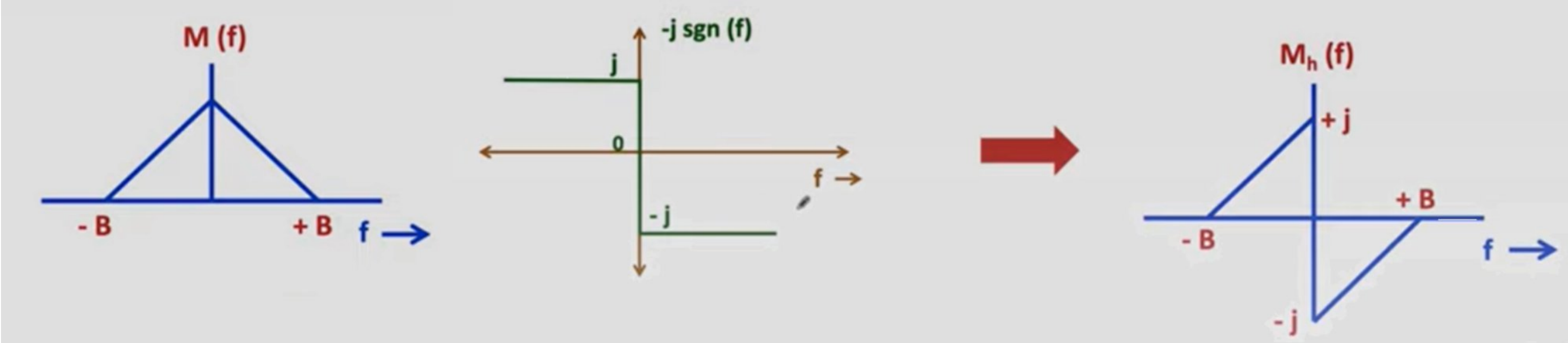
Hilbert transform functions as a wideband $\pi/2$ radians phase shifter

$$H_h(f) = -j\mathrm{sgn}(f) = \begin{cases} -j, & \text{if } f \geq 0; \\ +j, & \text{if } f < 0. \end{cases}$$

$$m(t) \longrightarrow \boxed{H_h(f)} \longrightarrow m_h(t)$$

Every frequency component in $m(t)$ experiences $\pi/2$−radians phase shift

$$\cos 2\pi f_0 t \longrightarrow \boxed{H_h(f)} \longrightarrow \cos(2\pi f_0 t - \frac{\pi}{2}) = \sin 2\pi f_0$$

