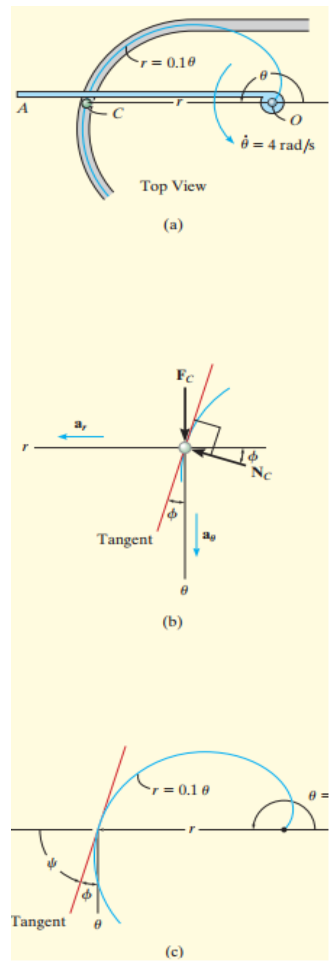
DYNAMICS

PROBLEM.29: A can C, having a mass of 0.5 kg, moves along a grooved horizontal slot shown in Fig.1–21a. The slot is in the form of a spiral, which is defined by the equation r = (0.1θ) m, where θ is in radians. If the arm OA rotates with a constant rate $\dot{\theta}$ = 4 rad/s in the horizontal plane, determine the force it exerts on the can at the instant θ = π rad. Neglect friction and the size of the can.



SOLUTION

Free-Body Diagram. The driving force $\mathbf{F}_C$ acts perpendicular to the arm OA, whereas the normal force of the wall of the slot on the can, $\mathbf{N}_C$, acts perpendicular to the tangent to the curve at θ = p rad, Fig. 1-21b. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the positive directions of r and u, respectively. Since the path is specified, the angle c, which the extended radial line r makes with the tangent, Fig. 1-21c, can be determined from Eq. 1-10. We have r = 0.1 θ , so that dr/dθ = 0.1, and therefore

$$\tan \psi = \frac{r}{dr/d\theta} = \frac{0.1\theta}{0.1} = \theta \text{ rad}$$

When θ = π, $\psi = \tan^{-1}\theta = \tan^{-1}\pi = 72.3^0$

, so that $\phi = 90 - \psi = 17.7^0$, as shown in Fig. 1–21c. Identify the four unknowns in Fig. 1-21b

Equations of Motion. Using ϕ= 17.7 and the data shown in Fig. 1–21b, we have

$$\left(\overset{\rightarrow}{+}\right) \quad \sum F_r = ma_r; \quad N_C \cos 17.7 = 0.5\, a_r \qquad 1$$

$(+\downarrow)\quad \sum F_\theta = ma_\theta;\ F_{C\,d} - N_C \sin 17.7 = 0.5a_\theta$ 2

Kinematics. The time derivatives of r and θ are

$\dot{\theta}= 4$ rad/s $\Rightarrow \ddot{\theta}= 0$, $r = 0.1\theta \Rightarrow \dot{r}= 0.1\dot{\theta} = 0.1(4)= 0.4 \Rightarrow \ddot{r}=0.1\,\ddot{\theta} = 0$

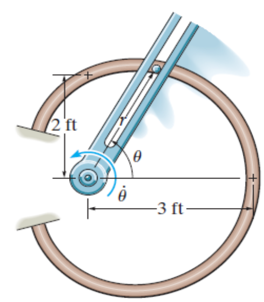
$a_r = \ddot{r} - r\,\dot{\theta}^2= = 0 - 0.1\pi(4)^2 = -5.03$ m/s^2 , $a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = 0 + 2(0.4)(4) = 3.20$ m/s^2

Substituting these results into Eqs. 1 and 2 and solving yields

$N_C = -2.64$ N, $F_C = 0.800$ N, What does the negative sign for N_C indicate?

