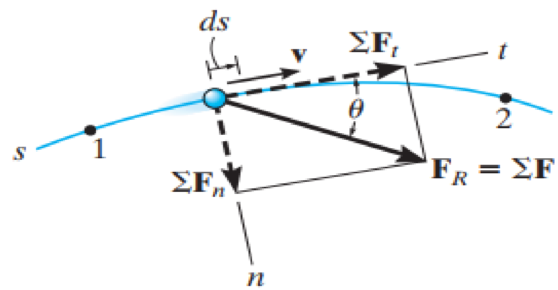
2.2 Principle of Work and Energy

Consider the particle in Fig. 2–7, which is located on the path defined relative to an inertial coordinate system. If the particle has a mass m and is subjected to a system of external forces represented by the resultant $\mathbf{F}_R = \mathbf{F}$, then the equation of motion for the particle in the tangential direction is $\sum F_t = ma_t$. Applying the kinematic equation $a_t = v\,dv/ds$ and integrating both sides, assuming initially that the particle has a position $S = S_1$ and a speed $v = v_1$, and later at $S = S_2 \Rightarrow v = v_2$, we have.



2-7

$$\sum \int_{S_1}^{S_2} F_t ds = \int_{V_1}^{V_2} mv\, dv$$

$$\sum \int_{S_1}^{S_2} F_t ds = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad (2\text{-}5)$$

From Fig. 2–7, note that $F_t = F\cos\theta$, and since work is defined from Eq. 2-1, the final result can be written as:

$$\sum U_{2\text{-}1} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad (2\text{-}6)$$

This equation represents the principle of work and energy for the particle. The term on the left is the sum of the work done by all the forces acting on the particle as the particle moves from point 1 to point 2. The two terms on the right side, which are of the form $T = \frac{1}{2}mv^2$, define the particle's final and initial kinetic energy, respectively. Like work, kinetic energy is a scalar and has units of joules (J) and ft.lb. However, unlike work, which can be either positive or negative, the kinetic energy is always positive, regardless of the direction of motion of the particle. When Eq. 2-6 is applied, it is often expressed in the form

(2-7)

which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to its final position is equal to the particle's final kinetic energy. As noted from the derivation, the principle of work and energy represents an integrated form of $F_t=ma_t$, obtained by using the kinematic equation $a_t = v\,dv/ds$. As a result, this principle will provide a convenient substitution for $F_t=ma_t$ when solving those types of kinetic problems which involve force, velocity, and displacement since these quantities are involved in Eq. 2–7. For application, it is suggested that the following procedure be used.

Work (Free-Body Diagram).

- Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that do work on the particle as it moves along its path.

Principle of Work and Energy.

- Apply the principle of work and energy, $T_1 + U_{1-2} = T_2$. (prove that by 2 way)
- The kinetic energy at the initial and final points is always positive, since it involves the speed squared $T = \frac{1}{2}mv^2$.
- A force does work when it moves through a displacement in the direction of the force.
- Work is positive when the force component is in the same sense of direction as its displacement, otherwise it is negative.

- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force displacement curve.
- The work of a weight is the product of the weight magnitude and the vertical displacement, $U_W = W.\mathbf{y}$ It is positive when the weight moves downwards.
- The work of a spring is of the form $U_{SP} = \frac{1}{2}ks^2$, where k is the spring stiffness and S is the stretch or compression of the spring.

Numerical application of this procedure is illustrated in the examples following.

If an oncoming car strikes these crash barrels, the car's kinetic energy will be transformed into work, which causes the barrels, and to some extent the car, to be deformed.

3-2 Principle of Work and Energy for a System of Particles

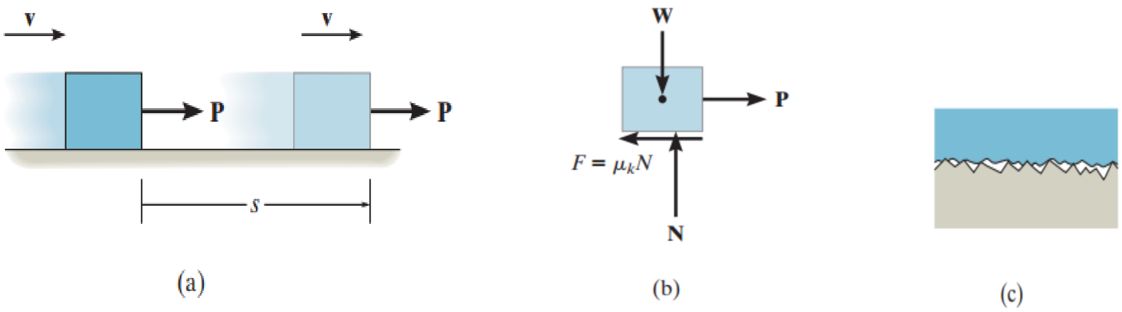
The principle of work and energy can be extended to include a system of particles isolated within an enclosed region of space as shown in Fig. 2–8. Here the arbitrary ith particle, having a mass mi, is subjected to a resultant external force **F**i and a resultant internal force **f**i which all the other particles exert on the ith particle. If we apply the principle of work and energy to this and each of the other particles in the system, then since work and energy are scalar quantities, the equations can be summed algebraically, which gives