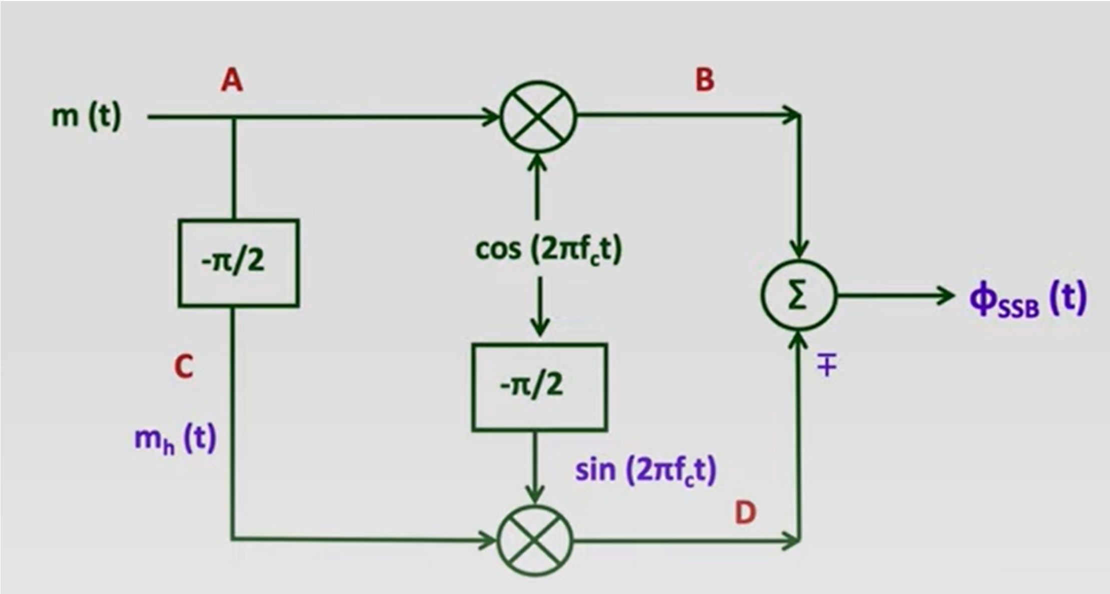


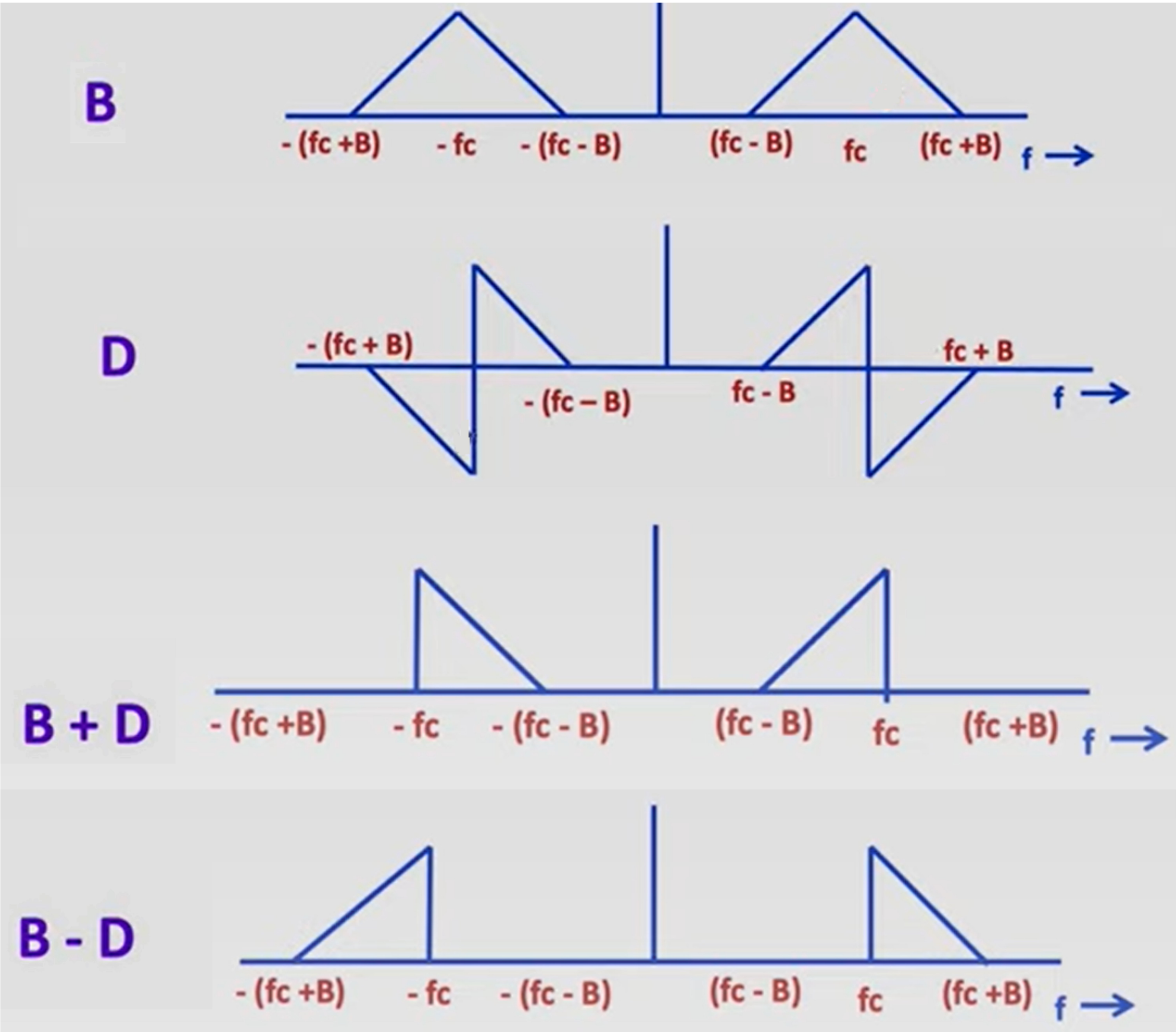
SSB Bandwidth

As only one sideband is transmitted in the SSB modulation , then the transmission bandwidth of a SSB modulated signal is defined by

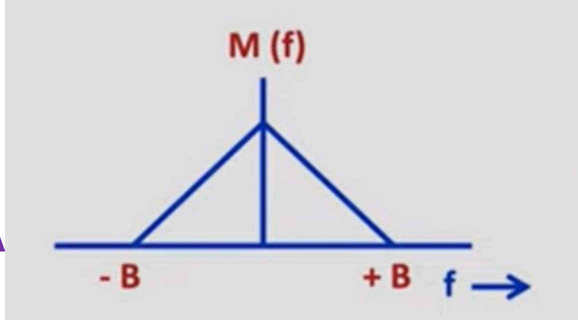
$$BW_T = B_m$$

Where B_m is the *message bandwidth*. We see that BW_T for SSB-AM is half that for DSB-AM.

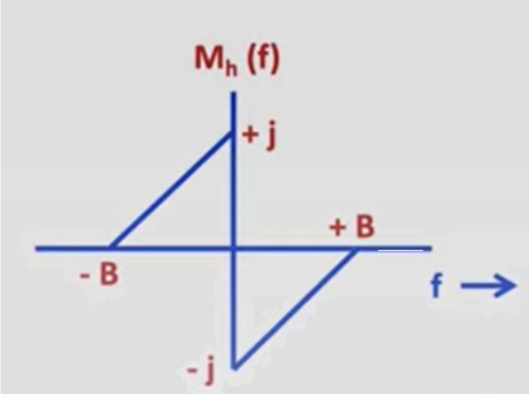

m (t)
A
B
-π/2
cos (2πf$_c$t)
Σ
ϕ$_{SSB}$ (t)
∓
C
m$_h$ (t)
-π/2
sin (2πf$_c$t)
D


B
- (fc +B)
- fc
- (fc - B)
(fc - B)
fc
(fc +B)
f →
D
- (fc + B)
- (fc – B)
fc - B
fc + B
f →
B + D
- (fc +B)
- fc
- (fc - B)
(fc - B)
fc
(fc +B)
f →
B - D
- (fc +B)
- fc
- (fc - B)
(fc - B)
fc
(fc +B)
f →

A


M (f)
- B
+ B
f →

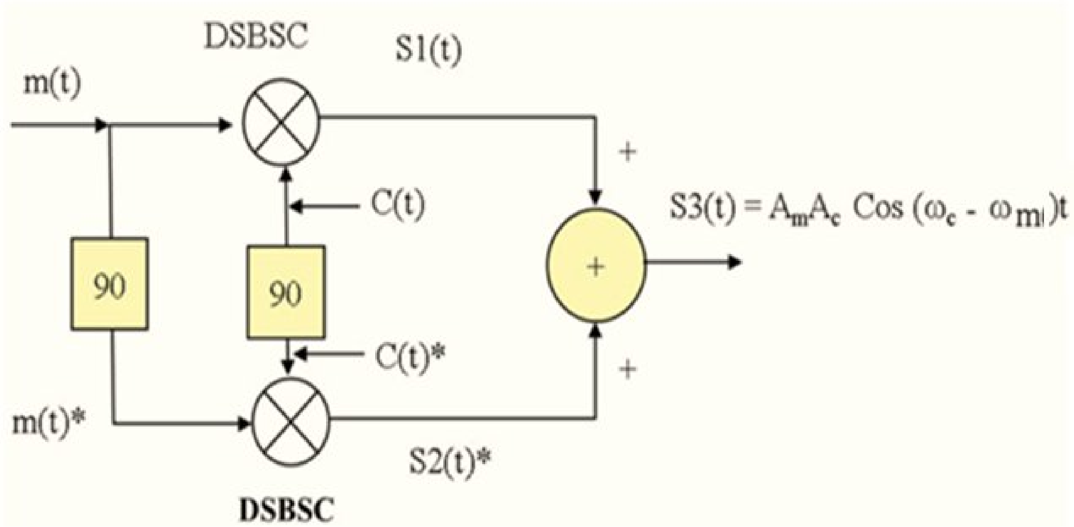
C


M$_h$ (f)
+ j
+ B
- B
f →
- j

Example: Design an SSB modulator to realize the lower sideband. Sketch the spectral response.

