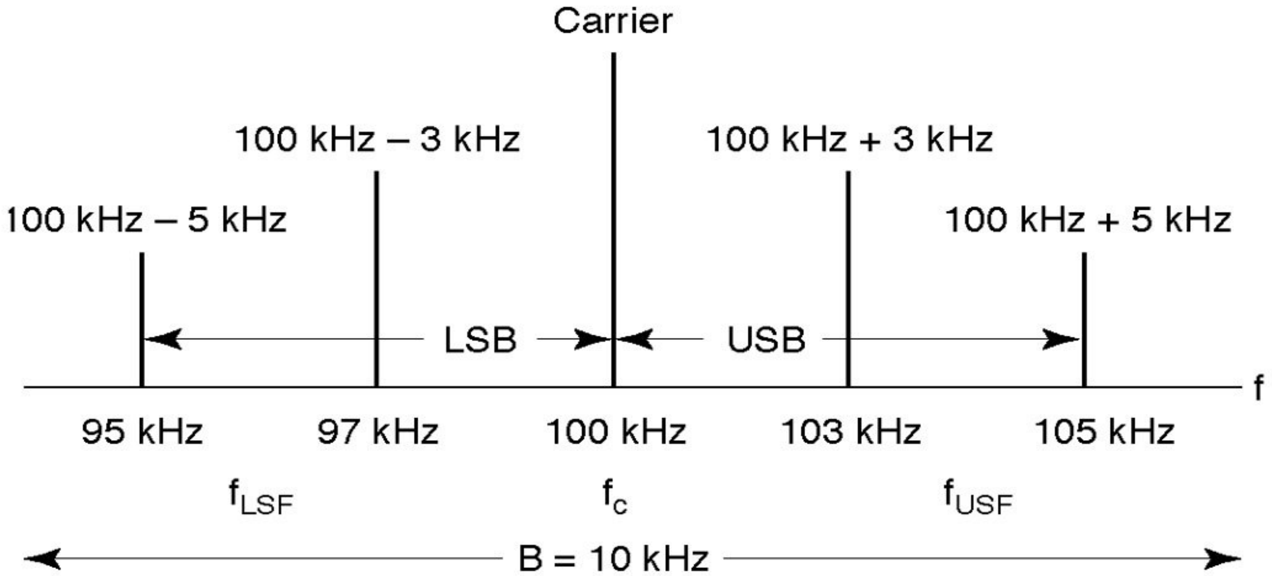
Solution : $f_c = 1oo\ KHz;\ f_{m(max)} = 5KHz$