$$\sum T_1 + \sum U_{2\text{-}1} = \sum T_2 \qquad (8\text{-}2)$$

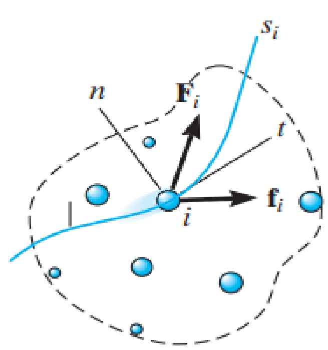
In this case, the initial kinetic energy of the system plus the work done by all the external and internal forces acting on the system is equal to the final kinetic energy of the system.

If the system represents a translating rigid body, or a series of connected translating bodies, then all the particles in each body will undergoes the same displacement.

Therefore, the work of all the internal forces will occur in equal but opposite collinear pairs and so it will cancel out. On the other hand, if the body is assumed to be



nonrigid, the particles of the body may be displaced along different paths, and some of the energy due to force interactions would be given off and lost as heat or stored in the body if permanent deformations occur. We will discuss these effects briefly at the end of this section and in Sec. 3.4. Throughout this text, however, the principle of work and energy will be applied to problems where direct accountability of such energy losses does not have to be considered



Inertial coordinate system

Work of Friction Caused by Sliding. A special class of problems will now be investigated which requires a careful application of Eq. 2-8. These problems involve cases where a body slides over the surface of another body in the presence of friction. Consider, for example, a block which is translating a distance *s* over a rough surface as shown in Fig. 2-9a. If the applied force **P** just balances the resultant frictional force $\mu_k N$, Fig. 2-9b, then due to equilibrium a constant velocity **v** is maintained, and one would expect Eq. 2-8 to be applied as follows:

$$P.S - F_r S = \Delta T$$

$$\frac{1}{2}mv_1^{\,2} + PS - \mu_k NS = \frac{1}{2}mv_2^{\,2}$$

Indeed this equation is satisfied if $P = \mu_K N$; however, as one realizes from experience, the sliding motion will generate heat, a form of energy which seems not to be accounted for in the work-energy equation. In order to explain this paradox and thereby more closely represent the nature of friction, we should actually model the block so that the surfaces of contact are deformable (nonrigid). *Recall that the rough portions at the bottom of the block act as “teeth,” and when the block slides these teeth deform slightly and either break off or vibrate as they pull away from “teeth” at the contacting surface, Fig. 2–9c.

As a result, frictional forces that act on the block at these points are displaced slightly, due to the localized deformations, and later other frictional forces replace them as other points of contact are made. At any instant, the resultant **F** of all these frictional forces remains essentially constant, i.e., $\mu_k N$; however, due to the many localized deformations, the actual displacement $\acute{S}$ of mkN is not the same as the displacement S of the applied force **P**. Instead, $\acute{S}$ will be less than S ($\acute{S} < S$), and therefore the external work done by the resultant frictional force will be $\mu_k N\acute{S}$ and not µkNS. The remaining amount of work, $\mu_k N\,(S- \acute{S})$, manifests it self as an increase in internal energy, which in fact causes the block’s temperature to rise In summary then, Eq. 2 -8 can be applied to problems involving sliding friction; however, it should be fully realized that the work of the resultant frictional force is not represented by mkNs; instead, this term represents both the external work of friction ($\mu_k N\acute{S}$) and internal work [$\mu_k N(S - \acute{S})$] which is converted into various forms of internal energy, such as heat.

Procedure for Analysis

Work (Free-Body Diagram).

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Principle of Work and Energy.

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- Forces that are functions of displacement must be integrated toobtain the work. Graphically, the work is equal to the area under the force-displacement curve.
- The work of a weight is the product of the weight magnitude andthe vertical displacement, UW = {Wy. It is positive when theweight moves downwards.
- The work of a spring is of the form Us = 1 2 ks2, where k is thespring stiffness and s is the stretch or compression of the spring.