Solution

- $m(t) = A_m \cos(\omega_m t)$
- $m(t)^* = A_m \sin(\omega_m t)$ − (Hilbert Transform)
- $C(t) = A_c \cos(\omega_c t)$
- $C(t)^* = A_c \sin(\omega_c t)$ − (Hilbert Transform)



Solving for S_1 and S_2, we obtain:

- $S_1(t) = A_c\, A_m \cos(\omega_m t) \cos(\omega_c t)$
- $S_2(t) = A_c\, A_m \sin(\omega_m t) \sin(\omega_c t)$

Obtain S_3 as:

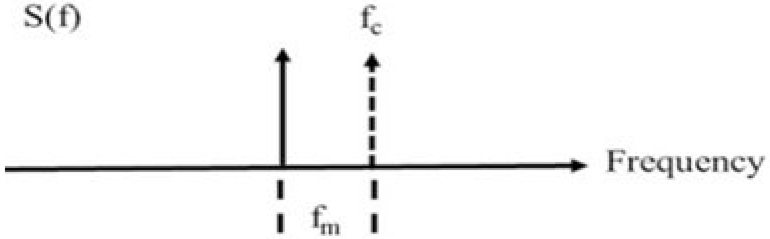
$$S_3(t) = S_1(t) + S_2(t)$$
$$= A_c A_m \cos(\omega_m t)\cos(\omega_c t) + A_c A_m \sin(\omega_m t)\sin(\omega_c t)$$

Using the following formula:

- $\cos A \cos B = 1/2\cos(A + B) + 1/2\cos(A - B)$
- $\sin A \sin B = 1/2\cos(A - B) - 1/2\cos(A + B)$

Solving for S_3, we get:

$$S_3 = A_c A_m \cos(\omega_c - \omega_m)t$$
$$= A_c A_m \cos 2\pi(f_c - f_m)t$$



In the above equation, S_3(t) is the desired SSB signal, which is the lower sideband only. The spectral response, showing the lower sideband, is presented

The SSB demodulator is identical to the synchronous demodulator used for DSB-SC

$$\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

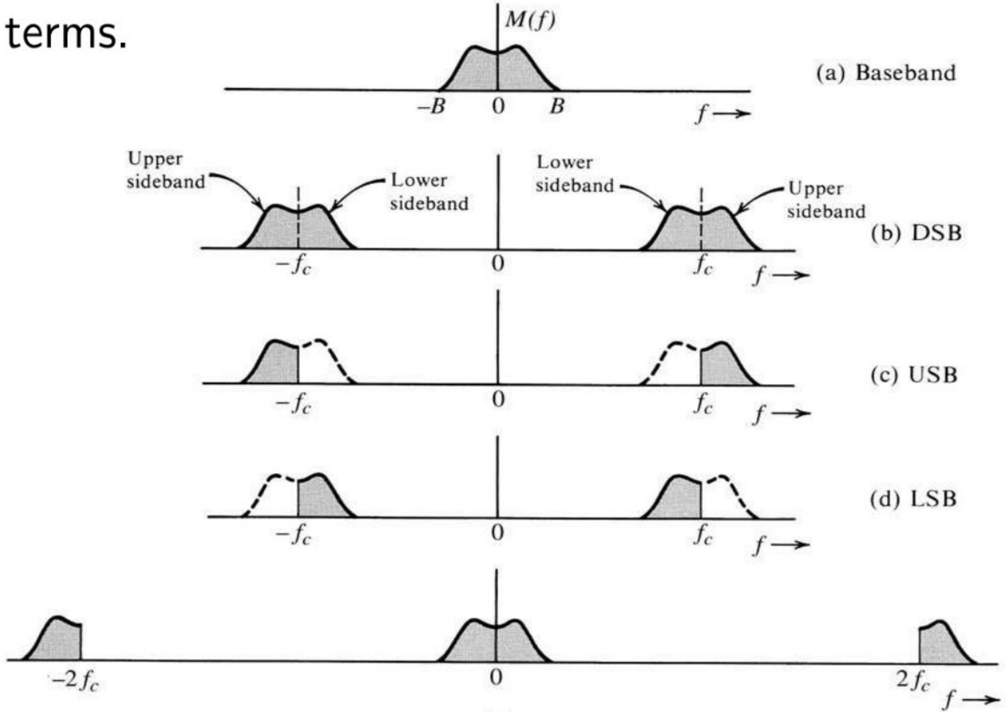
Hence

$$\begin{aligned}\phi_{SSB}(t)\cos(\omega_c t) &= \tfrac{1}{2}m(t)[1+\cos 2\omega_c t] \mp m_h(t)\sin 2\omega_c t \\ &= \tfrac{1}{2}m(t) + \tfrac{1}{2}[m(t)\cos 2\omega_c t \mp m_h(t)\sin 2\omega_c t]\end{aligned}$$

Hint: sin(2x) = 2 sin(x) cos(x)

Thus, the product $\phi_{SSB}(\omega)\cos(\omega_c t)$ yields the baseband signal and another SSB signal with carrier $2\omega_c$. A low-pass filter will suppress the unwanted SSB terms.