a. LSF= $f_c - f_{m(max)} = 100 - 5 = 95KHz$ ***:limits from 95 to 100 KHz***

USF= $f_c + f_{m(max)} = 100 + 5 = 105KHz$ ***:limits from 100 KHz to 105 KHz***

$b.\ \boldsymbol{B} = \mathbf{2} f_{m(max)} = 10KHz$

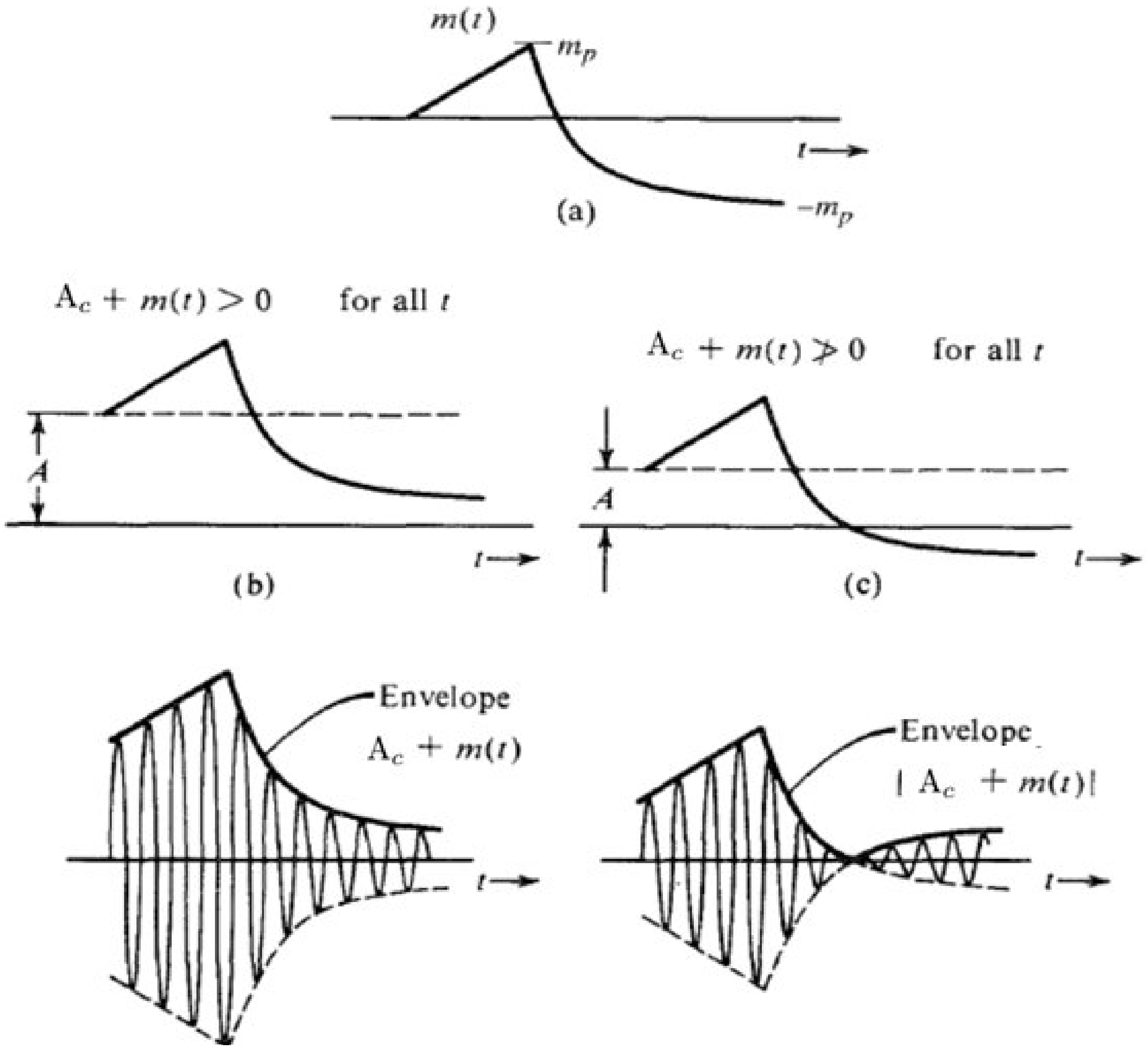
c. LSF= $f_c - f_{m(max)} = 100 - 3 = 97KHz$ and ***USF=*** $f_c - f_{m(max)} = 100 + 3 = 103KHz$



Two cases are considered

$$\varphi_{\mathrm{AM}}(t) = [A_c + m(t)] \cos \omega_c t$$

- In the first case, A_c is large enough so that $A_c + m(t) > 0$ (is nonnegative) for all values of t.
- In the second case, A is not large enough to satisfy this condition.



In first case (left side) , **the envelope** has the same shape as ***m(t)***
In the second case, the envelope shape is not same as ***m(t)*** because some parts get rectified.

This means we can detect the desired signal ***m(t)*** by detecting the envelope in the first case. Such detection is not possible in the second case.