PROBLEM.30: The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, r = (2 + cos θ) ft if θ = (0.5t^2) rad, where *t* is in seconds, determine the force which the rod exerts on the particle at the instant t= 1, The fork and path contact the particle on only one side.

SOLUTION



$r = 2 + \cos\theta \Rightarrow \dot{r}= -\sin\theta.\dot{\theta}$, $\theta= 0.5t^2 \Rightarrow \dot{\theta}=t \Rightarrow \ddot{\theta}= 1$

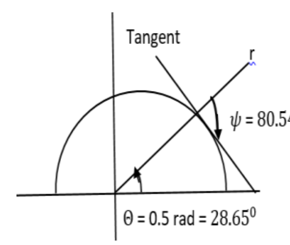
$\ddot{r}= -\sin\theta.\ddot{\theta}-\dot{\theta}\cos\theta$

At t=1, $\theta= 0.5(1)^2 =0.5$ rad and $\ddot{\theta}= 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r}= -\sin 0.5(1) = -0.4974$ ft/s^2

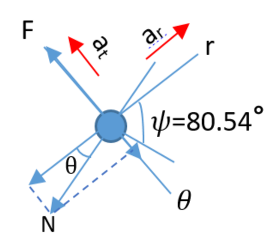
$\ddot{r}= -\cos 0.5(1)^2 - \sin 0.5.(1) = -1.357$ ft/s^2



$a_r = \ddot{r} - r\dot{\theta} = -1.375 - 2.8776(1)^2 = -4.2346$ ft/s^2

$a_\theta = r\ddot{\theta}+ 2\dot{r}\dot{\theta}= 2.8776(1) + 2(-0.4794)(1) = 1.9187$ ft/s^2

$\tan\psi= \dfrac{r}{dr/d\theta} = \dfrac{2+\cos(0.5)}{-\sin(0.5)} = -6.002$ rad $\Rightarrow \psi=tan^{-1}(-6.002) = 80.54^0$, θ= 90- ψ= 9.46^0



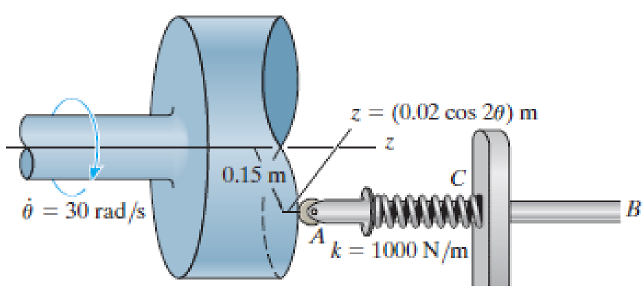
$(+\nearrow)\ \sum F_r=m\,a_r \Rightarrow -N\cos\theta=m.a_r \Rightarrow -N\cos(9.46^0) =(2/32.2)(-4.2346) \Rightarrow N =$ 0.2666 lb

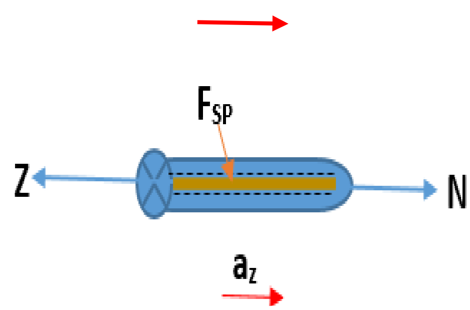
(+↖) $\sum F_\theta = m a_\theta \Rightarrow F - N\sin\theta = m.a_\theta \Rightarrow F - 0.2666 \sin(9.46^0) = (2/32.2)(1.9187)$, $F = 0.163$ lb

PROBLEM.31: The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where r = 0.15 m and z =(0.02 cos 2θ) m. If the cam is rotating at a constant rate of 30 rad/s, determine the force component Fz at the end A of the follower when θ = 30°. The spring is uncompressed when θ = 0°. Neglect friction at the bearing C.