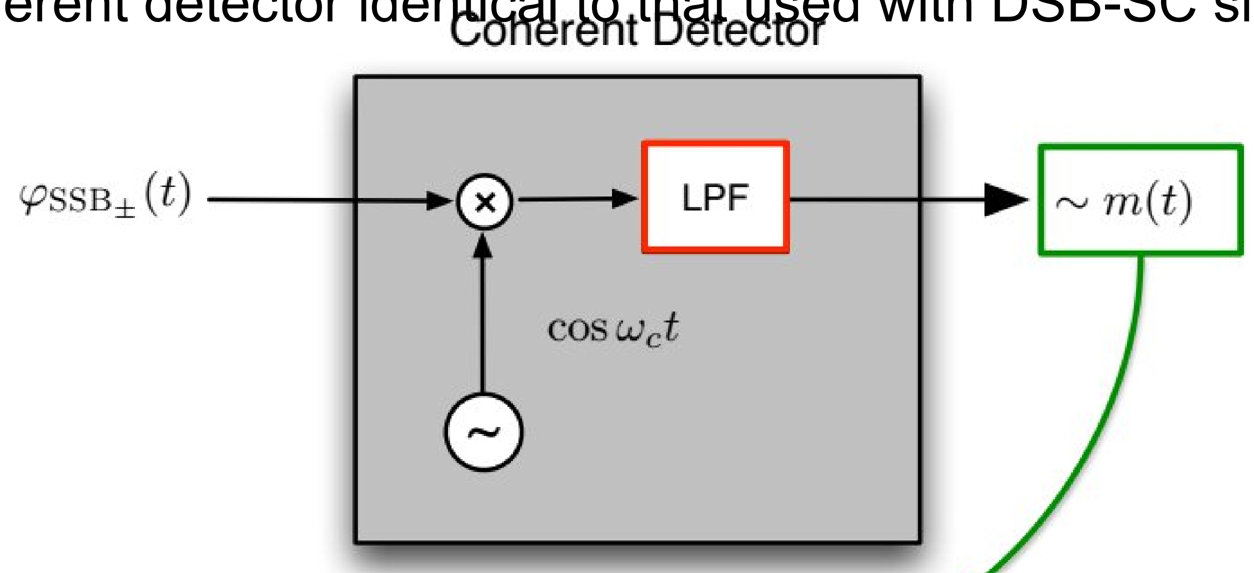
For example, multiplying of a USB signal by cos $(\omega_c t)$ shifts its spectrum to the left and right by ω_c. Low-pass filtering of this signal yields the desired baseband signal. The case is similar with LSB.



Thus, the product $\varphi_{SSB}(\omega)\ \cos(\omega_c t)$ yields the baseband signal and another SSB signal with carrier $2\omega_c$.

A low-pass filter will suppress the unwanted SSB terms, giving the desired baseband signal *m(t)/2.*

The use of a coherent detector identical to that used with DSB-SC signals.



$$\varphi_{\mathrm{SSB}_\pm}(t)\cos\omega_c t = \left[m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t\right]\cos\omega_c t$$

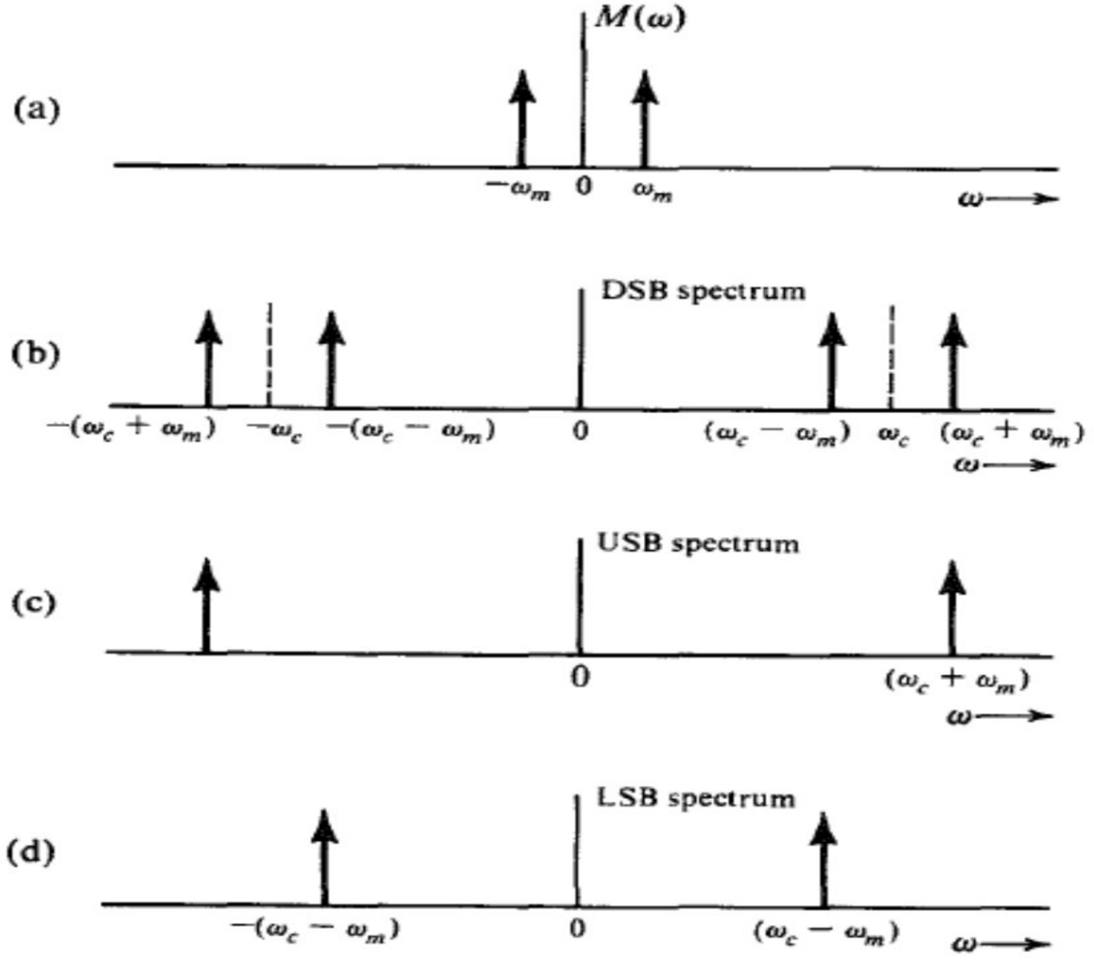
$$= m(t)\cos^2\omega_c t \mp m_h(t)\sin\omega_c t\cos\omega_c t$$