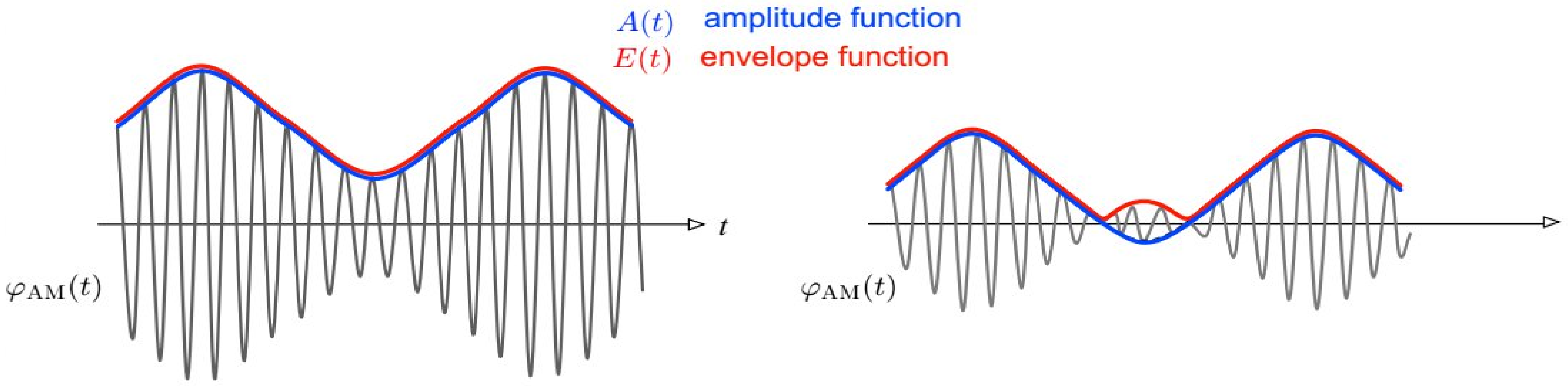
We shall see that the envelope detection is an extremely simple and inexpensive operation, which does not require generation of a local carrier for the demodulation. But as seen above the envelope of AM has the information about m(t) only **if the AM signal [Ac +m(t)] cos(ωct) satisfies the condition A_c+ m(t) > 0 for all t.**

Let m_p be the peak amplitude (positive or negative) of ***m(t)***. This means that $Max\ (m(t)) = m_p$, Hence, $A_c \geq m_p$

Thus, the minimum carrier amplitude required for the viability of envelope detection is m_p

Key elements of an AM signal $\varphi_{\text{AM}}(t) = [A_c + m(t)] \cos \omega_c t$ are the **amplitude** and **envelope** functions:

amplitude $= A_c + m(t)$ **envelope** $= |A_c + m(t)|$



Case 1: $A(t) \geq 0,\ \forall t$

Case 1: $A(t) \geq 0$, for some t and $A(t) < 0$, for some t.

$E(t) = A(t)$

$E(t) = |A(t)|$

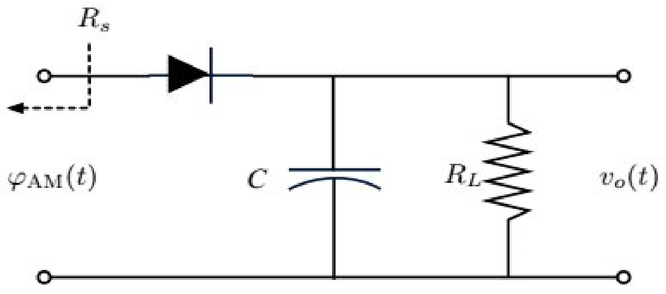
A simple and inexpensive **envelope detector** can demodulate AM signals if:

- **envelope** $\propto m(t)$
- **envelope** $= E(t) = \left|A_c + m(t)\right| = \left[A_c + m(t)\right] = A(t) =$ **amplitude**
- $\left[\, A_c + m(t) \,\right] \geq 0$