Solution

Kinematics. Using the chain rule, the first and second time derivatives of z are





$Z = (0.02 \cos 2\theta)$ m $\Rightarrow \dot{Z} = 0.02[-\sin 2\theta.\ 2\dot{\theta} = -0.04 \sin 2\theta.\dot{\theta}$ m/s =

$\ddot{Z} = -0.04\,[\sin 2\theta.\ddot{\theta} + \dot{\theta}\,(2\cos 2\theta.\dot{\theta})] = -0.04\sin 2\theta.\ddot{\theta} - 0.08 \cos 2\theta.\dot{\theta}^2$ m/s^2

Here $\ddot{Z} = 30^0$, $\dot{\theta} = 30$ rad/s, $\ddot{\theta} = 0$

$\ddot{Z} = -0.08 \sin 2(30).(30)^2 = -36$ m/s^2, $a_Z = \ddot{Z} = -36$ m/s^2

$x = 0.02 \cos 2(30) + 0.02 \cos 2(0) = 0.03$

Equation of Motion.

$(\overset{\rightarrow}{+})$ $\sum F_Z = m a_Z \Rightarrow$ Referring to the FBD of the follower, Fig. *a*, $F_{SP} = Kx = 1000(0.03)$

$\Sigma F_Z = m a_Z$; $N - F_{SP} = m a_Z \Rightarrow N - 30 = 0.5(-36.0) \Rightarrow N = 12.0$ N

DYNAMICS

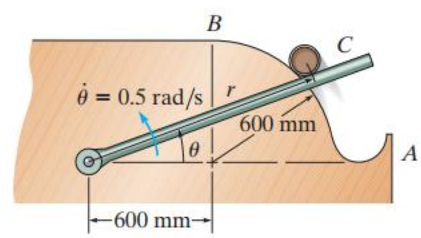
PROBLEM.32: A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot{\theta}$= 0.5 rad/s, determine the force which the rod exerts on the can at the instant θ = 30. Neglect the effects of friction in the calculation and the size of the can so that r = (1.2 cos θ) m. The ramp from A to B is circular, having a radius of 600 mm.

SOLUTION

$r = 2(0.6\cos\theta) = 1.2\cos\theta \Rightarrow \dot{r} = -1.2\sin\theta.\dot{\theta}$

$\ddot{r} = -1.2(\sin\theta.\ \ddot{\theta} + \cos\theta.\ \dot{\theta}^2)$

At $\theta = 30^0$, $\dot{\theta} = 0.5$ rad/s, $\ddot{\theta} = 0$

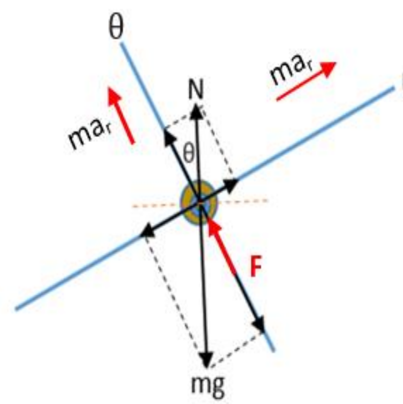


$r = 1.2\cos 30° = 1.0392$ m, $\dot{r} = -1.2\sin 30°(0.5) = -0.3$ m/s

$\ddot{r} = -1.2\cos 30°(0.5)^2 - 1.2\sin 30°(0) = -0.2598$ m/s^2

$a_r = \ddot{r} + r\,\dot{\theta}^2 = (-0.2598)^2 - 1.0392(0.5) = -0.5196$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} = 2(-0.3)(0.5) = -0.3$ m/s^2



(+↗) $\sum F_r = ma_r$; $N\sin\theta - mg\sin\theta = ma_r$

N SIN 30 -3(9.81) SIN 30 = 3(-0.5196)