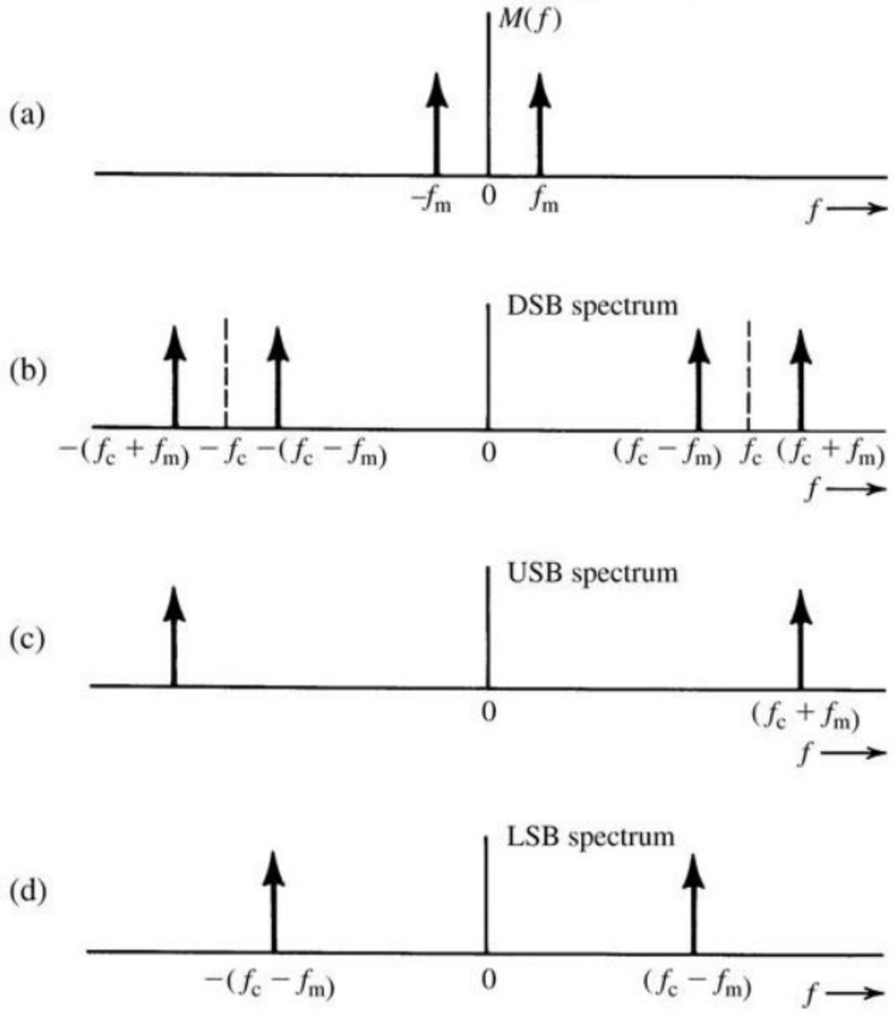
$$= \boxed{\frac{1}{2}m(t)} + \cancel{\frac{1}{2}m(t)\cos 2\omega_c t \mp \frac{1}{2}m_h(t)\sin 2\omega_c t}$$

Example: Find $\varphi_{SSB}(t)$ for a simple case of a tone modulation, that is, when the modulating signal is a sinusoid ***m(t) = cos(ω_mt).***

Sol: consider the spectrum of *m(t)* (Fig. a) and its DSB-SC (Fig. b), USB (Fig. c), and LSB (Fig. d) spectra

It is evident that the spectra in Fig. c and d do indeed correspond to the φ_{USB} (t) and φ_{LSB} (t)





Vestigial Sideband (VSB)

In certain applications (such as television broadcasting), a DSB modulation technique takes too much bandwidth for the (television) channel, and an SSB technique is too expensive to implement, although it takes only half the bandwidth. In this case, a compromise between DSB and SSB, called *vestigial sideband* (VSB), is often chosen. VSB is obtained by partial suppression of one of the sidebands of a DSB signal.

VSB modulation distinguishes itself from SSB modulation in two practical respects:

- Instead of completely removing a sideband, a trace or *vestige* of that sideband is transmitted; hence, the name "vestigial sideband."
- Instead of transmitting the other sideband in full, *almost* the whole of this second band is also transmitted.

Accordingly, the transmission bandwidth of a VSB modulated signal is defined by

$$\boldsymbol{BW_T = B_v + B_m}$$

Where $\boldsymbol{B_v}$ is the *vestige bandwidth* and $\boldsymbol{B_m}$ is the *message bandwidth*. Typically, $\boldsymbol{B_v}$ is 25 percent of $\boldsymbol{B_m}$, which means that the VSB bandwidth $\boldsymbol{BW_T}$ lies between the SSB bandwidth, $\boldsymbol{B_m}$, and DSB-SC bandwidth, $\boldsymbol{2B_m}$. Thus, The BW is typically 25% greater than that of SSB.

The VSB is proposed for the following reasons:

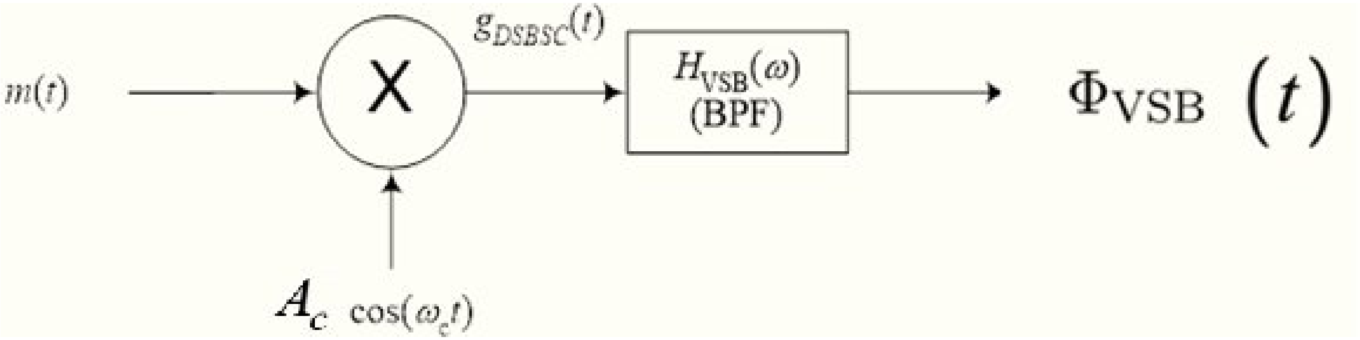
- Typically, the spectra of wideband signals (exemplified by television video signals and computer data) contain significant low frequencies, which make it impractical to use SSB modulation. VSB improves the low-frequency response and allows DC to pass undistorted.
- The spectral characteristics of wideband data befit the use of DSB-SC. However, DSBSC requires a transmission bandwidth equal to twice the message bandwidth, which violates the bandwidth conservation requirement. VSB has bandwidth efficiency advantages over DSB or AM, similar to that of SSB
- Simplifies the filter design

Hence, **this form of modulation is well suited for the transmission of wideband signals such as television signals that contain significant components at extremely low frequencies**. In commercial television broadcasting, a sizable carrier is transmitted together with the modulated wave, which makes it possible to demodulate the incoming modulated signal by an envelope detector in the receiver and thereby simplify the receiver design.