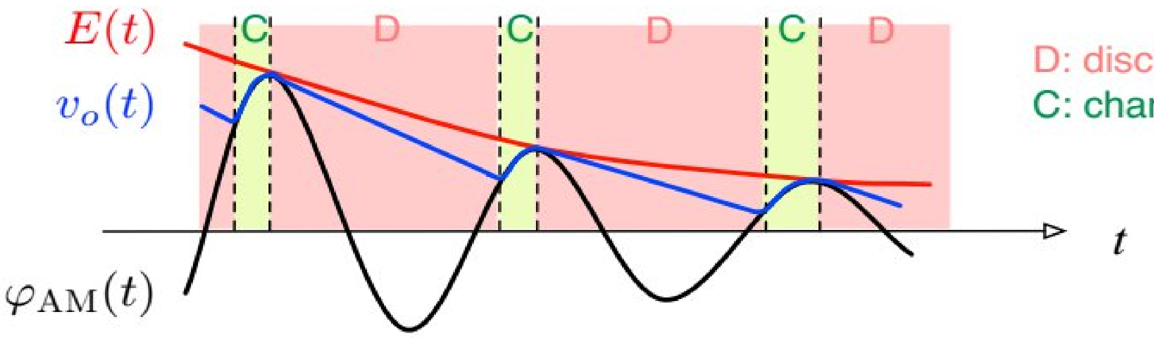
Conversely, if $E(t) = [A_c + m(t)] < 0$ for some t :

- The envelope will no longer be a scaled and shifted version of the modulating signal $m(t)$.
- $[A_c + m(t)]$ will experience phase reversals at zero crossings.
- We have to use a synchronous/coherent detector as with DSB-SC amplitude modulated signals.

If **envelope** = $E(t) = [A_c + m(t)] \geq 0$



R_L = load resistance
R_s = source resistance



D: discharge
C: charge

When $\varphi_{AM}(t) > 0$ and $\varphi_{AM}(t) > v_O(t)$:

- Diode is forward biased and conducts,
- C charges to the max value of $\varphi_{AM}(t)$

When $\varphi_{AM}(t) < 0$ and/or $v_O(t) > \varphi_{AM}(t)$:

- Diode is reverse biased, does not conduct
- C discharges over R_L

An envelope detector consists of a diode and an RC circuit, which is a simple lowpass filter. During the positive half-cycle of the input signal, the diode conducts, and the capacitor charges up to the peak value of the input signal. When the input falls below the voltage on the capacitor, the diode becomes reverse-biased and the input disconnects from the output. During this period, the capacitor discharges slowly through the resistor R. On the next cycle of the carrier, the diode again conducts when the input signal exceeds the voltage across the capacitor

AM: Modulation index

Definition

Modulation Index of an AM signal:

$$\boxed{\mu = \frac{m_p}{A_c}}$$

where $m_p = \max_t |m(t)|$

We can demodulate an AM signal using an envelope detector, if

$$\boxed{E(t) = A(t) = [A_c + m(t)] \geq 0, \text{ for all } t}$$

or equivalently when

$$\boxed{0 \leq \mu \leq 1}$$

In addition, we can define the percentage Modulation as

$$\textit{Percentage Modulation} = \mu \times 100\%$$

Modulation index for complex information signal

- When several frequencies simultaneously amplitude modulate a carrier, the combined coefficient of modulation is defined as:

$$\mu_{Total} = \sqrt{(\mu_1)^2 + (\mu_2)^2 + (\mu_3)^2 + \ldots(\mu_n)^2}$$

μ_{Total}= Total modulation index/coefficient of modulation

μ_1, μ_2, μ_3, μ_n= modulation index/coefficient of modulation for input 1, 2 ,3 , n

Example : A carrier is simultaneously modulated by two sine waves having modulation indices of 0.4 and 0.3. Find the total modulation index ?

Solution:

$$\mu_{Total} = \sqrt{(\mu_1)^2 + (\mu_2)^2} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5$$

Single-tone message signal $m(t) = A_m \cos \omega_m t$ such that the AM signal is:

$$\boxed{\varphi_{\text{AM}}(t) = \left[A_c + A_m \cos \omega_m t \right] \cos \omega_c t}$$

with $\omega_m \ll \omega_c$

a. Determine the modulation indices, and
b. Investigate the envelope for:
   i. $A_m / A_c < 1$
   ii. $A_m / A_c = 1$
   iii. $A_m / A_c > 1$
c. Sketch the spectrum of the AM signal.

Evaluate the parameters:

$$m_p = \max_t |m(t)| = A_m$$

$$A_{\max} = \max_t \left[A_c + m(t) \right] = A_c + A_m$$

$$A_{\min} = \min_t \left[A_c + m(t) \right] = A_c - A_m$$