N = 15.5 N

(+↖) $\sum F_\theta = m\,a_\theta \Rightarrow F - mg\cos\theta + N\cos\theta = ma_\theta$

F - 3(9.81)cos 30 + 15.5 cos 30 = 3(-0.3) ⇒F= 16.665 N

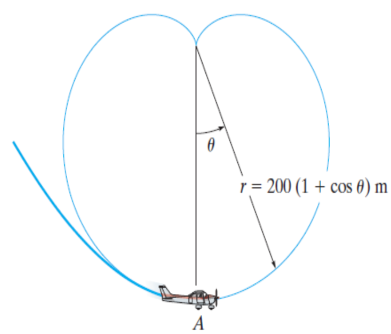
Differentiation of Trigonometric Function

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\sec x) = \sec x.\tan x$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\csc x) = -\csc x.\cot x$

DYNAMICS

PROBLEM.33: The pilot of the airplane executes a vertical loop which in part follows the path of a cardioid, $r = 200(1 + \cos\theta)$ m,where θ is in radians. If his speed at A is a constant, $v_p = 85$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He has a mass of 80 kg. Hint: to determine the time derivatives necessary to calculate the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 200(1 + \cos\theta)$. Then, for further information.

Solution



Kinematic. Using the chain rule, the first and second time derivatives of r are.

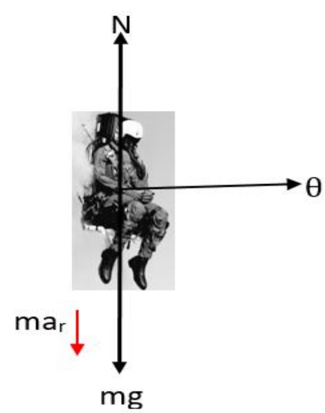
$r = 200(1 + \cos\theta) \Rightarrow \dot{r} = -200\sin\theta.\,\dot{\theta}$

$\ddot{r} = -200[\sin\theta.\,\ddot{\theta} + \dot{\theta}\,.\cos\theta.\,\dot{\theta} = -200\sin\theta.\,\ddot{\theta} - 200\cos\theta.\,\dot{\theta}^2$

When $\theta = 0°$; $r = 200(1 + \cos 0°) = 400$ m

$\dot{r} = -200(\sin 0°)\,\dot{\theta} = 0$

$\ddot{r} = -200[\sin 0°.\ddot{\theta} + 200\cos 0°.\,\dot{\theta}^2] = -200\,\dot{\theta}^2$



Using Eq. $V = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$

$V^2 = \dot{r}^2 + (r\dot{\theta})^2 \Rightarrow 85^2 = 0 + (400\dot{\theta})^2 \Rightarrow \dot{\theta} = 0.2125$ rad/s

Thus,

$a_r = \ddot{r} - r\dot{\theta}^2 - 200(0.21252) - 400(0.21252) = -27.09\ m/s^2$

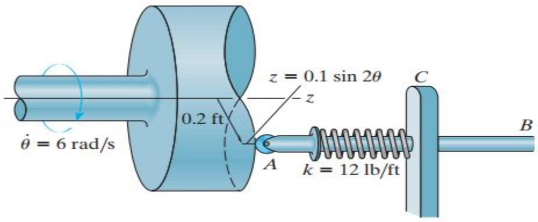
Equation of Motion. Referring to the FBD of the pilot, Fig. a,

$(+\downarrow)$ $\Sigma F_r = ma_r$; $80(9.81) - N = 80(-27.09) \Rightarrow N = 2952.3\ N = 2.95$ kN