VSB signals are generated using standard AM or DSB-SC modulation, then passing modulated signal through a sideband shaping filter.

The transmitted signal

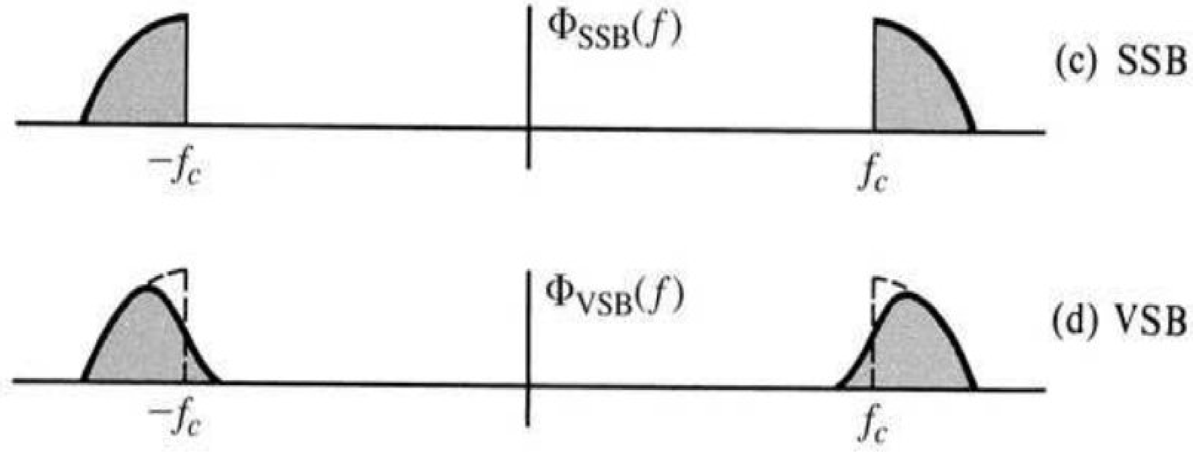
$$\Phi_{VSB}(t) = \left[A_c m(t)\cos 2\pi f_c t\right] * h(t) \leftrightarrow$$



The transmitted signal has spectrum

$$\Phi_{VSB}(f) = \frac{A_c}{2}\left[(M(f + f_c) + M(f - f_c))\right]H_i(f)$$

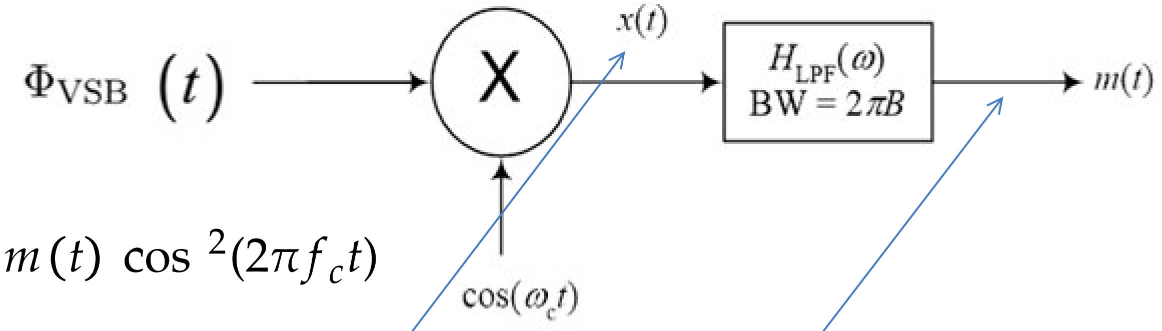
where $H_i(f)$ is the *shaping filter* for the VSB modulator.



Filter requirement:

$$H(f - f_c) + H(f + f_c) = \text{constant},$$

To demodulate the VSB signal, synchronous detector (frequency shifting plus filtering) can be used. The demodulated signal is



VSB Demodulator (receiver)

$$x(t) = \varnothing_{VSB}(t)\cos(2\pi f_c t) = A_c\, m(t)\, \cos^2(2\pi f_c t)$$

$$\Phi_{\mathrm{VSB}}(f) = \frac{A_c}{2}\left[(M(f+f_c)+M(f-f_c))\right]H_i(f)$$

At receiver (Rx)

Shifting the VSB spectrum to the right and left

$$X(f) = \frac{A_c}{4}[M(f-2f_c)+M(f)]H(f-f_c) + \frac{A_c}{4}[M(f)+M(f+2f_c)]H(f+f_c)$$

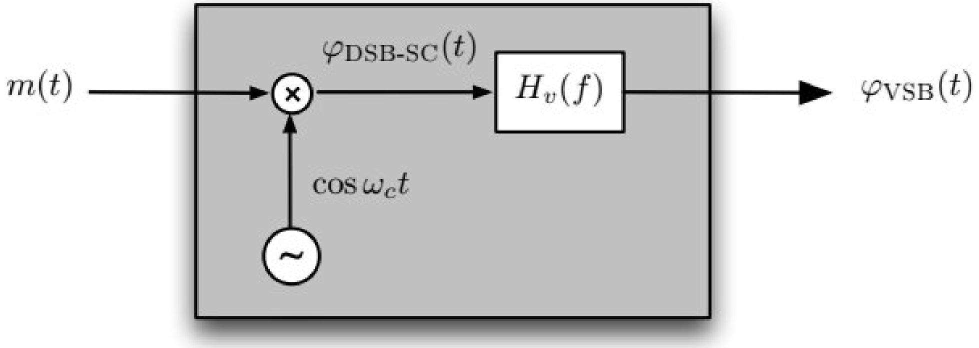
Constant

$$S_l(f) = \frac{A_c}{4} M(f)\left[H(f-f_c)+H(f+f_c)\right]$$

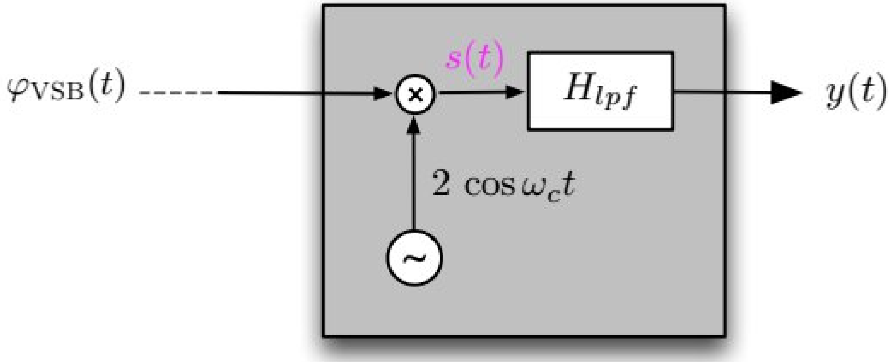
The result demodulated signal is a scaled version of the desired message signal

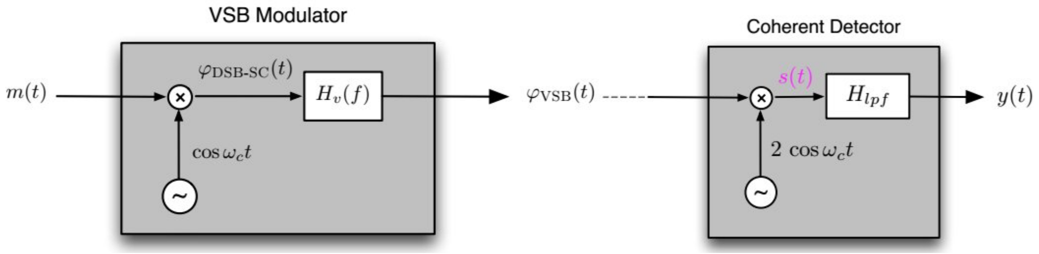
Vestigial Sideband (VSB)

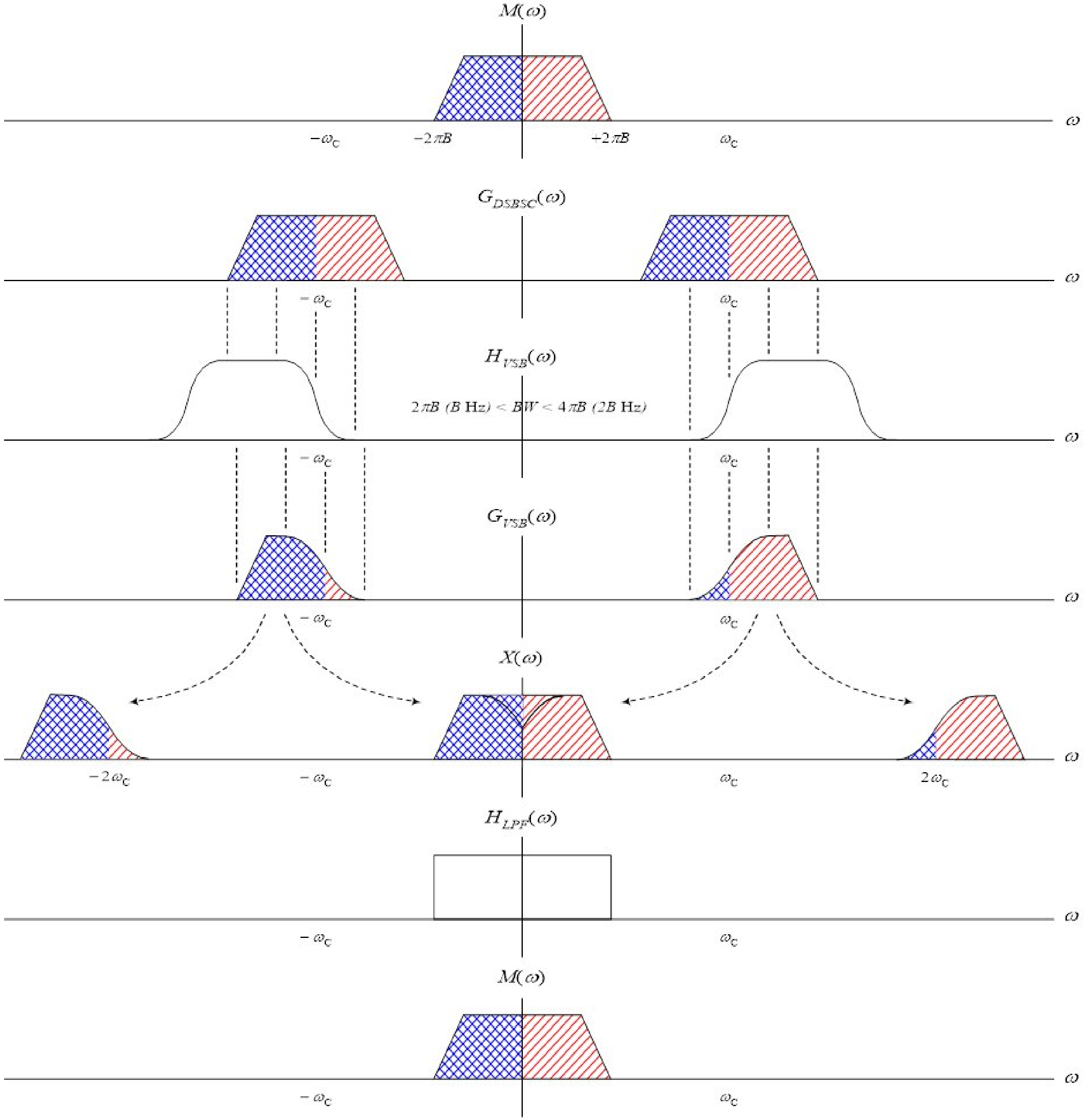
VSB Modulator



Coherent Detector







Comparison of conventional AM, DSB-SC, SSB and VSB.