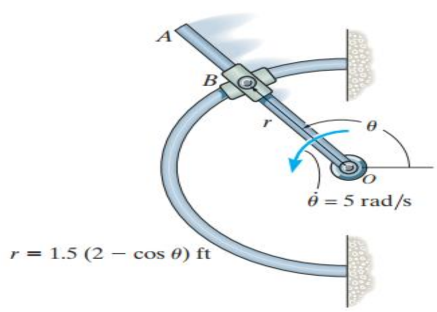
DYNAMICS

HOMEWORK: Do as required for solution of questions fellow

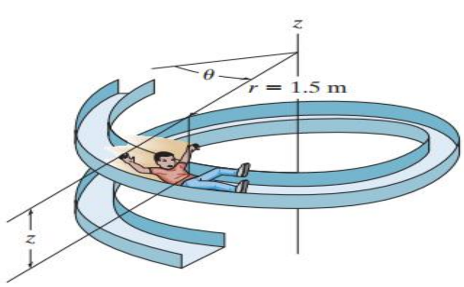
Q1. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where r = 0.2 ft and z = (0.1 sin 2θ) ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when θ = 45. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C



Q2. Rod OA rotates counterclockwise with a constant angular velocity of θ = 5 rad/s. The double collar B is pin connected together such that one collar slides over the rotating rod and the other slides over the horizontal curved rod, of which the shape is described by the equation r = 1.5(2 - cos θ) ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant θ = 120. Neglect friction.

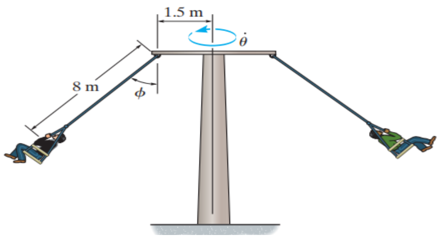


Q3. The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components r = 1.5 m, θ = (0.7t) rad, and z = (-0.5t) m, where t is in seconds. Determine the components of force **F**r, **F**u, and **F**z which the slide exerts on him at the instant t = 2 s. Neglect the size of the boy.

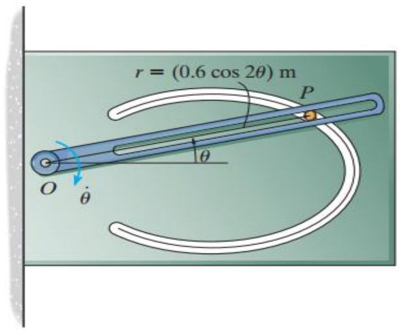


DYNAMICS

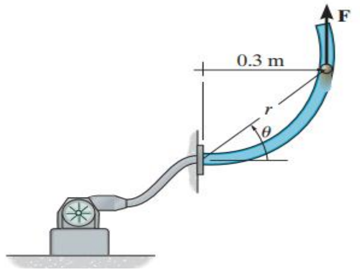
Q4. Determine the constant angular velocity $\dot{\theta}$ of the vertical shaft of the amusement ride if $\emptyset = 45$. Neglect the mass of the cables and the size of the passengers.



Q5. The 0.2-kg pin P is constrained to move in the smooth curved slot, which is defined by the lemniscate r = (0.6 cos 2θ) m. Its motion is controlled by the rotation of the slotted arm OA, which has a constant clockwise angular velocity of $\dot{\theta}$= -3 rad/s. Determine the force arm OA exerts on the pin P when θ = 0. Motion is in the vertical plane



Q6. The 0.2-kg ball is blown through the smooth vertical circular tube whose shape is defined by r=(0.6 sinθ)m, where u is in radians. If θ= (πt^2) rad, where t is in seconds, determine the magnitude of force **F** exerted by the blower on the ball when t= 0.5 s.



Q7. The 2-Mg car is traveling along the curved road described by r = ($50e^{2\theta}$) m, where θ is in radians. If a camera is located at A and it rotates with an angular velocity of $\dot{\theta}$ = 0.05 rad/s and an angular acceleration of $\ddot{\theta}$ = 0.01 rad/s^2 at the instant θ = π/6 rad, determine the resultant friction force developed between the tires and the road at this instant.