- **Conventional AM: simple to modulate and to demodulate, but low power efficiency and double the bandwidth.**
- **DSB-SC: high power efficiency, but more complex to modulate & demodulate, doubles the bandwidth.**
- **SSB: high power efficiency, the same (message) bandwidth, but more difficult to modulate & demodulate.**
- **VSB: lower power efficiency & larger bandwidth than SSB but easier to implement.**

Angle Modulation

Angle Modulation: either the phase or the frequency of the carrier wave is varied by ***m*(*t*)** while the amplitude of the carrier wave is constant.

- Whenever the frequency of a carrier varied, the phase is also varied and vice versa.
- FM and PM must both occur whenever either form of angle modulation is performed.
- If the carrier frequency is varied directly in accordance with the modulating signal, **FM.**
- If the phase of the carrier is varied directly in accordance with the modulating signal, **PM**.
- Therefore, direct FM is indirect PM and direct PM is indirect FM.
- Direct FM/*PM* – frequency/*phase* of the **constant amplitude carrier** is varied proportionally to the **amplitude of the modulating signal** at a rate equal to the frequency of the modulating signal.

$$c(t) = A_c \cos(2\pi f_c t + \theta)$$

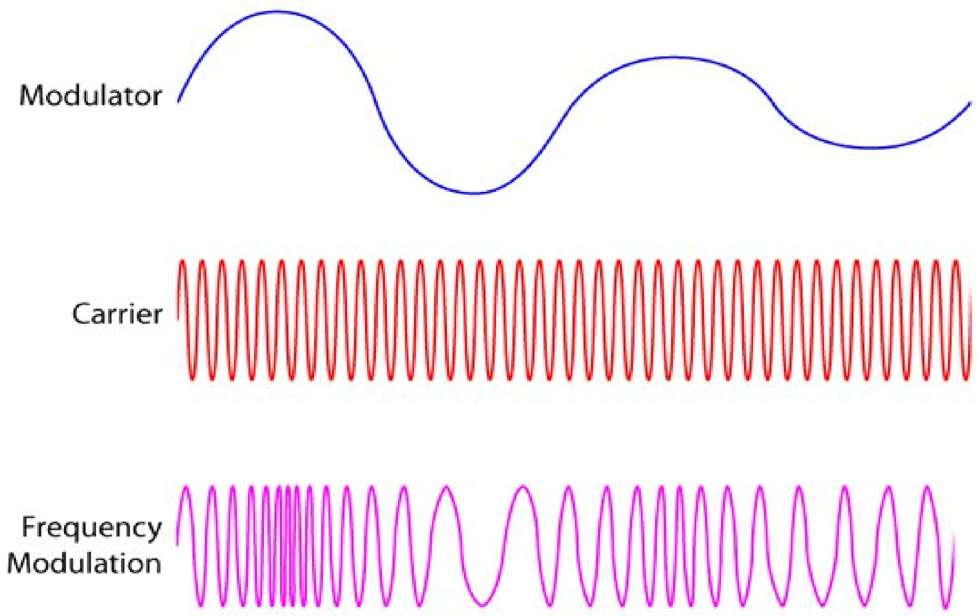
Why FM is more resistance to noise than AM?

Noise & interference usually corrupt the signals by changing their voltage (Amplitude), by adding spikes or some other ways by changing the shape of the signal; however, it cannot change its frequency or phase. **Since FM signals only change their frequency, not their amplitude,** it is possible to design a receiver, so it ignores amplitude changes.

For instance, an FM radio often contains a limiter circuit which clips the tops off the signal to make the signal the same height no matter what level it comes in. This won't work on very weak signals, since then there's nothing to clip, but for strong signal, FM can be remarkably free of noise

How carrier frequency change in accordance to modulating signal ***m*(*t*)**?

$$c(t) = A_c \cos(2\pi f_c t + \theta)$$



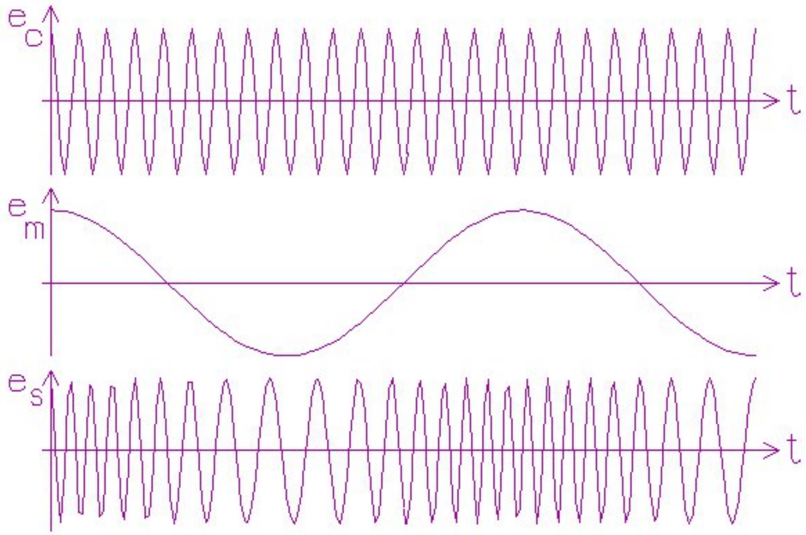
- +ve increase in modulating signal amplitude causes increase in carrier frequency.

- –ve increase in modulating signal amplitude causes decrease in carrier freq.

Carrier

Modulating Signal

FM signal



$$\varphi(t) = A_c \cos\theta(t)$$

where:

$$A_c = \text{constant}$$

$$\theta(t) = g\left[m(t)\right]$$

$$\omega_i(t) = \frac{d\theta(t)}{dt} \quad \boxed{\text{instantaneous frequency}} \quad f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

Phase Modulation (PM):

$$\theta(t) = 2\pi f_c t + K_p m(t)$$

Where K_p is the deviation sensitivity for phase or phase deviation constant for PM with unit volt^{-1}. **The modulated PM signal is given by**

$$\varphi_{\mathrm{PM}}(t) = A_c \cos\left(2\pi f_c t + K_p m(t)\right)$$

Information contained in $m(t)$ is embedded in the **time-varying phase**.